STABLE NUMERICAL SCHEMES FOR FRACTIONAL AND VARIABLE ORDER MATHEMATICAL MODELS

Thesis submitted in partial fulfillment for the

award of degree

DOCTOR OF PHILOSOPHY

By

Priyanka Rajput

DEPARTMENT OF MATHEMATICAL SCIENCES
INDIAN INSTITUTE OF TECHNOLOGY
(BANARAS HINDU UNIVERSITY)
VARANASI-221005, INDIA

Roll No. : 19121014

2025

CONCLUSION

Chapter 2 In this chapter, we have proposed a higher order numerical scheme for VO-TFRSDE by using L-123 approximation in time. The proposed scheme is tested on two numerical examples for various $\alpha(\mathrm{x}, \mathtt{t})$. The numerical results shown in Figures 2.1-2.4 and data presented in Tables 2.1-2.4 are observed to be highly accurate with higher order of convergence. The scheme reaches very high accuracy even on a very small number of grid points for some numerical examples. For the proposed numerical scheme, theoretical unconditional stability is shown. Several other variable-order time-fractional non-linear problems can be solved using the proposed numerical schemes, which is one of our objectives and a topic for further research.

Chapter 3 In this chapter, we have developed a higher order stable numerical approach for the variable-order time-fractional subdiffusion equation. For different values of the $\alpha(r, \mathtt{t})$, three numerical examples are used to validate the proposed scheme. The data reported in Table 3.1-3.9, as well as the numerical results shown in Figures 3.1-3.6, were found to be highly accurate with higher order of convergence. A comparison study with the present scheme is also provided in Table 3.3-3.5 and Table 3.9 to show the effectiveness and accuracy of the proposed scheme. For some numerical instances, this method provides very high precision even with a relatively small number of grid points. The theoretical unconditional stability of the proposed numerical scheme is also discussed. The scheme is applicable to a wide range of other variable-order temporal fractional nonlinear problems.

Chapter 4 In this chapter, we have presented a new difference scheme for multi-term time fractional electromagnetic wave model. The Caputo fractional derivatives of order α and β, where $1 < \beta < \alpha < 2$ have been approximated by L3 approximation, whereas spatial derivatives have been approximated by central difference scheme. The numerical scheme is validated on two numerical examples and the obtained results are highly accurate and of second order convergence with respect to space and time. The comparative study of the numerical results with [133] and [132] in Tables 4.1, 4.2, 4.5 and 4.6 prove the effectiveness of our scheme. The proposed approach can be extended for the nonlinear and higher dimensional time-fractional mathematical models.

In **chapter 5**,We have presented a new semi-discrete scheme for time fractional Black-Scholes model with a combination of finite difference and operational matrix approach. The Caputo derivative of order α, where $1 < \alpha < 2$, has been approximated by L-123 approximation, and the space derivatives have been approximated by operational matrix approach based on shifted Legendre polynomial. The algorithm is observed to be highly accurate and showing a higher order of convergence, $(4 - \alpha)$ in time for all alpha $0 < \alpha < 1$. The comparative study of the numerical results has shown that the proposed algorithm performs better in terms of error efficiency and convergence order. Therefore, this semi-discrete approach is found to be an effective numerical approach for solving other fractional and variable-order mathematical models.