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Name of the Student: Rakesh Ranjan Chand

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DEDICATION

To
my guru, *Pusparaj Dash*,
my baba, *Himanshu Sekhar Chand*, maa, *Jyotsna Rani Chand*,
and
Sushree Priyadarsini Mohanty

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LIST OF ABBREVIATIONS

AC	Alternating current
CMF	Calculated matched frequency
CV	Calculated frequency value
DC	Direct current
DSO	Digital storage oscilloscope
EM	Electro–magnetic
EMEH	Electro–magnetic energy harvester/harvesting
ES	Electro–static
ESH	Electro–static energy harvester/harvesting
FE	Finite element
FOSD	First–order shear deformation
FoM _v	Volume figure of merit
IC	Integrated circuit
LVPM	VPM calculated using linear formulation
MEMS	Micro–Electro–Mechanical System
MMS	Method of multiple scales
MOI	Moment of inertia
MS	Magneto–strictive
NPD	Normalized power density
NVPM	VPM calculated using nonlinear formulation
OC	Open–circuit
PVEH	Piezoelectric vibration energy harvester/harvesting
PD	Power density

PAF	Performance amplification factor
PMN-PT	Lead magnesium niobate–lead titanate
PPM	Power–per–total mass
PZN-PT	Lead zinc niobate–lead titanate
PZT	Lead zirconate titanate
RMS	Root-mean-square
RVEH	Rotational piezoelectric vibration energy harvester/harvesting
SDOF	Single degree–of–freedom
SF _{OMBW}	Figure of merit with bandwidth information
SRF	Simulated resonance frequency
TPMS	Tire pressure monitoring systems
VC	Voltage calculated using current formulations
VEH	Vibration energy harvester
VR	Voltage reported by original author
VPM	Voltage–per–total mass

LIST OF SYMBOLS

Even though all the primary symbols used in this thesis are defined in the text as they arise, a list of them is presented underneath for easy reference. The appropriate explanation of the symbol at its place of use is anticipated to eradicate the muddle.

$(a,0)$	Focus of the parabola
A	Cross-sectional area of the piezoelectric coupled beam
A_h	Cross-sectional area of the host beam at fixed end
A_p	Cross-sectional area of the piezoelectric patch at fixed end
$A_h(x)$	Cross-sectional area of the host beam at any standard section
$A_p(x)$	Cross-sectional area of the piezoelectric patch at any standard section
b_0	Width of the harvester at fixed end
$b(x)$	Width of the harvester at any standard section
$b(l)$	Width of the harvester at $x = l$
C_p	Capacitance of the piezoelectric patch
D_3	Electric displacement along Z -direction
e_{31}	Stress constant of the piezoelectric material
E_3	Electric field along Z -direction
E_h	Young's modulus of host material
E_p	Young's modulus of PZT-patch

f	Transverse exciting acceleration amplitude
g	Gravitational acceleration
$I_h(x)$	Area MOI of the host beam at any standard section
$I_p(x)$	Area MOI of the PZT–patch at any standard section
$J_p(x)$	Second moment of the cross-sectional area
K^e	Total kinetic energy of the system
l	Length of the piezoelectric–coupled beam
M	Total mass of the system
M_{LM}, M_L	Mass of the tip load
Q	Charge accumulated at the electrodes
r, r_{RH}	Radius of the mounting hub
r_0	Overall radius of rotation
R_L	Resistive load
s_1	Strain– displacement relation
S_1	Nonlinear unidirectional strain– displacement relation
$s_i(t)$	Spatial function of time
t_h	Thickness of the host beam
t_p	Thickness of the piezoelectric–patch
t_r	Thickness ratio (t_p/t_h)
T_1, σ_1	Stress in X –direction
U^e	Total potential energy of the system

$v(t)$	Voltage across the PZT–patch
V_h	Volume of the host beam
V_p	Volume of the host PZT–patch
$w(x,t)$	Transverse deflection of the tip
W_e	External workdone
$y(x)$	Equation of a parabola
Y_{11}^E	Elastic stiffness constants (Young’s modulus)
ϕ	Taper parameter
δW^e	Virtual workdone
ρ_h	Mass density of the host beam
ρ_p	Mass density of the PZT–patch
ξ	Bookkeeping parameter
σ	Detuning parameter
ω_d	Excitation frequency
ϵ_{33}^S	Dielectric permittivity constant
$\psi_i(x)$	Normalized admissible function

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