

ANALYTICAL STUDY OF CERTAIN NONLINEAR WAVE PROPAGATION PROBLEMS OF GAS-DYNAMICS



Thesis submitted in partial fulfillment for the
Award of Degree
Doctor of Philosophy

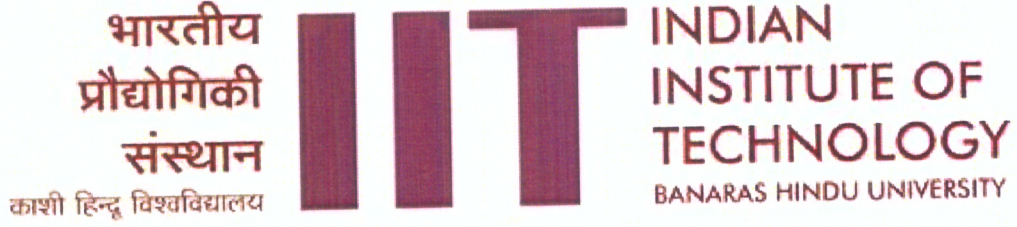
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I, **Pradeep**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. L. P. Singh** from **Jan, 2019** to **Sept, 2023** at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

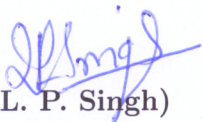
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*This thesis is dedicated
with love and deepest gratitude*

To my parents

Mr. Lautoo Prasad and Mrs. Vimala Devi

To my supervisor

Prof. L. P. Singh

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Pradeep
Pradeep

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