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my Parents and family, who instilled in me the virtue

of perseverance and commitment, and relentlessly encouraged me to strive for excellence…”

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It is certified that the work contained in the thesis titled "**STRUCTURAL RESPONSES OF FUNCTIONALLY GRADED COMPOSITE BEAMS REINFORCED WITH CARBON NANOTUBES**" by **Mr. Babar Abhijeet Babasaheb** has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

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Signature: Supervisor

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Associate Professor
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I, **Babar Abhijeet Babasaheb**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Rosalin Sahoo** from **January 2020 to August 2024** at the **Department of Civil Engineering, Indian Institute of Technology (BHU), Varanasi.** The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my knowledge.

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ACKNOWLEDGMENT

I started my Ph.D. just before the onset of the COVID-19 pandemic, a period that presented numerous challenges and uncertainties. Nonetheless, I made it to the finish line! I would like to express my deepest gratitude to the exceptional individuals who supported and guided me throughout this transformative journey, enabling me to successfully obtain my doctorate despite the many unforeseen obstacles that arose.

First and foremost, I thank the **almighty God** and **Baba Vishwanath** who showered his blessings, without which it would not have been possible for me to complete the dissertation in the present form.

त्वमेव माता च पिता त्वमेव ।

त्वमेव बंधुश्च सखा त्वमेव ।

त्वमेव विद्या द्रविणं त्वमेव ।

त्वमेव सर्वम् मम् देव देव ॥

Tvam-Eva Maataa Ca Pitaa Tvam-Eva |

Tvam-Eva Bandhush-Ca Sakhaa Tvam-Eva |

Tvam-Eva Viidyaa Dravinnam Tvam-Eva |

Tvam-Eva Sarvam Mama Deva Deva ||

It means 'Lord! You are my mother and father; relation and friend; knowledge and wealth; truly, my everything, my Lord!'

According to Vedas (Sacred Hindu Transcripts), the Guru is described as follows:

ॐ अज्ञान तिमिरान्धस्य ज्ञानाञ्जन शलाकया।

चक्षुरुन्मीलितं येन तस्मै श्री गुरवे नमः॥

OM Ajnyaana Timira Andhasya Jnyaana Aajana Shaalaakayaa |

Chakssur Unmilitam Yena Tasmai Shri Gurave Namah ||

it means

Salutations to the Guru who removes the darkness of ignorance from our (inner) eyes by applying the collyrium of the light of knowledge | By whom our (inner) eyes are opened; salutations to that Guru |

I would like to express my deep gratitude to my esteemed Ph.D. supervisor, **Dr. Rosalin Sahoo,** Associate Professor, Department of Civil Engineering, Indian Institute of Technology (BHU) Varanasi, for her invaluable guidance and encouragement throughout my Ph.D. journey. Under her mentorship, I gained deep insights and explored areas of study that I might not have ventured into otherwise. Her advice was always delivered with honesty and selflessness, rooted in my best interests, which I found truly humbling. She taught me the critical importance of attention to detail in numerical analysis and the interpretation of results, while also emphasizing the significance of individual contributions within a team. Her ability to offer timely guidance, often before I even recognized the need for it, is something I deeply admire and aspire to emulate. Her passion, strong work ethic, and positive outlook on both research and life have been a tremendous inspiration to me throughout my Ph.D. journey. I am especially thankful for her thorough review of my work and the trust she placed in me to navigate my own path. Her subtle yet powerful mentorship has helped me overcome numerous challenges, making her an exceptional teacher and mentor.

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In Sanskrit verse, the family is described as: "कुटुम्बं सर्वमिति" (Kuṭumbaṃ sarvaṃasti), which means "family is everything". I want to express my deepest gratitude to my family, particularly my parents, **Mr. Babasaheb Tukaram Babar** and **Mrs. Sunita Babar,** for their boundless love and unwavering support. This dissertation and all my efforts are a testament to their commitment to being the best parents anyone could ask for. Thank you for your encouraging words and for supporting me as I took what may have seemed an unconventional path. Despite the limited time I've spent with them since leaving home for my studies, they have never complained. I am especially grateful to my mother, who has been my constant source of inspiration and strength, always believing in me even during my weakest moments. I also want to acknowledge my sister, **Mrs. Madhuri Patil,** for shouldering her own responsibilities while also taking on mine to support our parents in my absence. Her continuous support, patience,

and positive outlook have been a tremendous help. No matter the circumstances, she has always stood by me and cared for our family with unwavering dedication. Lastly, I am profoundly thankful for my entire family, especially **Aai, Tatya, Mavashi, Mama, Mami, Snehal, Tejal, Isha, Nisha,** and **Jyotsna**. Their encouraging voices over the phone have played a crucial role throughout this journey.

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To everyone whose names may not be mentioned here, please accept my heartfelt thanks for your support and prayers. Your encouragement has been a tremendous help throughout this journey.

Babar Abhijeet

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LIST OF NOTATIONS

Symbol	Definition
x, z	Cartesian coordinate system
U, W	2D displacements in the global x and z-direction
u_0, w_0	Mid-plane displacement components in the x and z-direction
θ_x	Shear displacements of the beam at the mid-plane
$\{\sigma\}$	Stress vector in the global coordinate system
$\{\varepsilon\}$	Strain vector in the global coordinate system
$[Q]$	Transformed reduced stiffness matrix
$\{\bar{\sigma}\}$	Stress vector in the material coordinate axis
$\{\bar{\varepsilon}\}$	Strain vector in the material coordinate axis
E_{11}, E_{22}	Young's Modulus in the longitudinal and transverse direction to the CNTs direction
G_{12}	In-plane shear modulus
$G_{13}, , G_{23}$	Transverse shear modulus
β_w	Winkler stiffness of the foundation
β_s	Shear stiffness of the foundation
δU	Strain energy
δU_b	The virtual Strain energy of the carbon nanotube reinforced composite beam
δU_f	The virtual potential energy of the elastic foundation
δV	The total potential energy
δK	Kinetic energy of the carbon nanotube reinforced composite beam
E^m	Young's modulus of matrix
G^m	Shear modulus of matrix
ν	Poisson's ratio carbon nanotube reinforced composite beam
ν^{cnt}	Poisson's ratio of carbon nanotubes
ν^p	Poisson's ratio of polymer matrix
V^m	The volume fraction of the matrix
V_p	The volume fraction Polymer matrix.

E^p	Young's Modulus of Polymer matrix.
G^p	Modulus of rigidity of Polymer matrix.
V_{cnt}	The volume fraction of carbon nanotubes
υ_{12}^{cnt}	Poisson's ratio of carbon nanotubes
V_{cnt}^*	volume fraction of carbon nanotubes added
E_{11}^{cnt}	Young's modulus of carbon nanotube in the longitudinal direction
E_{22}^{cnt}	Young's modulus of carbon nanotube in the lateral direction
G_{12}^{cnt}	Shear modulus of carbon nanotube
W_{cnt}	Mass fraction of carbon nanotube
ρ^p	Density of matrix
P^{cnt}	Density of carbon nanotube
ρ	Density of carbon nanotube reinforced composite beam
η_1, η_2, η_3	Efficiency parameter
Ω	Ohm a constant
N_{xx}	In-plane stress resultants
M_{xx}	Moment stress resultants
Q_1,	Transverse shear stress resultants
h	Overall thickness of the carbon nanotube reinforced composite beam
L	Overall length of the carbon nanotube reinforced composite beam
q	Mechanical pressure
Q_m	The magnitude of the load
$\overline{I}_0$, $\overline{I}_1$, $\overline{I}_2$, $\overline{I}_3$,$\overline{I}_4$, $\overline{I}_5$,	Inertia components
[A], [B], [D], [E], [F], [H], [R], [L],	Rigidity submatrices relating the stress-resultants and derivatives of the primary variables
$\{\Delta_i\}$	Displacement vector
$\{q_i\}$	Mechanical force vector
ω	The natural frequency of the beam
[M]	Mass matrix
[K]	Global stiffness matrix of the carbon nanotube reinforced composite beam

$[G]$	The geometric stiffness matrix.
σ_{xx}	The normal stress in the x-direction
τ_{xz}	The shear stress acting in the x-direction on a plane whose normal is in the z-direction
ε_{xx}	The normal strain in the *x*-direction
γ_{xz}	The shear strain in the xz-plane
R_{EF}	The Pasternak's elastic foundation reaction
f(z)	Shape function
Nxx	Axially compressive force
U_{mx}, W_{mx}, X_{mx}	The arbitrary parameters.
$\bar{u}$	The Non-dimensional axial displacement
$\bar{\sigma}_{xx}$	The dimensionless axial stress
$\bar{w}$	The dimensionless transverse displacement
$\bar{N}_{x0}$	The nondimensional critical buckling load
$\bar{\omega}$	The dimensionless natural frequency

LIST OF ABBREVIATIONS

3D	Three Dimensional
2D	Two Dimensional
CNTs	Carbon nanotubes
SWCNTs	Single walled carbon nanotubes
MWCNTs	Multi-walled Carbon nanotubes
CNTR	Carbon nanotube reinforced
CNTRC	Carbon nanotube reinforced composite
FG-CNTRC	Functionally graded carbon nanotube reinforced composite
GPLRC	Graphene nanoplatelet-reinforced composite
CBT	Classical Beam Theory
CLBT	Classical Laminate Beam Theory
FSDT	First Order Shear Deformation Theory
HSDT	Higher-Order Shear Deformation Theory
IHSDT	Inverse Hyperbolic Shear Deformation Theory
TSDT	Trigonometric Shear Deformation Theory
SNDT	Shear and Normal Deformation Theory
TBT	Third-Order Beam Theory
ESDT	Extended Shear Deformation Theory
FRP	Fiber Reinforced Polymers
FGM	Functionally Graded Material
ESL	Equivalent Single Layer
LW	Layer wise
ZZ	Zigzag
CVD	Chemical vapor deposition
HiPCO	High-pressure carbon monoxide
PECVD	Plasma enhanced chemical vapour deposition method
MPECVD	Microwave plasma chemical vapour deposition method
RF-CVD	Radiofrequency chemical vapour deposition method

HFCVD	Hot-filament chemical vapour deposition method
FCCVD	Floating catalyst chemical vapour deposition method
PMMA	Poly methyl methacrylate
PmVA	{(mphenylenevinylene)-co-[(2, 5-dioctaxy-p-phenylene) viny-lene]}
PVC	Poly vinyl chloride
PHSDT	Polynomial Higher-Order Shear Deformation Theory
NPHSDT	Non-Polynomial Higher-Order Shear Deformation Theory
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
ROM	Rule of Mixture
MD	Molecular Dynamics
FEM	Finite Element Method
RVE	Representative volume element
XFEM	Extended Finite Element Method
FDM	Finite Difference Method
NSGT	Non-Local Strain Gradient Theory
DSC	Discrete Singular Convolution
DQM	Differential Quadrature Method
EE	Equilibrium Equations
RHZZT	Refined Higher-Order Zigzag Theory
RFSDT	Refined First Order Shear Deformation Theory
PSDT	Polynomial Shear Deformation Theory
UDL	Uniformly Distributed Load
SSL	Sinusoidal Load
UD	Uniformly Distributed

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It is certified that the work contained in the thesis titled "**STRUCTURAL RESPONSES OF FUNCTIONALLY GRADED COMPOSITE BEAMS REINFORCED WITH CARBON NANOTUBES**" by **Mr. Babar Abhijeet Babasaheb** has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

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DECLARATION BY THE CANDIDATE

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Date: 23/08/2024

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It is certified that the above statement made by the student is correct to the best of my knowledge.

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