Chapter 7

Prescribed-time adaptive neural consensus of uncertain nonlinear multi-agent systems

7.1 Introduction

Prescribed-time consensus is a critical research problem in various practical applications where achieving tasks within specific time frames is essential. However, limited research has been conducted on prescribed-time consensus, with most existing work focusing on systems with known or available dynamics in advance [82–86]. Consensus in uncertain nonlinear multi-agent systems presents greater challenges, despite its wide practical applications. The predominant methods for handling unknown terms in nonlinear models involve approximating them using neural networks (NNs), assuming the function's domain is within a compact set [87].

In this chapter, we propose an adaptive prescribed-time consensus control method for uncertain nonlinear MASs using Radial Basis Function (RBF) neural networks to approximate unknown nonlinearities. The method ensures stability through graph theory and Lyapunov stability theory, offering the advantage of achieving consensus within a predetermined time frame. Our approach has practical applications in robotics, communication networks, and power systems, addressing the need for effective control strategies in real-world multi-agent systems.

The contributions of this chapter are:

(i) Unlike the previous studies reported in [82–85] and the references therein, this chapter achieves consensus control for uncertain nonlinear MASs with a prescribed convergence time, which is a novel contribution.

(ii) RBF NNs are employed to approximate the unknown nonlinearities in the system, enabling the control method to adapt to changing system conditions and uncertainties.

(iii) The boundedness of all closed-loop signals is ensured through an extensive analysis using the Lyapunov direct method.

7.2 Preliminaries and problem formulation

7.2.1 Graph theory

Consider a weighted undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, \cdots, v_n\}$ represents the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the edge set, and $\mathcal{A} = [a_{ij}]_{n\times n}$ represents the adjacency matrix. In this graph, if $(v_j, v_i) \in \mathcal{E}$ and $(v_i, v_j) \in \mathcal{E}$, then the agent v_i can receive information from the agent v_j, and vice versa. Additionally, if $(v_j, v_i) \in \mathcal{E}$, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$. The in-degree of node v_i is defined as the sum of the elements in the i-th row of the adjacency matrix, denoted as $d_i = \sum_{j=1}^{n} a_{ij}$. The Laplacian matrix $\mathcal{L}$ is defined as $\mathcal{L} = \mathcal{B} - \mathcal{A}$, where $\mathcal{B} = \text{diag}\{d_1, \cdots, d_n\}$. A graph $\mathcal{G}$ is considered connected if there exists a path from any two distinct nodes $i, j \in \mathcal{V}$. An undirected path is a sequence of edges in $\mathcal{G}$ of the form $(i, i_1), (i_1, i_2), \cdots, (i_k, j) \in \mathcal{E}$, where the nodes $i_l; l = 1, \cdots, k$ are distinct.

Definition 7.1 *[17] The system* (2.1) *is said to be input-to-state stable in prescribed-time t_p if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function μ, for all $t \in [t_0, t_0 + t_p)$, such that*

$$|z(t)| \leq \beta\left(|z(t_0)|, \psi(t, t_0)\right) + \mu\left(\|d\|_{t_0,t}\right), \tag{7.1}$$

where $\psi(t, t_0)$ is the prescribed-time adjustment function and satisfies the following conditions:

1. $\psi(t, t_0) = \int_{t_0}^{t} \psi_1(\tau, t_0)d\tau, \quad \psi_1(t, t_0) \geq 1$

2. $\lim_{t\to t_0+t_p} \psi(t, t_0) = +\infty$.

Please note that the above conditions for $\psi(t, t_0)$ are satisfied by a wide range of time-varying functions $\psi_1(t, t_0)$. For instance, we take $\psi_1(t, t_0) = (\frac{t_p}{t_p+t_0-t})^{q^*}$, with $q^* \geq 2$ being an integer, then $\psi(t, t_0) = \frac{t_p}{q^*-1}\left[(\frac{t_p}{t_p+t_0-t})^{q^*-1} - 1\right]$.

Lemma 7.2 *[114] If there exists a radially unbounded positive differentiable function $V : \mathbb{R}^n \to \mathbb{R}_{>0}$ with positive constants C^*, K_1^* and K_2^* such that the inequality*

$$\dot{V}(t) \leq -2C^*\psi_1(t, t_0)V(t) + \psi_1(t, t_0)K_1^* + K_2^* \tag{7.2}$$

holds, then we can conclude that

$$V(t) \leq V(t_0)e^{\frac{-2C^* t_p}{q^*-1}\left(\left(\frac{t_p}{t_p+t_0-t}\right)^{q^*-1}-1\right)} + \frac{K^*}{2C^*} \tag{7.3}$$

where $K^* = K_1^* + K_2^*$.

Lemma 7.3 *[70] For an undirected connected graph* $\mathcal{G}$*, the* n *eigenvalues of the symmetric Laplacian matrix* $\mathcal{L}$ *are real and can be arranged in ascending order as* $0 = \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots \leq \lambda_n$*. In particular,* λ_2 *is commonly known as the algebraic connectedness of an undirected connected graph, and it provides insights into the convergence rate of consensus.*

Lemma 7.4 *[70] For any* $z_1, z_2 \in \mathbb{R}$*, one has* $z_1 z_2 \leq \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$.

The authors in [82–86] successfully achieved consensus within a prescribed finite time. However, achieving precise consensus in multi-agent systems (MASs) can pose challenges in practical scenarios due to disturbances and unknown dynamics. On the other hand, if the system's nonlinearities are completely unknown, the consensus protocol designed may exhibit performance degradation or even instability. Hence, a viable criterion is to ensure that the consensus error of the MASs remains uniformly ultimately bounded. In other words, the consensus error of each agent should converge to near zero within a preselected time, with a small residual error permissible.

Lemma 7.5 *[117] For any continuous function* $\mathcal{W}(z) : \mathbb{R}^l \to \mathbb{R}^m$ *defined on a compact set* $\Omega_z \subset \mathbb{R}^l$*, there exists an RBF NNs* $\mathcal{W}_{nn}(z) = \phi^\top S(z)$ *such that* $\mathcal{W}(z) = \phi^\top S(z) + \sigma(z)$*, where* $z \in \Omega_z \subset \mathbb{R}^l$ *is the input vector for the NNs,* $\sigma(z)$ *represents an approximation error satisfying* $\|\sigma(z)\| \leq \sigma$*,* $\phi \in \mathbb{R}^{p\times m}$ *is the ideal weight matrix, and* $S(z) = [S_1(z), S_2(z), \cdots, S_p(z)]^\top$ *is a vector of basis functions. The basis functions are defined as* $S_i(z) = exp\left[\frac{-(z-r_i)^\top(z-r_i)}{\bar{\mu}_i^2}\right]$*, where* $r_i = [r_{i1}, r_{i2}, \cdots, r_{il}]^\top$ *is the center and* $\bar{\mu}_i$ *is the width of the Gaussian function. The ideal weight matrix* ϕ *is determined as* $\phi = \mathrm{argmin}_{\phi\in\mathbb{R}^{p\times m}}\{\sup_{z\in\Omega_z}\|\mathcal{W}(z) - \phi^\top S(z)\|\}$.

7.3 Adaptive neural prescribed-time consensus control design

In this section, a prescribed-time adaptive NNs-based consensus control method is proposed to achieve the prescribed-time consensus for uncertain nonlinear MASs with unknown nonlinearities. Schematic block diagram of the proposed adaptive NN-based prescribed-time consensus control for uncertain nonlinear multi-agent systems. is illustrated in Figure 7.1.

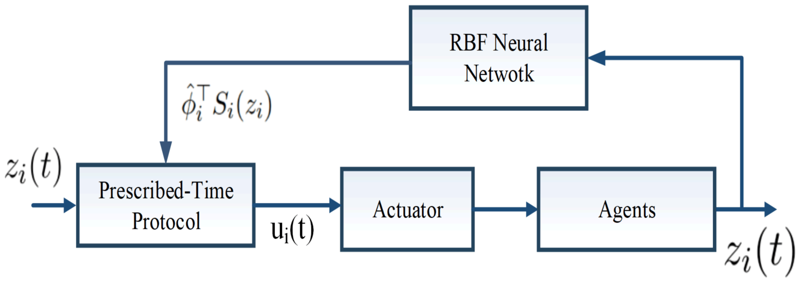


Figure 7.1: Schematic block diagram of the proposed adaptive NN-based prescribed-time consensus control for uncertain nonlinear multi-agent systems.

Let us consider MASs consisting of n agents, where the dynamics of the i-th agent ($i = 1, 2, \ldots, n$) are described as follows:

$$\dot{z}_i(t) = f_i\left(z_i(t)\right) + u_i(t), \quad z_i(t_0) = z_{i0} \tag{7.4}$$

where $z_i(t) \in \mathbb{R}^m$ and $u_i(t) \in \mathbb{R}^m$ represents the state and control input for the i-th agent respectively. The function $f_i(z_i(t))$ is a continuous unknown nonlinear function associated with the i-th agent.

Assumption 7 *The MASs* (7.4) *have an undirected, connected communication topology.*

Assumption 8 *The MASs* (7.4) *are input-to-state stable.*

The main objective here is to design a consensus protocol $u_i(t)$ that achieves consensus among the agents' states within a prescribed time, with a small residual error, i.e. $\lim_{t\to t_p} |z_i(t)-z_j(t)| \leq \sigma, \forall i, j \in \mathcal{V}$.

Now, the consensus error of the i-th agent can be defined as follows:

$$e_i(t) = \sum_{j=1}^{n} a_{ij}(z_i(t) - z_j(t)), \quad i = 1, ..., n \tag{7.5}$$

where a_{ij} represents the edge weight in the adjacency matrix corresponding to the communication topology.

In order to facilitate the analysis, the consensus error can be expressed in its concise form as

$$\Xi(t) = \mathcal{L}Z(t) \tag{7.6}$$

where $\Xi(t) = [e_1(t), e_2(t), ..., e_n(t)]^\top$ and $Z(t) = [z_1(t), z_2(t), ..., z_n(t)]^\top$. Now, we will utilize RBF NNs to approximate the unknown nonlinear dynamics in the MASs (7.4). Let us assume that $\mathcal{W}_i(z_i) = f_i(z_i(t))$. Given that $\mathcal{W}_i(z_i)$ is a continuous function, according to Lemma 7.5, we can utilize RBF NNs to approximate it with a random precision σ_i over a compact set $\Omega_{z_i} \subset \mathbb{R}^m$. The approximation is defined as $\mathcal{W}_i(z_i) = \phi_i^\top S_i(z_i) + \sigma_i(z_i)$, where $\phi_i \in \mathbb{R}^{p_i \times m}$ represents the true weight matrix of the RBF NNs.

Now, we propose the following prescribed-time NNs consensus protocol for the MASs (7.4)

$$u_i(t) = -\frac{1}{2}e_i(t) - \gamma_{1i}\psi_1(t, t_0)e_i(t) - \hat{\phi}_i^\top S_i(z_i) \tag{7.7}$$

with the weight updating law for $\hat{\phi}_i$ as

$$\dot{\hat{\phi}}_i = S_i(z_i)e_i^\top(t) - \gamma_{2i}\psi_1(t, t_0)\hat{\phi}_i \tag{7.8}$$

where $S_i(z_i)$ is the basis function vector, γ_{1i} and γ_{2i} are positive design constants. The estimation error in NNs weights is given by $\tilde{\phi}_i = \hat{\phi}_i - \phi_i$, where $\hat{\phi}_i$ is the estimate of the true weight ϕ_i.

In the present stage, we provide the following theorem to summarize the main results for the considered MASs (7.4) under the proposed consensus protocol.

Theorem 7.6 *Considering the nonlinear MASs described by equation (7.4) under assumptions 7-8, and assuming that $f_i(z_i)$ satisfies Lemma 7.5, we can draw the following conclusions by applying the proposed consensus protocol (7.7) with the weight updating law (7.8): 1) all the agents in the MASs will reach a common agreement within the prescribed time t_p, with a small residual error. 2) all the closed-loop signals of the MASs (7.4) are guaranteed to be bounded for all future time.*

Proof. Consider the Lyapunov candidate as

$$V(t) = V_1(t) + V_2(t) \tag{7.9}$$

where $V_1(t) = \frac{1}{2}Z^\top(t)\mathcal{L}Z(t)$ and $V_2(t) = \frac{1}{2}\sum_{i=1}^{n} \mathrm{Tr}\big(\tilde{\phi}_i^\top \tilde{\phi}_i\big)$.

First, we will demonstrate the positive definiteness of $V_1(t)$ in relation to the consensus error $\Xi(t)$. Assumption 7 and Lemma 7.3 lead to the conclusion that $\mathcal{L}$ is a symmetric matrix with $(n-1)$ positive eigenvalues and one zero eigenvalues. According to matrix theory, we can deduce the existence of an orthogonal matrix $\mathcal{Q}$ that satisfies

$$\mathcal{L} = \mathcal{Q}\Lambda\mathcal{Q}^\top \tag{7.10}$$

where $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ and $\lambda_i, i = 1, \cdots, n$ are the eigenvalues of $\mathcal{L}$ arranged in ascending order. Next, we can express $V_1(t)$ as

$$\begin{aligned} V_1(t) =& \frac{1}{2}Z^\top(t)\mathcal{Q}\Lambda\mathcal{Q}^\top Z(t) = \frac{1}{2}Z^\top(t)\mathcal{Q}\sqrt{\Lambda}\sqrt{\Lambda}\mathcal{Q}^\top Z(t) \\ =& \frac{1}{2}Z^\top(t)\mathcal{Q}\sqrt{\Lambda}\sqrt{\hat{\Lambda}}\sqrt{\hat{\Lambda}^{-1}}\sqrt{\hat{\Lambda}^{-1}}\sqrt{\hat{\Lambda}}\sqrt{\Lambda}\mathcal{Q}^\top Z(t) \\ =& \frac{1}{2}Z^\top(t)\mathcal{Q}\Lambda\mathcal{Q}^\top\mathcal{Q}\hat{\Lambda}^{-1}\mathcal{Q}^\top\mathcal{Q}\Lambda\mathcal{Q}^\top Z(t) \\ =& \frac{1}{2}Z^\top(t)\mathcal{L}^\top\mathcal{Q}\hat{\Lambda}^{-1}\mathcal{Q}^\top\mathcal{L}Z(t) = \frac{1}{2}\Xi^\top(t)\mathcal{D}\Xi(t) \end{aligned} \tag{7.11}$$

where $\hat{\Lambda} = \mathrm{diag}\{\rho, \lambda_2, \lambda_3, \cdots, \lambda_n\}$ with ρ being a positive constant between λ_2 and λ_n. The matrix $\mathcal{D} = \mathcal{Q}\hat{\Lambda}^{-1}\mathcal{Q}^\top$ represents a positive definite matrix. Hence, one can conclude that $V_1(t)$ is positive definite in relation to the consensus error $\Xi(t)$.

Taking the time derivative of $V(t)$ along the solutions of (7.4) for the interval $t \in [t_0, t_0+t_p)$ yields:

$$\begin{aligned} \dot{V}(t) =& \dot{V}_1(t) + \sum_{i=1}^{n}\mathrm{Tr}\big(\tilde{\phi}_i^\top\dot{\hat{\phi}}_i\big) \\ =& \sum_{i=1}^{n} e_i^\top(t)\dot{z}_i(t) + \sum_{i=1}^{n}\mathrm{Tr}\big(\tilde{\phi}_i^\top\dot{\hat{\phi}}_i\big) \\ =& \sum_{i=1}^{n} e_i^\top(t)\big(f_i(z_i) + u_i(t)\big) + \sum_{i=1}^{n}\mathrm{Tr}\big(\tilde{\phi}_i^\top\dot{\hat{\phi}}_i\big) \end{aligned} \tag{7.12}$$

Let us assume that the compact set Ω_{z_i} is large enough to include the working states of the MASs. Then, inserting (7.7) and (7.8) into (7.12) results in:

$$\begin{aligned} \dot{V}(t) =& \sum_{i=1}^{n} e_i^\top(t)\bigg(\phi_i^\top S_i(z_i) + \sigma_i(z_i) - \frac{1}{2}e_i(t) \\ & - \gamma_{1i}\psi_1(t,t_0)e_i(t) - \hat{\phi}_i^\top S_i(z_i)\bigg) \\ & + \sum_{i=1}^{n}\mathrm{Tr}\left(\tilde{\phi}_i^\top\left[S_i(z_i)e_i^\top(t) - \gamma_{2i}\psi_1(t,t_0)\hat{\phi}_i\right]\right) \end{aligned} \tag{7.13}$$

Further simplifying (7.13), we obtain

$$\begin{aligned} \dot{V}(t) =& \sum_{i=1}^{n}\bigg(-\frac{1}{2}e_i^\top(t)e_i(t) - \gamma_{1i}\psi_1(t,t_0)e_i^\top(t)e_i(t) \\ & + e_i^\top(t)\sigma_i(z_i)\bigg) - \sum_{i=1}^{n}\gamma_{2i}\psi_1(t,t_0)\mathrm{Tr}\left(\tilde{\phi}_i^\top\hat{\phi}_i\right) \end{aligned} \tag{7.14}$$

Utilizing Lemma 7.4, we can write

$$e_i^\top(t)\sigma_i(z_i) \leq \frac{1}{2}e_i^2(t) + \frac{1}{2}\sigma_i^2(z_i) \tag{7.15}$$

$$-\mathrm{Tr}\left(\tilde{\phi}_i^\top\hat{\phi}_i\right) \leq -\frac{1}{2}\mathrm{Tr}\left(\tilde{\phi}_i^\top\tilde{\phi}_i - \phi_i^\top\phi_i\right) \tag{7.16}$$

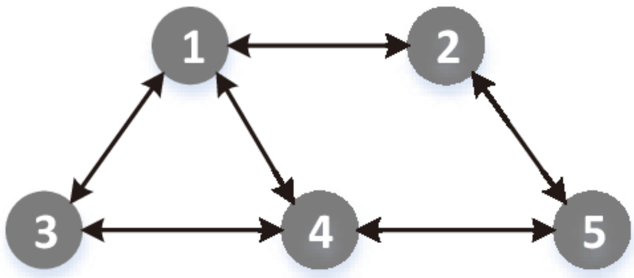


Figure 7.2: Communication topology.

By combining (7.15) and (7.16) with (7.14), results in

$$\begin{aligned}\dot{V}(t) \leq &\sum_{i=1}^{n} \Big[-\frac{1}{2}e_i^2(t) - \gamma_{1i}\psi_1(t,t_0)e_i^\top(t)e_i(t) + \frac{1}{2}e_i^2(t) \\ &+ \frac{1}{2}\sigma_i^2(z_i) - \frac{\gamma_{2i}}{2}\psi_1(t,t_0)\mathrm{Tr}\left(\tilde{\phi}_i^\top\tilde{\phi}_i - \phi_i^\top\phi_i\right)\Big] \end{aligned} \tag{7.17}$$

Further simplifying (7.17), we obtain

$$\begin{aligned}\dot{V}(t) \leq &-\mu_1 \sum_{i=1}^{n} \psi_1(t,t_0)e_i^\top(t)e_i(t) \\ &-\frac{\mu_2}{2}\sum_{i=1}^{n}\psi_1(t,t_0)\mathrm{Tr}\left(\tilde{\phi}_i^\top\tilde{\phi}_i\right) \\ &+\sum_{i=1}^{n}\left[\frac{1}{2}\sigma_i^2(z_i) + \frac{\gamma_{2i}}{2}\psi_1(t,t_0)\mathrm{Tr}\left(\phi_i^\top\phi_i\right)\right] \end{aligned} \tag{7.18}$$

where $\mu_1 = \min\{\gamma_{1i} : i = 1, 2, \ldots, n\}$, and $\mu_2 = \min\{\gamma_{2i} : i = 1, 2, \ldots, n\}$.

From (7.11), we have

$$\sum_{i=1}^{n} e_i^\top(t)e_i(t) = \Xi^\top(t)\Xi(t) \geq \frac{2}{\lambda_{max}(\mathcal{D})}V_1(t) \tag{7.19}$$

Then, it follows that

$$\begin{aligned}\dot{V}(t) \leq &-\frac{2\mu_1}{\lambda_{max}(D)}\psi_1(t,t_0)V_1(t) - \mu_2\psi_1(t,t_0)V_2(t) \\ &+\sum_{i=1}^{n}\left[\frac{1}{2}\sigma_i^2(z_i) + \frac{\gamma_{2i}}{2}\psi_1(t,t_0)\mathrm{Tr}\left(\phi_i^\top\phi_i\right)\right] \end{aligned} \tag{7.20}$$

Considering (7.9), (7.20) can be written as

$$\dot{V}(t) \leq -2C^*\psi_1(t,t_0)V(t) + \psi_1(t,t_0)K_1^* + K_2^* \tag{7.21}$$

where $C^* = \min\{\frac{\mu_1}{\lambda_{max}(D)}, \frac{\mu_2}{2}\}$, $K_1^* = \sum_{i=1}^{n}\frac{\gamma_{2i}}{2}\mathrm{Tr}\left(\phi_i^\top\phi_i\right)$ and $K_2^* = \sum_{i=1}^{n}\frac{1}{2}\sigma_i^2(z_i)$.

Furthermore, utilizing Lemma 7.2, we obtain

$$V(t) \leq V(t_0) e^{\frac{-2C^* t_p}{q^*-1}\left(\left(\frac{t_p}{t_p+t_0-t}\right)^{q^*-1}-1\right)} + \frac{K^*}{2C^*} \tag{7.22}$$

where $K^* = K_1^* + K_2^*$. Consequently, from Lemma 7.2 and (7.22), one can guarantee that $e_i(t)$ and $\tilde{\phi}_i$ are bounded for all $t \in [t_0, t_0 + t_p)$. As a result, $\lim_{t \to t_0 + t_p} e_i^2(t) \leq \frac{K^*}{2C^*}$, for all $t \geq t_p$ which further implies that prescribed-time consensus is achieved with a small residual error $\frac{K^*}{2C^*}$ within the a priori chosen time t_p. This completes the proof. ■

7.4 Simulation examples

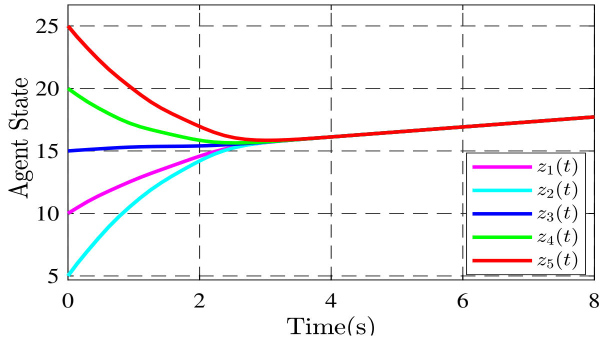
In this section, we provide an example to support the derived theoretical findings and demonstrate their applicability. We consider the nonlinear Kuramoto model of five coupled oscillators, where each oscillator's dynamics is given by

$$\begin{aligned} \dot{z}_i(t) &= w_i + \frac{P}{5}\sum_{j=1}^{5} \sin\big(z_j(t) - z_i(t)\big) + u_i(t) \\ y_i(t) &= z_i(t) \end{aligned} \tag{7.23}$$

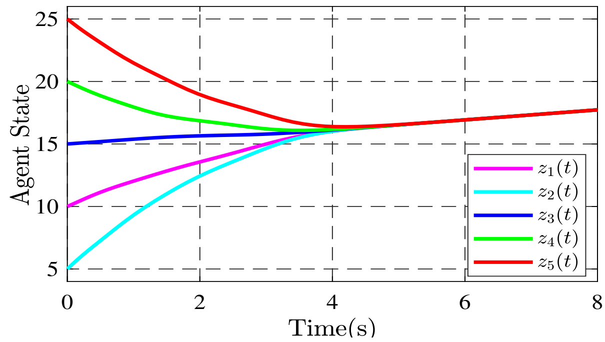
for $i = 1, 2, \cdots, 5$. Here, $z_i(t) \in \mathbb{R}$ represents the phase of the i-th oscillator, $P \in \mathbb{R}$ is the coupling gain, w_i denotes the natural frequency, $u_i(t)$ is the control input for the i-th agent, and $y_i(t)$ represents the model output of the i-th agent.

The simulation results were obtained by applying the proposed prescribed-time consensus law (7.7) with the adaptation law (7.8). The communication topology is illustrated in Figure 7.2. The initial conditions of the agents are set as follows: $z_1(t_0) = 10$, $z_2(t_0) = 5$, $z_3(t_0) = 15$, $z_4(t_0) = 20$, and $z_5(t_0) = 25$. The system parameter values are chosen as $w_i = 0.4$ and $P = 1$. The desired consensus time is selected as $t_p = 4s$ and $t_p = 5s$. The design parameters are assigned values of $\gamma_{1i} = 1.5$ and $\gamma_{2i} = 3$. Additionally, the RBF NNs $\phi_i^\top S_i(z_i)$ $(i = 1, 2, 3, 4, 5)$ consist of 100 nodes, with the centers r_i evenly spaced within the range [-1,1], and the widths $\bar{\mu}$ set to 2.

Figure 7.3-7.6 presents the results obtained using the proposed Adaptive NNs based prescribed-time consensus protocol with the selected prescribed times of $t_p = 4s$ and $t_p = 5s$. Figure 7.3(a) displays the evolution of the agents' state trajectories for $t_p = 4s$. The corresponding control effort and consensus error are illustrated in Figure 7.4(a) and 7.5(a), respectively. It can be observed from the results that both the control effort and consensus error remain bounded for all time, indicating the effectiveness of the proposed protocol.

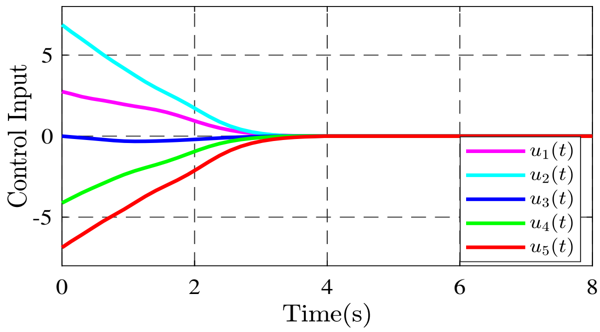


(a)

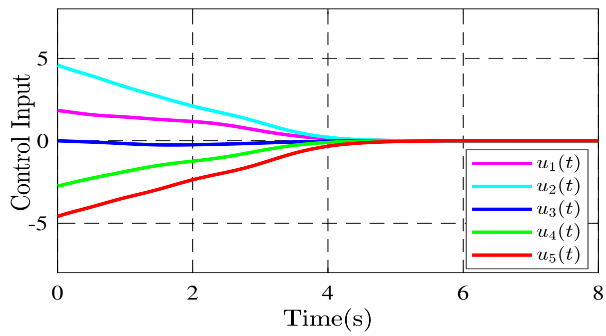


(b)

Figure 7.3: State evolution under protocol (7.7). (a) under $t_p = 4s$. (b) under $t_p = 5s$.

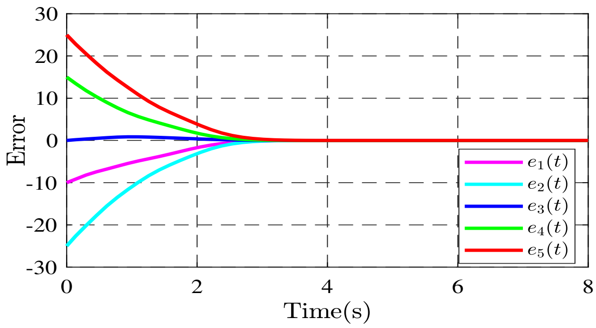


(a)

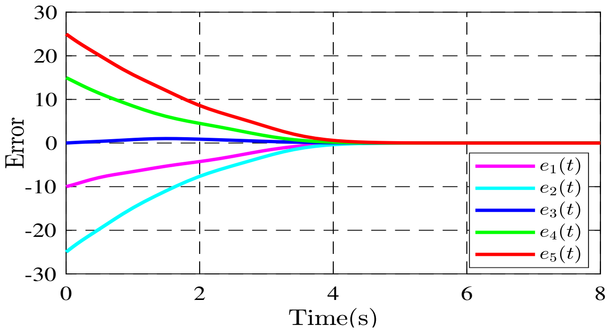


(b)

Figure 7.4: Control input $u_i(t)$. (a) under $t_p = 4s$. (b) under $t_p = 5s$.

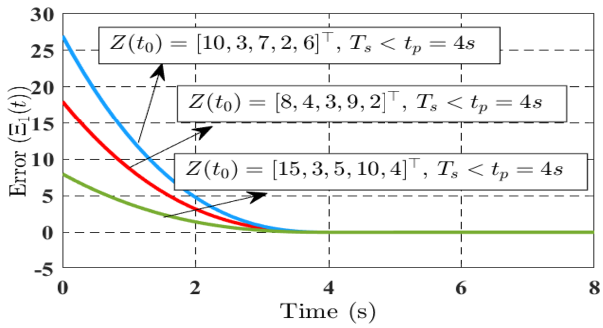


(a)

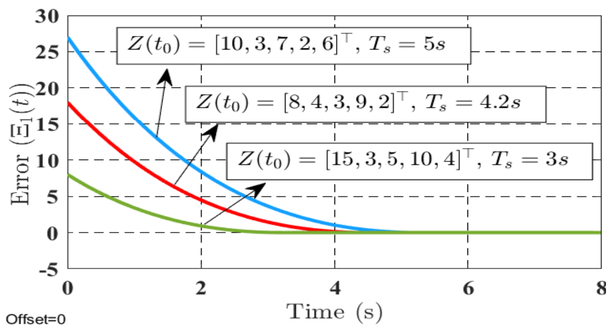


(b)

Figure 7.5: Consensus error $e_i(t)$. (a) under $t_p = 4s$. (b) under $t_p = 5s$.



(a)



(b)

Figure 7.6: Performance comparison under different initial conditions. (a) $\Xi_1(t)$ under consensus law (7.7)-(7.8). (b) $\Xi_1(t)$ under consensus law in [77].

Similarly, simulation results are obtained using the same initial conditions and control parameters for the prescribed time of $t_p = 5s$. Figure 7.3(b) demonstrates the consensus control of the state trajectories of the agents within the prescribed time. Furthermore, Figure 7.4(b) and 7.5(c) depict the curves of the control effort and consensus error, respectively, for the $t_p = 5s$ case.

In addition, the performance comparison between the proposed consensus law and the one developed by [77] involves testing three different initial conditions. The results, depicted in Figure 7.6, reveal that the consensus achieved under (7.7)-(7.8) is independent of the initial states $Z(t_0)$ and other design parameters. Conversely, the convergence time of the consensus law in [77] varies with different initial parameters.

Remark 7.7 *Please note that the proposed consensus scheme remains valid until the designated time* t_p*. A switched-gain strategy is employed to ensure the continuity of consensus beyond the specified time.*

Remark 7.8 *The simulation results reveal an important observation regarding the choice of the prescribed-time* t_p*. When a smaller value of* t_p *is selected, it leads to larger control inputs for the agents during the transient stage, as evident from Figure 7.4. This behavior is expected as a shorter prescribed time requires faster convergence, necessitating more significant control actions. However, it is crucial to consider practical limitations on the control inputs of the agents.*

7.5 Conclusion

In this chapter, we have discussed an adaptive NN-based prescribed-time consensus problem for uncertain nonlinear multi-agent systems. Our approach involved the use of RBF NNs to approximate the unknown nonlinearities in the multi-agent system. Through extensive theoretical analysis, we have proven that our proposed scheme guarantees consensus among the agents within a prescribed time, which is a significant feature of our consensus control scheme. Additionally, we have shown that all signals in the closed-loop system remain bounded for all future time. In the presented example, we have applied our proposed protocol to a nonlinear Kuramoto model of coupled phase oscillators, demonstrating its effectiveness.

For future research, our findings can be extended to more complex network topologies and practical scenarios that involve communication interferences, delays, and actuator faults.