

**AMPLITUDE COEFFICIENTS FOR APERTURE ELECTRIC FIELD OF
THE WATER-LOADED CONVENTIONAL AND IMPROVED METAL
DIAGONAL HORN**

A.1 Introduction

Amplitude coefficient for a wave-mode in a waveguide/horn is a non-negative scalar measure of the amplitude of a wave in the waveguide/horn during one wave cycle for that mode. This appendix contains the amplitude coefficients for aperture electric field of water-loaded conventional metal diagonal horns and improved metal diagonal horns used in chapters 2 and 3 respectively.

A.2 Amplitude coefficients for aperture electric field of the water-loaded conventional metal diagonal horn

The x- and y-components of electric field at the aperture of a water-loaded conventional metal diagonal horn antenna [Love (1976)] are represented by

$$E_{x_1}(x, y, 0) = a_{10} \cos\left(\frac{\pi y}{d}\right) \quad (\text{A.1})$$

$$E_{y_1}(x, y, 0) = a_{01} \cos\left(\frac{\pi x}{d}\right) \quad (\text{A.2})$$

where a_{10} and a_{01} are amplitude coefficients *i.e.* normalizing constants for TE₁₀ and TE₀₁ modes respectively.

The input power P_{in} can be written as the superposition of power levels for TE₁₀-mode (P_{10}) and TE₀₁-mode (P_{01}) *i.e.*

$$P_{10} + P_{01} = P_{in} \quad (\text{A.3})$$

Normalizing P_{10} and P_{01} with respect to the input power P_{in} , results in equation A.4

$$\frac{P_{10}}{P_{in}} + \frac{P_{01}}{P_{in}} = 1 \quad (\text{A.4})$$

Normalized power levels for TE₁₀- and TE₀₁-modes are given by

$$\frac{P_{10}}{P_{in}} = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} |E_{x_1}|^2 dx dy \quad (\text{A.5})$$

$$\frac{P_{10}}{P_{in}} = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} a_{10}^2 \cos^2\left(\frac{\pi y}{d}\right) dx dy \quad (\text{A.6})$$

$$\frac{P_{01}}{P_{in}} = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} |E_{y1}|^2 dx dy \quad (\text{A.7})$$

$$\frac{P_{01}}{P_{in}} = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} a_{01}^2 \cos^2\left(\frac{\pi x}{d}\right) dx dy \quad (\text{A.8})$$

Therefore, equation A.4 can be written as

$$\int_{-d/2}^{d/2} \int_{-d/2}^{d/2} a_{10}^2 \cos^2\left(\frac{\pi y}{d}\right) dx dy + \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} a_{01}^2 \cos^2\left(\frac{\pi x}{d}\right) dx dy = 1 \quad (\text{A.9})$$

Equation A.9 was solved to obtain

$$a_{10} = a_{01} = 1/d$$

since power is divided equally between the two modes [Love (1976)].

A.3 Amplitude coefficients for aperture electric field of the water-loaded improved metal diagonal horn

The x- and y-components of electric field at the aperture of a water-loaded improved metal diagonal horn antenna [Silver (1976)] are represented by

$$E_{x1}(x, y, 0) = a_{10} \cos\left(\frac{\pi y}{d}\right) \exp(-j\beta_{10}d_2) + a_{30} \cos\left(\frac{3\pi y}{d}\right) \exp(-j\beta_{30}d_2) \quad (\text{A.10})$$

$$E_{y1}(x, y, 0) = a_{01} \cos\left(\frac{\pi x}{d}\right) \exp(-j\beta_{01}d_2) + a_{03} \cos\left(\frac{3\pi x}{d}\right) \exp(-j\beta_{03}d_2) \quad (\text{A.11})$$

where a_{10} , a_{01} , a_{30} , a_{03} are amplitude coefficients and β_{10} , β_{01} , β_{30} , β_{03} are phase constants for TE₁₀, TE₀₁, TE₃₀ and TE₀₃ modes respectively.

The input power P_{in} can be written as the sum of the powers (= Input power/2) carried by TE₁₀-mode (P_{10}) and TE₃₀-mode (P_{30}) is equal to sum of powers (= Input power/2) carried by TE₀₁-mode (P_{01}) and TE₀₃-mode (P_{03}) *i.e.*

$$P_{10} + P_{30} = \frac{P_{in}}{2} \quad (\text{A.12})$$

$$P_{01} + P_{03} = \frac{P_{in}}{2} \quad (\text{A.13})$$

Normalizing P_{10} , P_{30} , P_{01} , and P_{03} with respect to the input power P_{in} , we get

$$\frac{P_{10}}{P_{in}} + \frac{P_{30}}{P_{in}} + \frac{P_{01}}{P_{in}} + \frac{P_{03}}{P_{in}} = 1 \quad (\text{A.14})$$

Normalized power levels for TE₁₀, TE₃₀, TE₀₁, and TE₀₃ modes are given by

$$\frac{P_{10}}{P_{in}} + \frac{P_{30}}{P_{in}} = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} |E_{x1}|^2 dx dy \quad (\text{A.15})$$

$$\frac{P_{10}}{P_{in}} + \frac{P_{30}}{P_{in}} = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} a_{10}^2 \cos^2\left(\frac{\pi y}{d}\right) + a_{30}^2 \cos^2\left(\frac{3\pi y}{d}\right) dx dy \quad (\text{A.16})$$

$$\frac{P_{01}}{P_{in}} + \frac{P_{03}}{P_{in}} = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} |E_{y1}|^2 dx dy \quad (\text{A.17})$$

$$\frac{P_{01}}{P_{in}} + \frac{P_{03}}{P_{in}} = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} a_{01}^2 \cos^2\left(\frac{\pi x}{d}\right) + a_{03}^2 \cos^2\left(\frac{3\pi x}{d}\right) dx dy \quad (\text{A.18})$$

Therefore equation A.14 can be written as

$$\begin{aligned} & \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} a_{10}^2 \cos^2\left(\frac{\pi y}{d}\right) + a_{30}^2 \cos^2\left(\frac{3\pi y}{d}\right) dx dy \\ & + \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} a_{01}^2 \cos^2\left(\frac{\pi x}{d}\right) + a_{03}^2 \cos^2\left(\frac{3\pi x}{d}\right) dx dy = 1 \end{aligned} \quad (\text{A.19})$$

Equation A.19 was solved to obtain

$$a_{10} = a_{01} = 1/d\sqrt{1.81} \text{ and}$$

$$a_{30} = a_{03} = 0.9 \times a_{10} \text{ (Obtained through optimization of field distributions)}$$

REFLECTION COEFFICIENT AT THE INTERFACE BETWEEN WATER-LOADED CONVENTIONAL/IMPROVED METAL DIAGONAL HORN AND BIO-MEDIUM**B.1 Introduction**

Reflection at any interface (separating the two media) is the result of impedance mismatch, which occurs due to change in physical properties of two media. Amount of reflection depends on the dielectric properties of the media. Mathematically, it is calculated using the reflection coefficient parameter. Reflection coefficient determines either the amplitude or the intensity of the reflected wave with respect to the incident wave. In this Appendix, the formula for reflection coefficient at the interface between water-loaded conventional/improved metal diagonal horn and bio-medium is given which is used in chapters 2 and 3.

B.2 Reflection coefficient at the interface between water-loaded conventional/improved metal diagonal horn and bio-medium

Since water-loaded conventional metal diagonal horn (MDH) supports TE_{10} and TE_{01} modes, the reflection coefficient at the interface between the water-loaded conventional MDH antenna (operating at a specific ISM frequency) and a bio-medium can be written as:

$$\Gamma_{10} = \Gamma_{01} = \frac{Z_A - Z_0^{10}}{Z_A + Z_0^{10}} \quad (\text{B.1})$$

where $Z_A (=1/Y_A)$ is the aperture impedance of the water-loaded conventional MDH antennas and Z_0^{10} is the wave impedance of the water-loaded conventional MDH antenna for TE_{10} mode.

However water-loaded improved metal diagonal horn supports TE_{10} , TE_{01} , TE_{30} , and TE_{03} modes. Hence, the reflection coefficient at the interface between the water-loaded improved MDH antenna (operating at a specific ISM frequency) and a bio-medium can be written as [Rizzi (1988)]:

$$\Gamma_{10} = \Gamma_{01} = \frac{Z_A - Z_0^{10}}{Z_A + Z_0^{10}} \quad (\text{B.2})$$

$$\Gamma_{30} = \Gamma_{03} = \frac{Z_A - Z_0^{30}}{Z_A + Z_0^{30}} \quad (\text{B.3})$$

where $Z_A (= 1/Y_A)$ is the aperture impedance of the water-loaded improved MDH antennas, and Z_0^{10} and Z_0^{30} are the wave impedances of the water-loaded improved metal diagonal horn antenna for TE₁₀ and TE₃₀ modes respectively.

B.3 Aperture impedance of water-loaded conventional/improved metal diagonal horn terminated in bio-medium

The aperture admittance of a water-loaded conventional/improved diagonal horn antenna terminated in a phantom muscle is given by [Compton (1964)]

$$Y_A = \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} \left\{ \bar{E}_{x1}(x, y, 0) \cdot \bar{H}_{y1}^*(x, y, 0) - \bar{E}_{y1}(x, y, 0) \cdot \bar{H}_{x1}^*(x, y, 0) \right\} dx dy \quad (\text{B.4})$$

where $\bar{E}_{x1}(x, y, 0)$, $\bar{E}_{y1}(x, y, 0)$ and $\bar{H}_{x1}^*(x, y, 0)$, $\bar{H}_{y1}^*(x, y, 0)$ are the x and y components of electric and magnetic fields at the horn aperture ($z = 0$) respectively, which can be obtained by using equations (2.5), (2.6), (2.11) and (2.12) for water-loaded conventional/improved MDH.

$$Y_A = \frac{1}{2\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[-jk_{z1} I_{\psi_1} + jk_{z1} R_{\psi_1} \right] \left\{ \frac{1}{j\omega\mu_0} \left\{ [(k_1^2 - k_y^2)(I_{\psi_1} + R_{\psi_1})] - k_x k_y (I_{\phi_1} + R_{\phi_1}) \right\} \right\}^* \right. \\ \left. - \left[jk_{z1} I_{\phi_1} - jk_{z1} R_{\phi_1} \right] \left\{ \frac{1}{j\omega\mu_0} \left\{ [(k_1^2 - k_x^2)(I_{\phi_1} + R_{\phi_1})] - k_x k_y (I_{\psi_1} + R_{\psi_1}) \right\} \right\}^* \right\} \quad (\text{B.5})$$

For obtaining aperture admittance of water-loaded conventional MDH, the values of R_{ψ_1} , R_{ϕ_1} , I_{ϕ_1} and I_{ψ_1} can be obtained by using equations (2.17), (2.18) and (2.30). And for obtaining the aperture admittance of water-loaded improved MDH, the values of R_{ψ_1} , R_{ϕ_1} , I_{ϕ_1} and I_{ψ_1} can be obtained by using the expressions given in equations (3.11) and (3.12) and (2.30).

Thus, aperture impedance of water-loaded conventional/improved MDH terminated in a bio-medium is given by

$$Z_A = \frac{1}{Y_A} \quad (\text{B.6})$$

It is to be noted that value of Z_A for water-loaded conventional MDH is different from that of water-loaded improved MDH.

B.4 Wave impedance of water-loaded conventional/improved metal diagonal horn terminated in a bio-medium

The wave impedances of water-loaded conventional and improved MDH terminated in a bio-medium (TE₁₀ and TE₀₁ modes in conventional MDH) and (TE₁₀, TE₀₁, TE₃₀, and TE₀₃ modes in improved MDH) may be computed from the relations [Rizzi (1988)]:

$$Z_o^{10} = Z_o^{01} = 120\pi \sqrt{\frac{\mu'_{rw}}{\epsilon'_{rw}}} \cdot \frac{\lambda_{g\epsilon}^{10}}{\lambda_{rw}} \quad (\text{B.7})$$

$$Z_o^{30} = Z_o^{03} = 120\pi \sqrt{\frac{\mu'_{rw}}{\epsilon'_{rw}}} \cdot \frac{\lambda_{g\epsilon}^{30}}{\lambda_w} \quad (\text{B.8})$$

where ϵ'_{rw} and μ'_{rw} are the real part of relative permittivity and permeability of water filling the conventional/improved MDH. $\lambda_{g\epsilon}^{10}$ is the guide wavelength for TE₁₀-mode in aperture section of water-loaded conventional and improved MDH. $\lambda_{g\epsilon}^{30}$ is the guide wavelength for TE₃₀-mode in aperture section of water-loaded improved MDH. The expressions for $\lambda_{g\epsilon}^{10}$ and $\lambda_{g\epsilon}^{30}$ are given as [Rizzi (1988)]

$$\lambda_{g\epsilon}^{10} = \frac{\lambda_w}{\sqrt{1 - \left(\frac{f_{c10}}{f}\right)^2}} \quad (\text{B.9})$$

$$\lambda_{g\epsilon}^{30} = \frac{\lambda_w}{\sqrt{1 - \left(\frac{f_{c30}}{f}\right)^2}} \quad (\text{B.10})$$

λ_w is the wavelength in water and can found from the relation [Rizzi (1988)]

$$\lambda_w = \frac{\lambda}{\sqrt{\epsilon'_{rw}\mu'_{rw}}} = \frac{c/f}{\sqrt{\epsilon'_{rw}\mu'_{rw}}} = \frac{c}{f\sqrt{\epsilon'_{rw}\mu'_{rw}}} \quad (\text{B.11})$$

$\mu'_{rw}(= 1)$ is the real part of relative permeability of water, c is the velocity of microwave in free space = 3×10^8 m/s, λ is wavelength in free space, f is the design frequency, and f_{c10} and f_{c30} are the cut off frequencies for TE₁₀- and TE₃₀-modes respectively. f_{c10} and f_{c30} can be computed from the relations:

$$f_{c10} = c/2d\sqrt{\epsilon'_{rw}} \quad (\text{B.12})$$

$$f_{e_{30}} = 3c/2d\sqrt{\epsilon'_{rw}} \quad (\text{B.13})$$

where d is the aperture size of the applicator.

PREPARATION AND MEASUREMENT OF DIELECTRIC PROPERTY OF BIOLOGICAL PHANTOM

C.1 Biological phantom preparation

Different types of phantom human muscle tissue have been reported in the literature [Stuchly *et al.* (1980)]. In this work, a jelly based biological phantom was used in the study at the frequency of 2450 MHz. The material compositions of the planar biological phantom are 30% gelatin + 69% water + 1% NaCl. For preparing a planar biological phantom of size 112 mm × 112 mm × 80 mm, 759 ml of water, 11 g of NaCl and 330 g of gelatin were taken.

Following procedure was adopted for preparation of biological phantom: Distilled water was heated so that it becomes warm and stirred in a container. After this specified amount of NaCl was added into 759 ml of distilled water and stirred in the container. Once the salt is dissolved in water, 330 g of gelatin was added and stirred until jelly like structure is formed. Prepared biological phantom is shown in Figure C.1.

C.2 Measurement of dielectric property of biological phantom

The complex relative permittivity of the prepared biological phantom was measured at 2450 MHz with the help of Keysight make E5071C network analyser and 85070E dielectric probe kit. The experimental determination of complex permittivity is based on the measurement of reflection coefficient S_{11} for the biological phantom. The dielectric probe kit was connected to the E5071C network analyser through a coaxial cable. The experimental setup for measuring the complex permittivity of the phantom material is shown in Figure C.1.

Before the measurement can be performed, the system was calibrated by connecting three known impedance standards (open, short and water) to the system and to obtain systematic internal errors. Then coaxial probe was placed on the sample (biological phantom) and the value of permittivity of the phantom was read from the display of network analyser.

The measured complex relative permittivity of the phantom is found to be $\sim 52 - j18$ + measurement errors. The measurement errors would be the sum of the errors due to probe model accuracy (typically about 2% to 5%), air, and water. The measured value is nearly in agreement with that available in the literature [Stuchly *et al.* (1980)].



Figure C.1: Experimental setup for measuring the permittivity of prepared biological phantom.

MEASUREMENT OF DIELECTRIC PROPERTY OF PERSPEX MATERIAL

The dielectric constant of the Perspex material used in the fabrication of the metal-dielectric wall diagonal horn (chapter 4) was measured using two techniques described in the reference [Altschuler (1963)]. Following techniques were used for measuring the dielectric constant of Perspex dielectric at 2450 MHz:

- (i) Two point method involving solution of transcendental equation, and
- (ii) Method involving two reactive terminations

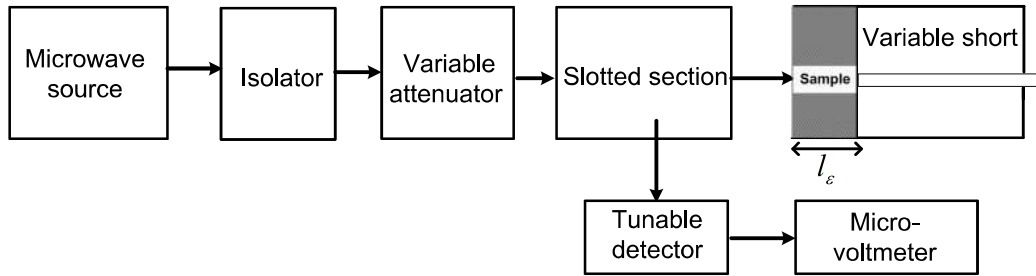


Figure D.1: Set-up for measuring dielectric constant of Perspex sheet.

(i) Two point method

Two point method is used for the measurement of dielectric constant of lossless dielectrics or dielectrics with medium loss by following the procedure given in reference [Altschuler (1963)]. The experimental setup for measurement of dielectric constant of the Perspex material is shown in Figure D.1. The measurement procedure for determination of dielectric constant is given in the reference [Altschuler (1963)].

The transcendental equation which is used for determination of dielectric constant experimentally is given below

$$\frac{\tan k(D_R - D + l_\epsilon)}{kl_\epsilon} = \frac{\tan k_\epsilon l_\epsilon}{k_\epsilon l_\epsilon} = \frac{\tan X}{X} \quad (D.1)$$

where $k = \frac{2\pi}{\lambda_g}$, $k_\varepsilon = \frac{2\pi}{\lambda} \sqrt{\varepsilon_r - \left(\frac{\lambda}{\lambda_c}\right)^2} = \frac{2\pi}{\lambda_{g\varepsilon}}$, l_ε ($= 6.33$ mm) is the thickness of dielectric sample, λ is the free space wavelength, λ_c is the cut-off wavelength for dominant TE₁₀ mode in the rectangular waveguide, $a = (72.136$ mm) is the broad dimension of rectangular waveguide, λ_g is the guide wavelength in the empty waveguide which can be measured by measuring the distance between alternate minima in the slotted section, D_R is the reference minimum position in the slotted line with short circuit load and D is the position of minimum in the slotted line with back face of the sample touching the short circuit.

By solving the transcendental equation D.1 for X , where $X = k_\varepsilon l_\varepsilon$, the value of ε_r of the Perspex material is computed.

The measured value of dielectric constant is found to be 2.85.

(ii) Method involving two reactive terminations

The dielectric constant of a sample can also be measured very simply when two different terminations are available. The advantage of this method is that the solution of ε_r does not involve transcendental equation. Measurement details are provided in reference [Altschuler (1963)]. In the present method, the dielectric constant is derived from the admittance determinant Y_ε .

$$\varepsilon_r = \frac{v_\varepsilon + \left(\frac{\lambda_g}{2a}\right)^2}{1 + \left(\frac{\lambda_g}{2a}\right)^2} \quad (\text{D.2})$$

$$Y_\varepsilon = Y_{oc} Y_{sc} \quad (\text{D.3})$$

Y_{oc} and Y_{sc} are the input admittances normalized to Y_0 (characteristic admittance) when output face of the sample are respectively open- and short-circuited. Normalized input admittances Y_{oc} and Y_{sc} , which are inverse of their corresponding normalized impedances were determined experimentally through standard slotted line method, which requires the measurement of

VSWR and position of first minimum from unknown impedance to obtain the value of unknown normalized admittance.

The computed value of dielectric constant is found to be 2.77.

Hence, the average value of the measured dielectric constant of Perspex material obtained through the two aforesaid methods is 2.81.