Chapter 1

Introduction

One of the primary goals of modern cosmology is to study the evolution of the Universe from the very beginning to the present era. In the recent past, various cosmological models to explain different stages in the evolution of the Universe have been presented (Peebles, 1993). The current technological developments have enabled us to observe the state of the Universe in the distant past. Shreds of evidence provided by the measurements of the Cosmic Microwave Background radiation (CMBR) - the relic radiation believed to be coming from the infant Universe, sheds light on the formation of the Universe and its dominant constituents: ordinary (baryonic) matter, dark matter, and dark energy (Hinshaw et al., 2013; Planck Collaboration et al., 2016). The CMB maps infer the density fluctuations in the very early Universe, which, under the influence of gravitational pull and cosmic expansion, later evolved to form various spatial structures in the Universe (Springel et al., 2005). Our understanding of the astrophysics of galaxies and other structures formed due to matter density fluctuations in the local Universe has also improved significantly with the help of the detailed study of the nature and spatial distribution of these structures (Cole et al., 2005). Although the Big Bang Cosmology, supported mainly by the CMB experiments and other observations, is the most successful theory about the evolution of the Universe, a detailed spatiotemporal map of the Universe as a whole is required, which can help us understand the many unanswered questions and constrain the parameters of various cosmological models.

1.1 Structure Formation and Evolution of the Universe

According to the standard model of cosmology or the ΛCDM model, the Universe was in an extremely hot and dense state in the very beginning. The matter, coupled with radiation, was in the form of sub-atomic particles and ions, forming the cosmic soup. With the evolution of time, the Universe expanded adiabatically and cooled. As the temperature dropped sufficiently, the radiation decoupled from matter. The radiation that became free to travel in the Universe is known today as Cosmic Microwave Background Radiation (CMBR). The ions (electrons and protons) combined together, around a redshift of $z \sim 1100$, and form neutral Hydrogen (Peebles, 1993) (henceforth H I). This process is known as Recombination, and the era is called the Epoch of Recombination. During this epoch, the Universe was filled with mostly neutral Hydrogen, with tiny fluctuations in the densities. From the recombination era afterward, the Universe remained utterly dark as there were no luminous sources. This period in the evolution of the Universe is known as the "Dark-Ages". The temperature power spectrum of the CMBR measures the statistical properties of the tiny fluctuations in matter density at the epoch of Recombination. Following the recombination era, small fluctuations in the dark matter grew through gravitational instability. Eventually, the baryonic matter density rose to the critical point, and the first luminous objects formed around a redshift $z \sim 30$. This cosmological evolution era is called the cosmic dawn (CD). The X-ray photons and the UV- radiations from the first sources heated and started ionizing the neutral Hydrogen in the surrounding medium. As time progressed, the bubbles of ionized Hydrogen increasingly overlapped, resulting in an entirely ionized universe. This process of phase transition of the Universe from a neutral state to an almost entirely ionized state is known as Reionization, and the epoch is hence termed as the Epoch of Reionization (EoR). Observation of quasar absorption spectra (Becker et al., 2001; Fan et al., 2003, 2006), the optical depth for Thomson scattering from CMB anisotropy (Hinshaw et al., 2013; Komatsu et al., 2011; Page et al., 2007; Planck Collaboration et al., 2016), intergalactic medium (henceforth IGM) temperature measurements (Bolton et al., 2010; Theuns et al., 2002), etc., suggests the EoR lasted around a redshift range of 15–6 for the EoR. At the end of this epoch, almost all the Hydrogen gas in the Universe became ionized. In the post-reionization Universe, only a tiny fraction of neutral Hydrogen is left, which can be found today in shelf-shielded dense, compact objects, like galaxies, etc. Despite its pivotal role, the EoR is one of the least understood epochs in the Universe's evolution. Many theoretical efforts guided by limited observational evidence are dedicated to understanding the process involved in this epoch and its evolution. Studying the Dark Ages, CD, and EoR through observations is the key to understanding many fundamental questions in cosmology like large-scale structure formation, the nature of the first luminous object, the first galaxies, their properties, and radiation mechanism, etc.

1.2 H I 21- cm signal as a Probe of EoR

The formation of structures, the nature of the first luminous objects, and the physics of Reionization can be traced by studying the evolution of the brightness temperature of H I (Furlanetto et al., 2006; Madau et al., 1997; Pritchard and Loeb, 2012; Shaver et al., 1999; Zaroubi, 2013). As neutral Hydrogen was the most abundant element of the Universe during the CD, EoR, and post-EoR era, studying the evolution of neutral Hydrogen (H I) through cosmic time can reveal the crucial mysteries of the Universe. Fortunately, we have a very unique and exciting probe to study the Hydrogen gas distribution in the Universe and the early structure formation from the Dark Ages, when ions first decoupled from Cosmic Microwave Background Radiation and recombined to form Hydrogen through

CD, EoR to present epoch, called the H I 21- cm signal. The H I 21- cm signal originates from the hyperfine spin-flip transition between the two levels because of the spin-spin interaction of the electron and proton of the Hydrogen atom. This radiation resulting from the line transition of the H I is peaked at a frequency of $\sim$ 1420 MHz or equivalently at the $\sim$ 21- cm wavelength, hence getting its famous name "21- cm signal". Observations of the redshifted 21- cm signal provide a powerful tool for learning about the first stars and galaxies and the reionization process and can also help us constrain the properties of the IGM. In addition to that, they will also provide information about active galactic nuclei (henceforth AGN), such as quasars, by observing the ionized bubbles surrounding individual AGNs.

The evolution of H I , and hence the matter density, can be studied by observing the 21-cm line at different redshifts. The neutral hydrogen density mostly tracks the underlying dark matter density field (Bharadwaj and Sethi, 2001) with a bias factor, with regions of higher density giving rise to a higher emission intensity. H I 21- cm line intensity mapping is a good way to trace the matter field (Battye et al., 2004; Bharadwaj and Sethi, 2001; Bharadwaj et al., 2001; Chang et al., 2008; Loeb and Wyithe, 2008; Santos et al., 2015; Villaescusa-Navarro et al., 2014), and can therefore be used to reconstruct the power spectrum of matter fluctuations. In the post-reionization era, after the Universe became ionized, the remaining neutral Hydrogen is mainly stored in self-shielded dense gas clouds, protected from ionizing UV radiation. These are predominantly hosted in galaxies, so the H I signal is effectively a tracer of the galaxy distribution. Intensity mapping observations, as with galaxy redshift surveys, can be used to measure the geometry and expansion rate of the Universe, therefore putting constraints on the properties of the dark energy (Bull et al., 2015). In contrast to high-resolution redshift surveys, intensity mapping surveys can be carried out much faster than redshift surveys, and a survey of very large cosmological volumes is possible in a fast and efficient manner. Intensity mapping has been proposed to measure phenomena on extremely large scales. A relatively low-resolution H I Intensity map can detect features found at large scales like primordial non-Gaussianity from inflation (Camera et al., 2013). The 21- cm intensity maps are also helpful for testing modifications to general relativity (Hall et al., 2013), and general relativistic corrections to the matter correlation function (Maartens et al., 2013). Overall, the 21- cm line is a golden tool in the arsenal of modern cosmologists.

1.3 Intensity mapping and 21- cm Power Spectrum

Currently, two types of redshifted 21- cm experiments attempt to observe the EoR. The first measures the global (mean) radio signal from a redshift $z \sim$ [6 - 20], mainly at an observation frequency range of $\nu = [70 - 200]$ MHz averaged over the whole sky (hemisphere) as a function of frequency. The global 21- cm signal is expected to be constant over large patches of the sky; the experimental efforts to measure it do not need high angular resolution and can be carried out with the instruments involving a single dipole (Pritchard and Loeb, 2012). Given the amount of foreground contamination, especially from our Galaxy, radio frequency interference (RFI), noise and calibration errors, and the limited amount of information in the data (mean intensity as a function of redshift), such experiments are much more challenging. Various experimental efforts are underway to detect the global 21- cm signal, such as the Experiment to Detect the Global EoR Signature (EDGES; Bowman et al., 2018) and Shaped Antenna measurement of the background RAdio Spectrum (SARAS; Singh et al., 2018). Bowman et al. (2018) has reported the first tentative detection of the 21- cm absorption from the CD at a redshift of 17. However, the claimed detection is still a matter of discussion as the detected signal is much larger than the predicted signal strength and has a different profile. Recently SARAS-3 results show the global 21- cm signal detected by the EDGES is not of astrophysical origin; it might be due to an uncalibrated systematics (Singh et al., 2022).

The other type of experiment that sheds light on the EoR is done using the principle of interferometry and is carried out in the frequency range of $\nu = 100 - 200$ MHz, corresponding to a redshift range of $z \sim 6 - 12$. These experiments are insensitive to the global signal but have the promise to measure the scale dependence of the fluctuations of 21-cm signal (Furlanetto et al., 2006). The interferometers pick the fluctuating component of the 21- cm signal brightness temperature in the form of visibility, related to the sky brightness distribution. Thus, we can study the sources and their effects on the IGM in more detail. Several interferometric observations are dedicated toward detecting the radio fluctuations in the redshifted 21- cm background arising from variations in the amount of neutral Hydrogen. These instruments are the upgraded Giant Metrewave Radio Telescope (uGMRT) (Gupta et al., 2017; Swarup et al., 1991), Low-Frequency Array (LOFAR; van Haarlem et al., 2013), Murchison Widefield Array (MWA; Bowman et al., 2013; Tingay et al., 2013), the Donald C. Backer Precision Array for Probing the Epoch of Reionization (PAPER; Ali et al., 2015; Parsons et al., 2010), the Hydrogen Epoch of Reionization Array (HERA; DeBoer et al., 2017), the New Extension in Nançay Upgrading loFAR (NenuFAR; Zarka et al., 2012; Zarka et al., 2015), the Square Kilometer Array (Koopmans et al., 2015; Mellema et al., 2013). etc.

The 21- cm signal itself is very weak, and given the relatively lower sensitivity of the instruments, the direct mapping of the redshifted 21- cm sky is rather difficult. Hence statistical estimators are needed to quantify the brightness temperature fluctuations of the CD-EoR 21- cm signal fields. One such statistical estimate of the signal is the power spectrum, which is the two-point correlation of the fluctuating fields (brightness temperature fluctuations in this case) in Fourier space. Let $\delta_{21}(\mathbf{x} = [T_b(\mathbf{x}) - \bar{T}_b]/\bar{T}_b$, be the fractional perturbation to the brightness temperature, a zero-mean random field, the power spectrum can be defined as (Bharadwaj and Sethi, 2001; Bharadwaj and Ali, 2005;

Furlanetto et al., 2006; Pritchard and Loeb, 2012)

$$\langle \tilde{\delta}_{21}(\mathbf{k_1})\tilde{\delta}_{21}(\mathbf{k_2})\rangle = (2\pi)^3\delta_D(\mathbf{k_1}-\mathbf{k_2})P_{21}(\mathbf{k_1}) \tag{1.1}$$

where angular brackets denotes the ensemble average, $\tilde{\delta}_{21}(\mathbf{k})$ is the Fourier transform of the $\delta_{21}(\mathbf{x})$ and $\delta_D(x)$ is the three dimensional Dirac delta function.

Here we focus on the experiments that aim to detect the redshifted 21- cm signal from EoR by measuring its power spectrum. We present a discussion on the power spectrum and various power spectrum estimators, especially the one we have used in this thesis, are presented in chapter 2.

1.4 Foregrounds

The strength of the cosmological 21- cm signal is relatively weak, and its fluctuations are expected to be of the order of $\sim 10mK$ (Furlanetto et al. (2006); Pritchard and Loeb (2012); Zaroubi et al. (2012), etc.). The detectable signal in the frequency range of observation that corresponds to the EoR is composed of many components, each with its own physical origin and statistical properties. These components are: (1) the 21- cm signal coming from the high redshift Universe, (2) Galactic and extragalactic foregrounds, (3) Ionospheric influences, (4) Telescope response effects, (5) Radio frequency interference (RFI), and (6) thermal noise. The intensity of this redshifted 21- cm signal is very low, and the presence of a very high brightness foreground which is 3 to 4 orders of magnitude higher than the H1 21- cm signal, makes its observation very challenging (Bernardi et al. (2010); Santos et al. (2005); Shaver et al. (1999), etc.). The major component of the foreground is Diffuse Radio Synchrotron Radiation and Galactic and Extragalactic bright point sources where the former contributes $\sim$ 70 % and later $\sim$30 % of the total foreground emission (Ali et al., 2008; Di Matteo et al., 2002; Ghosh et al., 2012; Jelić et al., 2008). The small contribution also comes from the galactic and extragalactic free-free emission (Cooray and Furlanetto, 2004; Oh and Mack, 2003). Therefore an accurate removal of bright foregrounds becomes essential for detecting redshifted 21- cm signals. Keeping the necessity of foreground removal in mind, various techniques are developed and discussed in the literature to mitigate the effect of foreground: these are the foreground avoidance (Datta et al., 2010; Morales et al., 2012; Vedantham et al., 2012), foreground subtraction (Bowman et al., 2009; Choudhuri et al., 2017b; Morales et al., 2006), and foreground suppression (Bharadwaj et al., 2019; Choudhuri et al., 2016, 2019). Apart from the orders of magnitude larger foregrounds, the instrumental effects also present a considerable challenge. The main challenge of the experiments in the low-frequency regime aiming to observe EoR is to distill the cosmological signal from this complicated mixture of influences discussed above. A detailed discussion on the foregrounds and their removal methods is presented in chapter 2.

1.5 Current status of CD/EoR Experiments

Currently, many new generation radio telescopes like uGMRT, LOFAR, MWA, etc., are being used to progress towards the H I 21- cm signal to study the Universe from the Dark Ages to the present. Upcoming telescopes like HERA and SKA are also being built with the promise to detect the EOR H I 21- cm signal. Especially the SKA will have significantly better sensitivity, resolution, frequency coverage, and improved instruments compared to the present telescopes, which will hopefully detect the redshifted 21- cm signal from Reionization. Several efforts have been made to measure the redshifted 21- cm signal with the currently existing observational facilities. These measurements not only significantly improved our understanding of various challenges in observing the redshifted 21- cm signal but have also been able to pose upper limits on the strength of the redshifted 21- cm signal. Ghosh et al. (2012) subtract the foreground compact sources and estimate the power spectrum of the sky brightness distribution at 150 MHz with the GMRT. At the wavenumbers of $k \sim 0.12 - 1.2$ h Mpc^{-1}, where the effect of the galactic synchrotron radiation is subdominant, they measure that the power spectrum amplitude as ~ 1000 mK^2 with ~ 10 hours of observations, which is much larger than the expected redshifted 21-cm signal. They attribute it to the systematics of visibility measurements. Hence, these measurements pose an upper limit to the redshifted 21- cm power spectrum amplitude. Paciga et al. (2013) used the GMRT to observe the 150 MHz sky for 40 hours and report an upper limit of $(248\ mK)^2$ at wavenumbers of 0.5 h Mpc^{-1}. Barry et al. (2019) have reported an upper limit of power spectral amplitude as 3900 mK^2 at the wavenumber of 0.2 h Mpc^{-1} with 21 hours of the MWA observations at the redshift 7.1. Mertens et al. (2020) gives an upper limit of $(73\ mK)^2$ at the wavenumber of 0.075 h Mpc^{-1} at the redshift of 9.1 with 141 hours of observations with the LOFAR. Recently Abdurashidova et al. (2022) have reported the upper-limits of $(30.76\ mK)^2$ at a wavenumber of 0.192 h Mpc^{-1} at $z = 7.9$, and $(95.74\ mK)^2$ at a wavenumber 0.256 h Mpc^{-1} at $z = 10.4$ with 18 nights of data from Phase I of the HERA; these limits are the most sensitive to-date by over an order of magnitude.

1.5.1 Effect of Gain Errors

All the above estimates of the upper limits posed using various interferometric observations are well above the theoretical 21- cm signal strength. This shows that, at present, the observations are limited by different systematics, including uncorrected instrumental effects. The interferometers measure the differential brightness temperature of the sky signal in the form of visibility. These measured visibilities differ from true sky visibility and have the sky signal modified by instrumental and line-of-sight effects. These effects are collectively known as gain. Various techniques, almost always with a known sky model (Pearson and Readhead, 1984; van der Tol et al., 2007; Wieringa, 1992; Wijnholds and van der Veen, 2009), are used to estimate the modifications in the sky signal. This process of estimating the instrumental and ionospheric effects and correcting the measured visibilities is known as calibration. In the case of ideal calibration, one can expect that the gains have been determined accurately and the corrected visibility will be the same as the true sky visibility. However, the estimation of the gains is subjected to the sky model's accuracy, the telescope's sensitivity, etc. These limits the accuracy of the gains which results in residual calibration/gain errors. The residual gains are usually small and can be ignored. For detecting the cosmological 21-cm signal, however, the residual gains can overwhelm the signal due to the presence of a much larger foreground. Hence the presence of bright foreground at the EoR frequencies additionally introduces the need for accurate calibration to achieve the high dynamic range measurements required in EoR experiments. The calibration errors often restrict the observations to reduce the expected thermal noise by integrating for a longer time.

Several sources of calibration/gain errors that lead to restrictions in detecting the redshifted 21-cm signal are investigated in the literature. Morales et al. (2012) have demonstrated that the simple frequency-independent calibration errors lead to residual power spectrum shapes contaminating nearly all k modes. Patil et al. (2016) studied the systematic effects arising due to calibration and subtraction of bright point sources in the LOFAR-EoR residual data. These effects include foreground suppression, which can cause the suppression of 21- cm signal, and excess noise with small scale fluctuations in frequency, causing the loss of sensitivity and a measurement bias in the 21- cm signal power spectrum. Gehlot et al. (2018) have studied various wide field and calibration effects such as gain errors, polarized foregrounds, and ionospheric effects in power spectral analysis for LOFAR-LBA. They have also reported an excess power in the stokes I power spectrum, which might be due to incomplete sky-model or imperfect calibration. In end-to-end simulations of full EoR power spectrum analysis, Barry et al. (2016) found that in the presence of an incomplete calibration catalog, the traditional per-frequency antenna calibration introduces contamination in the EoR window outside the wedge. Ewall-Wice et al. (2017) have also studied the impact of sky-based calibration errors for inaccurate sky models. Their work found that the unmodelled components of the foregrounds contaminate the EoR window by introducing a small frequency structure into gain solutions. The calibration errors associated with an incomplete sky model affect redundant calibration even in the case of perfect redundancy and identical antenna beams and can exceed the predicted EoR signal (Byrne et al., 2019). In the case of redundancy calibration, errors in gain solutions are also introduced due to the non-redundancy of the arrays. Liu et al. (2010) shows that non-redundant baseline distributions result in spectral structures contaminating the EoR detections. Dillon et al. (2020) have also studied the effect of non-redundancy on the gain solutions for a redundant array such as HERA. This introduces characteristic patterns into the gain solutions, affecting the calibrated visibilities and power spectra. Joseph et al. (2018) investigated the effect of sky flux distribution and antenna position offsets in redundancy calibration. The position offsets introduce a bias into the complex gain solutions phase. They notice an enhancement in the bias as the distance between bright radio sources and the pointing center, and the flux density of the source increase. The deviations from perfect redundancy due to the antenna-to-antenna variations in redundant-baseline calibration produce considerable foreground power leakage from the wedge contaminating a considerable fraction of the EoR window (Orosz et al., 2019). Using simulations, the effect of primary beam non-redundancy is also studied by Choudhuri et al. (2021). They find that an additional temporal structure is induced in the gain solutions.

Various hybrid calibration approaches to minimize the effect of calibration errors are also being developed and applied in different EoR experiments. Byrne et al. (2021) have presented a hybrid calibration framework that unifies both the sky-based and redundant calibration, showing an improvement in calibration performance through simulations. A hybrid correlation calibration (CorrCal) scheme, to address the issue of sky-model errors and imperfect array redundancy, is also presented and applied on PAPER experiments data (Gogo et al., 2022). A similar calibration approach is also discussed and applied to the MWA phase II data in Zhang et al. (2020). A precision bandpass calibration method namely CALAMITY is presented in Ewall-Wice et al. (2021). These studies try to minimize the errors in the gain solutions by introducing new calibration techniques or by improving the available ones.

Usually, the calibration errors are relatively small, and the residual gain errors can be neglected for the bulk of the interferometric observations. However, accurate calibration of instrumental effects is much needed for high dynamic range observations such as the redshifted 21-cm signal in the presence of bright foreground.

1.6 Aim of this thesis

In the previous sections, we have briefly discussed the process of cosmological structure formation and the evolution of the Universe. We have seen the importance of studying the EoR, a missing puzzle in the history of the evolution of the Universe. Solving this puzzle can give us in-depth knowledge of the processes involved in the process of structure formation and can answer other relevant questions. We also have discussed how the redshifted 21- cm signal from the neutral Hydrogen opens the door of possibilities to study the various eras of the Universe. The intensity mapping of the redshifted 21- cm signal has the potential to address many unanswered questions of Cosmology and Astrophysical origin. Numerous efforts are underway to probe the Dark Ages, CD, and EoR using global 21- cm experiments and radio interferometric telescopes designed to detect the 21- cm line of neutral Hydrogen from the EoR. As discussed previously, there are many challenges that these experiments need to overcome in order to reliably detect the cosmological 21- cm signal, such as bright foreground emission, complex ionospheric and instrumental effects, and RFI.

In recent times, there have been mammoth efforts on the theoretical and observational front to explore the various data components of the EoR experiments to understand the various challenges better and detect the signal by mitigating them. Various foreground mitigation techniques exist, and improved algorithms are being developed to remove the foregrounds accurately. However, if not treated well and mitigated efficiently, the above-mentioned systematic effects combined with the prominent foregrounds can severely contaminate the 21- cm signal from the Reionization era, obstructing its detection, irrespective of the accuracy of the foreground removal methods. Advanced calibration algorithms are also being applied to the observed data to accurately solve the ionospheric and instrumental effects. The present calibration techniques are efficient and give reasonable gain solutions; there always remains some error in the calibration process due to various practical difficulties. Although the residual gain errors left in the data due to uncalibrated effects are small, studying their properties and effects becomes of paramount importance in the context of high dynamic range observations like EoR.

As discussed in section 1.5, various studies have been done on the aspect of assessing the effect of calibration/gain errors in the context of EoR experiments. Datta et al. (2009, 2010) have discussed the bright point source subtraction requirements for EoR observations and present the calibration accuracy limits to detect the 21- cm signal from Reionization for MWA. They have studied the extragalactic point source contamination due to position errors in the sky model for bright sources and the frequency-independent residual gain errors in interferometric calibration. Building upon these works, in this thesis, we focus on the problem of time and frequency-dependent residual gain errors and study their effect in the EoR 21- cm signal power spectrum measurements. The aims of this thesis are as follows:

- To understand the effect of residual gain errors in the primary calibration.
- To study the effect of time and frequency correlated residual gain errors in the presence of bright foregrounds in high dynamic range observations such as EoR observations.
- To develop a framework to study the effect of time and frequency correlated gain errors analytically in the detection of redshifted 21- cm signal from Reionization using power spectrum estimation methods in the presence of bright foregrounds.

So far we have presented a brief introduction of the cosmological evolution, H I 21- cm signal and intensity mapping, and related challenges. In the coming chapters, we focus on the objectives stated above and try to investigate the effect of time and frequency-correlated gain errors in the context of EoR observations. The rest of the thesis is organized as follows. In chapter 2 we discuss the H I power spectrum and its relation with the dark matter power spectrum, the basic radio interferometric observation, and various H I power spectrum estimators highlighting the one used in this thesis. The foreground models and their removal methods are also discussed in chapter 2. In chapter 3 we present a detailed discussion on the time and frequency-dependent gains, various calibration methods and the origin of residual gain errors, and the effect of residual gain errors in primary calibration considering a simple case of a point source on the center of the field of view of observation. The effect of time-correlated residual gain errors through simulated observations for GMRT baseline configuration with the Poisson distribution of point sources as foreground model is discussed in chapter 4. Residual gain errors introduce a bias in the power spectrum estimations. In chapter 5 we present a framework to calculate the bias and variance of the power spectrum in the presence of time-correlated residual gain errors analytically and study various effects of gain errors considering a Gaussian model of time-correlated gain errors. The effect of frequency-correlated gain errors and the analytical expressions to estimate the bias and variance of the power spectrum for frequency-correlated gain errors along with the time correlation is presented in chapter 6. The thesis concludes with the summary and future prospects of the study in chapter 7. Some supplementary materials related to the chapter 4 are given in appendix A, and the detailed calculations of the chapter 5 and 6 are presented in appendix B.

1.7 Technical details

The numerical results discussed in this thesis are obtained with in-house developed numerical codes and simulations using computing languages like C and python. Shown figures are generated using the python matplotlib plotting tool. All the simulations are performed at the workstation and other computing facilities in the department of physics, computer center, and the PARAM SHIVAY computing facility at IIT(BHU) Varanasi, India 221005.

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