Study of Reaction-Advection-Diffusion Equations arising in Porous Media

Thesis submitted in partial fulfillment for the Award of degree

Doctor of Philosophy

By

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I, "***Chetna Biswas***", certify that the work embodied in this thesis is my own bona fide work and carried out by me under the supervision of "***Prof. Subir Das***" from " ***July 2019***" to "***April 2024***" at the "***Department of Mathematical Sciences***", Indian Institute of Technology (Banaras Hindu University), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not include them in this thesis and have not cited as my own work.

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Date:

Place: Varanasi **(Chetna Biswas)**

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Abbreviations

PDE	**P**artial **D**ifferential **E**quation
IDE	**I**ntegro **D**ifferential **E**quation
FPDE	**F**ractional **P**artial **D**ifferential **E**quation
LW	**L**egendre **W**avelets
RADE	**R**eaction-**A**dvection-**D**iffusion **E**quation
FRADE	**F**ractional-order **R**eaction- **A**dvection-**D**iffusion **E**quation
ADE	**A**dvection-**D**iffusion **E**quation
FRDE	**F**ractional-order **R**eaction-**D**iffusion**E**quation
SLCM	**S**hifted **L**egendre **C**ollocation **M**ethod
BFE	**B**urgers-**F**isher **E**quation