Chapter 7

Conclusion and Future Works

Conclusion

This thesis tackles the challenges related to stability, synchronization, and global dissipativity within neural networks, specifically focusing on higher-dimensional neural networks with various controllers. It is emphasized that conducting an in-depth mathematical analysis is crucial for examining synchronization problems. A central aspect of the synchronization of neural networks involves the stability analysis of nonlinear differential equations. Thus, the effective methods for the stability analysis of error systems for synchronization problems have been explored and derived less conservative results than existing ones. The primary focus has been given to investigating the synchronization of general models of neural networks and extending stability analyses to a higher dimensional model and extending to the higher order, i.e., the INN model. The first chapter details artificial neural networks' origin, motivation, and mathematical modeling. The fundamental definitions and properties of

DDE are also presented. Additionally, a physical interpretation of Hopfield, Cohen-Grossberg, and higher dimensional and higher-order models of neural networks, further extending these models to incorporate time delays, have been provided. The first chapter introduces Lyapunov theory and matrix measure theory, the tool applied in various chapters of this thesis for stability, stability of error analysis, and global dissipativity analyses.

The second chapter is focused on addressing the fixed-time synchronization of QVNNs. Acknowledging the non-commutative nature of quaternions, the strategy employed involved separating the quaternion model into four distinct real-valued systems. A well-designed controller, coupled with a Lyapunov function and the application of the fixed-time lemma, facilitated the achievement of synchronization results while mitigating the challenges posed by non-commutativity. Given the impact of time delays on the dynamics of neural networks, as discussed in Chapter 1, the study extended its scope to consider mixed-type delays in this synchronization framework. The numerical example provided empirical evidence, demonstrating that the system trajectory indeed reached synchronization within a fixed time interval. This distinctive form of synchronization, achieved within a predetermined timeframe, is aptly termed fixed-time synchronization.

In the third chapter, the focus shifted towards investigating quasi-projective synchronization within CVRNNs while considering parameter mismatch. The methodology adopted involved the matrix measure approach, distinct from the conventional Lyapunov approach, leading to less conservative results. Considering non-identical parameters in the drive and response systems aligns with practical applications where mismatches are prevalent. This chapter defines the error based on the difference between the response and a scaling factor times the drive system. The study delves into the analysis of the error system and establishes sufficient criteria for quasi-projective synchronization using the matrix measure. Different forms of synchronization have been identified by exploring various scaling factor values. Including proportional delays, representing infinite types of time delays, adds a layer of realism to the model. The chapter further examines the error bounds in quasi-projective synchronization, revealing fluctuations in the error of the systems. Graphical representations in the numerical section visually depict these fluctuations under different scenarios. Importantly, the approach of this chapter, considering more realistic parameters and results, stands in contrast to the previous chapter, where identical parameters and errors were assumed, limiting further deductions.

In the fourth chapter, the investigation is centered on establishing global exponential stability criteria for CGINNs. This exploration has employed the Lyapunov method and inequality techniques and adopted a non-reduction order approach. The preceding two chapters were rooted in traditional neural network models and their synchronization criteria. In contrast, this chapter delved into INNs, characterized by second-order differential equations encapsulating rich biological backgrounds, as discussed in the introduction section of this thesis. Historically, most studies on INNs have favored an order reduction approach, transforming second-order systems into first-order systems through variable substitution. However, such approaches often lead to increased complexity in the original systems, with more variables. In a departure from these methodologies, chapter 4 introduces a non-reduction order approach. This involves defining a newly developed Lyapunov function encompassing state variables and their derivative terms. The resulting stability criteria are more straightforward as compared to previous methods. Additionally, the chapter specifically focused on CGINNs containing a more general model than other neural network models. Notably, the existing results in the literature can be viewed as particular cases within the broader framework of the model.

In the fifth chapter, the focus of the investigation shifted to the anti-synchronization of QVINNs featuring unbounded discrete delays. In contrast to the separation technique discussed in the first chapter, where QVNNs were separated into four real-valued systems, this chapter adopts a non-separation approach. By maintaining the originality of the systems, the study provides more compact results and reduces theoretical complexity. Unlike the conventional process of order reduction, this chapter, akin to the previous one, addresses INNs without utilizing order reduction methods. A notable aspect of this chapter is exploring anti-synchronization results using two distinct controllers viz., feedback and adaptive controllers. The introduction of adaptive controllers for QVINNs represents a novel contribution, enabling the derivation of results without relying on order reduction or separation approaches. The unbounded time delays in this chapter add to its generality as compared to previous chapters, where time delays were bounded. This choice is particularly relevant as neurons consider the entire past rather than a partial history within bounded time delays. The proposed results are validated through numerical examples, and the application of QVNNs is showcased. They have demonstrated their superior performance over CVNNs and illustrated their effectiveness in storing true color image patterns.

The sixth chapter focuses on establishing the global and exponential dissipativity of QVINNs with mixed time delays. Building upon the methodology of the fifth chapter, which explored QVINNs with unbounded delays using a non-reduction and non-separation approach for QVINNs, this chapter extends the investigation to encompass unbounded discrete and distributed time delays. The inclusion of distributed time delays broadens the scope, demonstrating that the previous chapter is a particular case of the present one. A significant advancement in this chapter is considering the global and exponential attractive set for QVINNs. While the previous study focused solely on unbounded discrete delays, this chapter includes both discrete and distributed time delays, expanding the results' applicability. Unlike the fourth and fifth chapters, where criteria for exponential-type stability were constrained to a convergence rate $\lambda \in (0, 1)$, this chapter reveals that the convergence rate is unbounded. This insight is significant as a higher convergence rate it indicates a faster convergence of state trajectories to their desired targets. Furthermore, the chapter concludes by showcasing the application of QVNNs in associative memory. This application requires only 144 neurons to store true color image patterns, emphasizing the efficiency and practical utility of QVNNs.

Future Works

In the upcoming research endeavors, a focus may be given to the impact of impulsivity on projective synchronization within QVINNs featuring unbounded time delays. The focus will revolve around examining impulsive control mechanisms, incorporating both continuous and discontinuous components, to effectively manage the response system during impulsive events across various impulse magnitudes. Drawing inspiration from the findings of the thesis in higher dimensions, particularly in the context of QVNNs and QVINNs showcasing superior practical performance as compared to RVNNs and CVNNs, the aim is to concentrate extensively on higher-dimensional and higher-order neural network models in the foreseeable future. Additionally, the investigations will encompass unbounded time delays as a crucial factor. Various potential challenges will be addressed in future research, and the intricacies associated with higher-dimensional and higher-order neural network models, considering the influence of unbounded time-varying delays, will be explored.

(i) This thesis primarily emphasizes utilizing feedback and adaptive controllers in different neural network models to attain the objectives. Future research endeavors will explore diverse dynamical studies. Specifically, the aim is to investigate the effectiveness of sliding mode, event trigger, and impulsive controllers in achieving the desired results for different neural network models. This shift in focus will provide a comprehensive understanding of alternative control methodologies and their applicability in the research domain.

(ii) The upcoming research will explore fixed-time synchronization for QVINNs by leveraging norm properties. The main aim is to achieve this without reducing order or adopting the separation approach. The investigation will extend to include unbounded and nondifferentiable delays, exemplified by functions such as $\tau(t) = \sqrt{t} + |\sin(t)|$, $\sqrt{t}$, etc., which must emphasize that these represent nondifferentiable time delays in the future study.

(iii) The upcoming research will explore stability outcomes in higher-dimensional networks, specifically focusing on octonion-valued neural networks and octonion-valued inertial neural networks. A non-separation approach for the former and a non-reduction-order approach may be adopted in the near future. The objective is to demonstrate the practical applications of these stability results, particularly in associative memory, and to elucidate their advantages compared to other network architectures.

(iv) The upcoming research will investigate the conditions to determine the existence of higher-order neural networks other than INNs and traditional first-order neural network models. Notably, this remains an open problem for researchers. Subsequently, it may be delved into a comprehensive study of their distinct dynamical behaviors.

(v) The upcoming research will examine fixed-time and projective fixed-time synchronization for INNs. It will be planned to employ the matrix measure approach, diverging from order reduction methods. To date, there is a scarcity of work directly addressing INNs through the matrix measure method. Consequently, this presents a challenging yet pioneering study to navigate this investigation without relying on order reduction approaches.

(vi) All chapters of this thesis contain continuous-type activation functions; future research will focus on investigating the dynamical behavior of neural networks featuring discontinuous activation functions. This exploration is motivated by the broader class of models that can be encompassed through discontinuous activation functions. The intent is to delve into the unique dynamics and implications introduced by such non-continuous elements in neural network architectures.

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