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I, ***Madhushi Verma***, research scholar under the supervision of ***Prof. K. K. Shukla***, Professor, Department of Computer Science and Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi, hereby declare that the work incorporated in the present thesis entitled "***Fuzzy Models and Heuristic Algorithms for a Class of Graph Problems***" submitted by me for the degree of ***Doctor of Philosophy*** is a record of first-hand research work done by me during the period of study.

Further, I do undertake the responsibility for the mistakes, error of facts and misinterpretations (if any) in the thesis which is entirely original and my own work.

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Place: IIT(BHU), Varanasi (Madhushi Verma)

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CANDIDATE'S DECLARATION

I, ***Madhushi Verma***, certify that the work embodied in this Ph.D. thesis is my own bonafide work carried out by me under the supervision of ***Prof. K. K. Shukla***, for a period of 4 years 2 months from July, 2011 to September, 2015 at the *Department of Computer Science and Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi*. The matter embodied in this Ph.D. thesis has not been submitted for the award of any other degree / diploma.

I declare that I have faithfully acknowledged, given credit to and referred to the researchers whenever their works have been cited in the text and the body of the thesis. Further, I certify that I have not specified some other's work, para, text, data, results, etc. reported in the journals, books, magazines, reports, dissertations, thesis, documents available at websites etc. in this Ph.D. thesis and cited as my own work.

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CERTIFICATE FROM THE SUPERVISOR

This is to certify that the statement made by the candidate is correct to the best of my knowledge.

Signature of the Supervisor
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Professor and Head
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Signature of the HOD
(**Prof. K. K. Shukla**)
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ANNEXURE-F
(see Clause XIII.1 (c) and XIII.2 (b) (iv))

COURSE WORK / COMPREHENSIVE EXAMINATION COMPLETION CERTIFICATE

This is to certify that **Ms. Madhushi Verma**, a bonafide research scholar of Department of Computer Science and Engineering, Indian Institute of Technology (Banaras Hindu University), has worked for 16 credits and successfully completed the Ph.D. course work examination which is a part of her Ph.D. programme.

Date:

(Signature of the Head of the Department)

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ANNEXURE-F
(see Clause XIII.1 (c) and XIII.2 (b) (iv))

PRE-SUBMISSION COMPLETION CERTIFICATE

This is to certify that **Ms. Madhushi Verma**, a bonafide research scholar of Department of Computer Science and Engineering, Indian Institute of Technology (Banaras Hindu University), has successfully completed the pre-submission seminar on 30th July 2015 entitled "Fuzzy Models and Heuristic Algorithms for a Class of Graph Problems" which is a part of her Ph.D. programme.

Date:

(Signature of the Head of the Department)

Place: IIT(BHU), Varanasi

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ANNEXURE-G
(see Clause XIII.2 (b) (v))

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Title of the Thesis: ***Fuzzy Models and Heuristic Algorithms for a Class of Graph Problems.***

Candidate's Name: ***Madhushi Verma***

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The undersigned hereby assigns to IIT(B.H.U), Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the Ph.D. degree.

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PREFACE

Graphs can represent the pair wise relations between the objects from a collection and can be denoted as $G(V,E)$, where V and E signifies the set of vertices and set of edges respectively. Graphs find application in several real life areas like in the GPS system, social networking sites, telecommunication industry, computer science and mathematics etc. In most of the applications, weighted graphs are used, where the weights associated with the edges and the nodes represent the parameters like delay, cost, time, score, distance, capacity etc., which cannot be determined precisely. Therefore, the best way to deal with prevailing uncertainty, is to use fuzzy numbers (instead of crisp numbers) to represent these parameters.

Here, two major graph problems have been considered, namely the orienteering problem and the constrained shortest path problem. Some fuzzy and intuitionistic fuzzy models have been proposed to deal with the problem. Since the orienteering problem is NP-Hard and the constrained shortest path problem is NP-Complete, some heuristics / meta-heuristic have also been proposed to tackle the two problems.

Firstly, a new ranking method has been proposed for the Quasi-Gaussian fuzzy number and applied on the shortest path problem, minimum spanning tree problem and the Steiner tree problem. Few latest ranking methods, available for the trapezoidal fuzzy numbers, have been applied on the constrained shortest path problem with the aim to determine the ranking method, which is appropriate for practical applications. A max-min formulation has been presented for the fuzzy version of the orienteering problem. Also, a parallel formulation has been presented for the fuzzy orienteering problem.

Next, a new type of fuzzy number called the Quasi-Gaussian intuitionistic fuzzy number (QGIFN) has been defined. Also, a centroid based ranking method has been proposed for QGIFN. A centroid based ranking method has been suggested for the trapezoidal intuitionistic fuzzy number (TIFN) that uses a more generalized, eight parameter representation. Another, centroid of centroids method of ranking has been proposed for TIFN. These intuitionistic fuzzy numbers (IFN) and their respective ranking methods have been applied on the constrained shortest path problem. A max-min formulation has also been stated for the intuitionistic fuzzy version of the orienteering problem. A theoretical analysis, called the work-depth analysis has been performed for the intuitionistic fuzzy orienteering problem (IFOP) to determine its parallelism. A new intuitionistic fuzzy metric space using the concept of intuitionistic fuzzy points has been proposed. Further, the distance metric proposed has been applied on IFOP.

Finally, a roulette wheel selection heuristic (RWS_OP) has been proposed for the orienteering problem that can be applied on incomplete as well as complete graphs. A flower pollination meta-heuristic (FPA_OP) have been stated that can be applied on complete graphs only and a bidirectional search heuristic for the constrained shortest path problem has been presented.