

**Copyright © Indian Institute of Technology
(Banaras Hindu University),
Varanasi, India, 2015.
All rights reserved.**



Department of Computer Science & Engineering

Phone: 0542- 2307055-56, 0542-6702764

Fax: 0542-2368428

Ref.No.: IIT(BHU)/CSE/2015-2016/

Date:

UNDERTAKING FROM THE CANDIDATE

I, **Madhushi Verma**, research scholar under the supervision of **Prof. K. K. Shukla**, Professor, Department of Computer Science and Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi, hereby declare that the work incorporated in the present thesis entitled “**Fuzzy Models and Heuristic Algorithms for a Class of Graph Problems**” submitted by me for the degree of **Doctor of Philosophy** is a record of first-hand research work done by me during the period of study.

Further, I do undertake the responsibility for the mistakes, error of facts and misinterpretations (if any) in the thesis which is entirely original and my own work.

Date:

(Signature of the candidate)

Place: IIT(BHU), Varanasi

(Madhushi Verma)



Department of Computer Science & Engineering

Phone: 0542- 2307055-56, 0542-6702764

Fax: 0542-2368428

Ref.No.: IIT(BHU)/CSE/2015-2016/

Date:

CANDIDATE'S DECLARATION

I, **Madhushi Verma**, certify that the work embodied in this Ph.D. thesis is my own bonafide work carried out by me under the supervision of **Prof. K. K. Shukla**, for a period of 4 years 2 months from July, 2011 to September, 2015 at the *Department of Computer Science and Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi*. The matter embodied in this Ph.D. thesis has not been submitted for the award of any other degree / diploma.

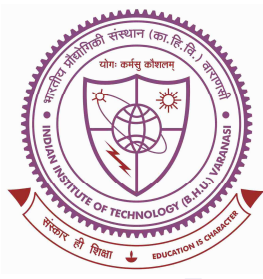
I declare that I have faithfully acknowledged, given credit to and referred to the researchers whenever their works have been cited in the text and the body of the thesis. Further, I certify that I have not specified some other's work, para, text, data, results, etc. reported in the journals, books, magazines, reports, dissertations, thesis, documents available at websites etc. in this Ph.D. thesis and cited as my own work.

Date:

(Signature of the candidate)

Place: IIT(BHU), Varanasi

(Madhushi Verma)



भारतीय
प्रौद्योगिकी
संस्थान
काशी हिन्दू विश्वविद्यालय



INDIAN
INSTITUTE OF
TECHNOLOGY
BANARAS HINDU UNIVERSITY

Department of Computer Science & Engineering

Phone: 0542- 2307055-56, 0542-6702764

Fax: 0542-2368428

Ref.No.: IIT(BHU)/CSE/2015-2016/

Date:

CERTIFICATE FROM THE SUPERVISOR

This is to certify that the statement made by the candidate is correct to the best of my knowledge.

Signature of the Supervisor
(**Prof. K. K. Shukla**)
Professor and Head
Department of Computer Science and
Engineering
Indian Institute of Technology (B.H.U)
Varanasi-221005

Signature of the HOD
(**Prof. K. K. Shukla**)
Professor and Head
Department of Computer Science and
Engineering
Indian Institute of Technology (B.H.U)
Varanasi-221005



Department of Computer Science & Engineering

Phone: 0542- 2307055-56, 0542-6702764

Fax: 0542-2368428

Ref.No.: IIT(BHU)/CSE/2015-2016/

Date:

ANNEXURE-F

(see Clause XIII.1 (c) and XIII.2 (b) (iv))

COURSE WORK / COMPREHENSIVE EXAMINATION COMPLETION CERTIFICATE

This is to certify that **Ms. Madhushi Verma**, a bonafide research scholar of Department of Computer Science and Engineering, Indian Institute of Technology (Banaras Hindu University), has worked for 16 credits and successfully completed the Ph.D. course work examination which is a part of her Ph.D. programme.

Date:

(Signature of the Head of the Department)

Place: IIT(BHU), Varanasi



Department of Computer Science & Engineering

Phone: 0542- 2307055-56, 0542-6702764

Fax: 0542-2368428

Ref.No.: IIT(BHU)/CSE/2015-2016/

Date:

ANNEXURE-F

(see Clause XIII.1 (c) and XIII.2 (b) (iv))

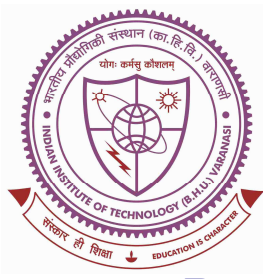
PRE-SUBMISSION COMPLETION CERTIFICATE

This is to certify that **Ms. Madhushi Verma**, a bonafide research scholar of Department of Computer Science and Engineering, Indian Institute of Technology (Banaras Hindu University), has successfully completed the pre-submission seminar on 30th July 2015 entitled “Fuzzy Models and Heuristic Algorithms for a Class of Graph Problems” which is a part of her Ph.D. programme.

Date:

(Signature of the Head of the Department)

Place: IIT(BHU), Varanasi



Department of Computer Science & Engineering

Phone: 0542- 2307055-56, 0542-6702764

Fax: 0542-2368428

Ref.No.: IIT(BHU)/CSE/2015-2016/

Date:

ANNEXURE-G (see Clause XIII.2 (b) (v))

COPYRIGHT TRANSFER CERTIFICATE

Title of the Thesis: *Fuzzy Models and Heuristic Algorithms for a Class of Graph Problems.*

Candidate's Name: *Madhusi Verma*

Copyright Transfer

The undersigned hereby assigns to IIT(B.H.U), Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the Ph.D. degree.

Signature of the Candidate

ACKNOWLEDGEMENT

Successful completion of any task gives great satisfaction and internal strength regarding the future problems, but at any stage, a person never exists alone. He or she is always accompanied by some people who provide support and suggestion for successful completion of any work. Therefore, it is a matter of great pleasure for me to thank all those people who inspired me and gave their kind hearted support at every stage of my research work.

First and foremost, I bow in great reverence to almighty God, the most gracious, the most merciful, whose bounteous blessings enabled me to accomplish this thesis.

I would like to honor my great institute “*Indian Institute of Technology (BHU), Varanasi*”, where I got a platform to learn the new techniques and concepts that enhanced my skills.

I would like to express my gratitude and sincere thanks to my supervisor and Head of the Department, *Prof. K.K. Shukla*. I am indebted to my guide for his seamless support, advice and guidance throughout my doctoral study. Without his active guidance, valuable suggestions and consistent support, this work would not have been possible. I am extremely grateful to him for giving me his precious time, in spite of his busy schedule, to carry out the fruitful discussions regarding the research work.

I am very grateful to my aunt, *Prof. Ranjana Patnaik*, School of Biomedical Engineering, for her blessings and the encouragement that she gave me to carry out this research work.

I am extremely thankful to *Prof. Rekha Srivastava*, Department of Mathematical Sciences for giving me lessons on fuzzy mathematics.

I would also like to convey my sincere gratitude to the other faculty members of the Department of Computer Science and Engineering, *Prof. R. B. Mishra, Prof. A. K. Tripathi, Prof. A. K. Agarwal, Prof. Rajeev Srivastava, Dr. S. K. Singh, Dr. R. S. Singh, Dr. Bhaskar Biswas* and *Dr. Vinayak Srivastava* for their extensive and inspiring guidance throughout this tenure.

I am enormously grateful to all the members of RPC, *Prof. K. K. Shukla, Prof. Rajeev Srivastava, Dr. Bhaskar Biswas* and *Dr. Neeraj Sharma* for their valuable suggestions, appreciation and encouragement.

My memory of the study period at IIT(BHU) can never be complete without mentioning my fellow research scholars. Special thanks to *Ms. Mridula Verma* and *Mr. Jayadeep Pati* for their great help and cooperation.

I extend special thanks to the non-teaching staff in the Department, particularly, *Mr. R. S. Tripathi, Mr. Pramod Kumar, Mr. Rajendra Kumar, Mr. Kanhaiya Lal, Mr. Ravi Kumar Bharti, Mr. Bharat Pandey* and *Mr. Tribhuvan* for their consistent support.

My acknowledgements would not be complete unless I express my heartfelt gratitude and deepest appreciation to *my grandparents, my parents, my sisters and all my family members* and *friends* for their blessings, support, love, encouragement and sacrifices.

Madhushi Verma

PhD. Research Scholar,

Enroll. No. 338916

Dept. of Computer Science and Engineering, IIT (BHU),
Varanasi (India).

CONTENTS

Acknowledgement

List of Figures

List of Tables

Preface

1. Introduction	1
1.1 Graphs	2
1.1.1 Types of Graphs	2
1.1.2 Application of Graphs	3
1.1.3 Graph Problems	4
1.2 Uncertainty in Real Life	7
1.3 Fuzzy Sets	7
1.4 Other Concepts Modelling Uncertainty	10
1.4.1 Rough Sets	10
1.4.2 Intuitionistic Fuzzy Sets or Vague Sets	11
1.4.3 Soft Sets	12
1.5 Related Work	13
1.6 Motivation	19
1.7 Research Contribution	19
1.8 Layout of Thesis	20
2. Fuzzy Numbers – Their Ranking Methods and Applications	22
2.1 Introduction	23
2.2 Fuzzy Numbers	26
2.2.1 Quasi-Gaussian Fuzzy Number (QGFN)	27

2.2.2 Trapezoidal Fuzzy Number (TFN)	53
2.3 Max – Min Formulation for Orienteering Problem	77
2.3.1 Basic Definitions	78
2.3.2 Problem Definition	79
2.3.3 Fuzzy Formulation of OP	80
2.3.4 FOP Algorithm	83
2.3.5 Illustrative Example	84
2.3.6 Parallel Formulation of FOP	91
2.4 Conclusion	97
3. Intuitionistic Fuzzy Numbers and Intuitionistic Fuzzy Point	100
3.1 Introduction	102
3.2 Intuitionistic Fuzzy Numbers (IFN)	103
3.2.1 Trapezoidal Intuitionistic Fuzzy Number (TIFN)	104
3.2.2 Quasi-Gaussian Intuitionistic Fuzzy Number (QGIFN)	113
3.3 Max – Min Formulation for Intuitionistic Fuzzy Orienteering Problem	121
3.3.1 IFOP Algorithm	122
3.3.2 Illustrative Example	123
3.3.3 Work-depth Analysis of IFOP	128
3.4 Intuitionistic Fuzzy Metric Space using Intuitionistic Fuzzy Point	132
3.4.1 Intuitionistic Fuzzy Metric Space	133
3.4.2 Intuitionistic Fuzzy Orienteering Problem using Intuitionistic Fuzzy Points	137
3.5 Conclusion	143

4. Heuristic Algorithms for Graph Problems	145
4.1 Introduction	147
4.2 Comparison of Selection Methods for Orienteering Problem	148
4.2.1 Selection Methods	149
4.2.2 Algorithm for Incomplete and Complete Graphs	151
4.2.3 Experimental Analysis	154
4.3 Roulette Wheel Selection based Heuristic Algorithm for the Orienteering Problem	161
4.3.1 Algorithm RWS_OP	163
4.3.2 Experimental Analysis	169
4.4 Flower Pollination Algorithm for Orienteering Problem	184
4.4.1 Algorithm <i>FPA_OP</i>	186
4.4.2 Experimental Analysis	188
4.5 Bidirectional Shortest Path Algorithm for the Constrained Shortest Path Problem with Good Average-Case Behavior	189
4.5.1 Algorithm	190
4.5.2 Experimental Analysis	194
4.6 Conclusion	197
5. Conclusion and Future Directions	199
5.1 Concluding Remarks	200
5.2 Future Scope	203

BIBLIOGRAPHY AND REFERENCES

LIST OF PUBLICATIONS

COPIES OF PUBLISHED PAPERS

CURRICULUM VITAE

LIST OF FIGURES

Fig. No.	Figure Title	Page No.
Fig. 1.1	Pictorial Representation of degree of membership	9
Fig. 2.1	Different shapes obtained by varying the value of fuzzification factor m	28
Fig. 2.2	Link Preference Index diagram	32
Fig. 2.3	Network for FSPP	36
Fig. 2.4	Ranking of paths in the network	40
Fig. 2.5	Network for FMST	44
Fig. 2.6	Fuzzy Minimum Spanning Tree for the given network	47
Fig. 2.7	Network for FSTP with Steiner points shown as double circles	48
Fig. 2.8	Fuzzy complete undirected distance graph G_1	50
Fig. 2.9	Fuzzy Minimal Spanning Tree T_1 of G_1	53
Fig. 2.10	Final Fuzzy Steiner Tree(T_{FS})	53
Fig. 2.11(a)	Behavior in terms of cost of shortest path shown by different ranking methods by varying the delay requirement on a random graph with 250 nodes generated by gengraph-win	66
Fig. 2.11(b)	Behavior in terms of path discretization error shown by different ranking methods by varying the delay requirement on a random graph with 250 nodes generated by gengraph-win	66
Fig. 2.11(c)	Behavior in terms of CPU Execution time shown by different ranking methods by varying the delay requirement on a random graph with 250 nodes generated by gengraph-win	67
Fig. 2.12	The point considered as Circumcenter of Centroid (COC) is shown by X	67
Fig. 2.13	A surface plot with the delay requirement, cost and CPU execution time of WF	68

Fig. 2.14	Box and Whisker plot showing the fuzzy cost as a TFN with four parameters at the delay requirement = 30 units	68
Fig. 2.15	Block diagram of a typical Wireless Sensor node (mote)	71
Fig. 2.16	A Wireless Sensor Network (WSN) represented as a Unit Disc Graph (UDG)	71
Fig. 2.17	Pictorial representation of trapezoidal fuzzy number	72
Fig. 2.18	(a) Behaviour of the CFSP algorithm when applied on a graph with 200 nodes generated using <i>gengraph-win</i> with source (s) = 45 and target (t) = 68, (b) Comparison of the same behaviour for four different network sizes with source (s) = 4 and target (t) = 45	74
Fig. 2.19	Progress of the CFSP algorithm in terms of energy consumption with a strict and a relaxed delay constraint when applied on a graph with 200 nodes generated using <i>gengraph-win</i>	76
Fig. 2.20	Progress of the CFSP algorithm in terms of path delay with delay constraint= 350 when applied on a graph with 50, 100, 150, 200 nodes generated using <i>gengraph-win</i>	76
Fig. 2.21	Membership Function for total collected score of a path	82
Fig. 2.22	Membership Function for total time taken to traverse a path	82
Fig. 2.23	Fuzzy decision set Z and Z^*	83
Fig. 2.24	Input Graph $G(V, E)$ with source vertex = 1 and destination vertex = 5	84
Fig. 2.25	Several steps of the parallel formulation of FOP	92
Fig. 3.1	Trapezoidal intuitionistic fuzzy number (TIFN)	104
Fig. 3.2	The point of reference used for ranking a TIFN	107
Fig. 3.3	Behaviour shown by the cost of the shortest path on varying the input delay constraint for a graph with 200 nodes generated using <i>gengraph-win</i>	113
Fig. 3.4	Quasi-Gaussian Intuitionistic Fuzzy Number	115
Fig. 3.5	Centroid method of ranking for QGIFN	116

Fig. 3.6	Trend observed in the cost of the shortest path on varying the input delay constraint for a graph with 100 nodes generated using gengraph-win	121
Fig. 3.7	The input graph with $N = 5, v_1 = 1, v_N = 5$ and the time and score values associated with each edge and vertex respectively	123
Fig. 3.8	The sequential module executing step 1 of IFOP that computes all the distinct paths in the given graph G	130
Fig. 3.9	The parallel version of IFOP along with its work-depth analysis stating the work and depth value of each step	131
Fig. 3.10	Input graph G with number of nodes (N) = 5 , source (v_1) = 1, target (v_N) = 5 and the co-ordinate values (intuitionistic fuzzy points) and the score values (trapezoidal intuitionistic fuzzy numbers) of each node	139
Fig. 4.1	Comparison of the maximum value of the total collected score obtained by four different selection methods for different T_{max} values (160 cities)	158
Fig. 4.2	Comparison of the maximum value of the total collected score obtained by four different selection methods for different T_{max} values (306 cities)	159
Fig. 4.3	Graph for (a) 160 cities and (b) 306 cities instance	160
Fig. 4.4	The process of selecting a path using roulette wheel selection function where the number in () denotes the probability of node selection	162
Fig. 4.5	Progression of RWS_OP algorithm for a graph with 25 nodes with source (V_1) = 1, destination (V_N) = 25 and $T_{max} = 70$	169
Fig. 4.6	Comparison of (a) maximum score and (b) mean score of each method with respect to time budget (T_{max})	173
Fig. 4.7	Comparison of execution time of each method with respect to time budget (T_{max}) based on 30 runs at $\alpha = 0.6$ for Real Road Network database with 306 cities of Poland	174
Fig. 4.8	Comparison of score with respect to α for (a) $T_{max} = 1500$ and (b) $T_{max} = 2500$ for a Real Road Network database with 306 cities of Poland	175
Fig. 4.9	Plots showing (a) utilization of the time budget and (b) increase in the total collected score at $\alpha = 0.6$ and $T_{max} = 1500$ for a Real Road Network database with 160 cities of Poland (c) Progression of RWS_OP algorithm	179

Fig. 4.10	Plots showing the observation of three different runs of RWS_OP with $\alpha = 0.2$ and $T_{max} = 1500$ for a Real Road Network database with 160 cities of Poland. As the algorithm progresses, it results in (a) decrease in the time budget and (b) increase in the total collected score as shown above	180
Fig. 4.11	Plots showing (a) utilization of the time budget and (b) increase in the total collected score for three different α values at $T_{max} = 1500$ for a Real Road Network database with 160 cities of Poland	181
Fig. 4.12	Plots showing (a) the percentage of nodes explored with the increase in T_{max} values at $\alpha = 0.6$ and (b) percentage of nodes explored and unexplored for different values of α at $T_{max} = 7000$ for a Real Road Network database with 160 cities of Poland for 30 runs	182
Fig. 4.13	Plot showing that RWS_OP can achieve higher total collected score for larger T_{max} values as compared to Ostrowski_CG and Ostrowski_IG methods when implemented on a Real Road Network database with 306 cities of Poland at $\alpha = 0.6$	184
Fig. 4.14	Comparison of the total collected score value achieved by GRASP and FPA algorithms for different T_{max} values when applied on a graph with 102 nodes, source=1, destination=102	189
Fig. 4.15	Comparison of the average execution time (s) of the CSPP algorithm suggested by Chen <i>et al.</i> (2008) with the bidirectional search algorithm for different network sizes	197

LIST OF TABLES

Table No.	Table Title	Page No.
Table 2.1	LPI value and Ranks of different paths	39
Table 2.2	Initial values of the two arrays	46
Table 2.3	Initializing the LPI value of root node	46
Table 2.4	Status of the two arrays after the neighbours of root are explored	46
Table 2.5	Status of the two arrays after exploring the next node in the priority queue Q	46
Table 2.6	Final values returning the connections in MST	47
Table 2.7	Initial status of the two arrays	51
Table 2.8	LPI value for the root is initialized	52
Table 2.9	Showing the status of the two arrays after the neighbours of the root are explored	52
Table 2.10	Showing the final connections in the fuzzy minimal spanning tree T_1 of G_1	52
Table 2.11	The energy deviation from unconstrained optimum for increasing delay constraint	75
Table 2.12	The value of edge weights (time taken to travel from one node to another)	85
Table 2.13	The value of node weights (score values)	85
Table 2.14	The total time taken and total collected score for each of the paths	88
Table 2.15	The expected value of the total time taken to traverse the path and the total collected score for each of the paths	89

Table 2.16	The grade of membership of each possible path for both the membership functions of time and score	90
Table 2.17	Showing the ranks of the desirable paths	90
Table 3.1(a)	The values of total time taken and total collected score obtained for each possible path	125
Table 3.1(b)	The expected value for the total time taken and the total collected score of each possible path	126
Table 3.1(c)	The membership value for the total time taken and the total collected score for each possible path	127
Table 3.2	Ranks of the desirable paths	128
Table 3.3	The d_{ij} value of each edge	140
Table 3.4	The value of total distance covered and total score collected on traversing each path	141
Table 3.5	The solution set after discarding those paths that do not satisfy the distance bound (D_{max})	142
Table 3.6	Ranks assigned to the paths to determine the most desirable path	142
Table 4.1	Comparison of the mean and maximum value of the total collected score obtained by <i>SEL_OP</i> when executed with four different selection procedures for 160 cities	156
Table 4.2	Comparison of the mean and maximum value of the total collected score obtained by <i>SEL_OP</i> when executed with four different selection procedures for 306 cities	157
Table 4.3	Comparison of maximum, mean and confidence Interval (CI) for mean of scores obtained by <i>RWS_OP</i> (keeping $v_1 = v_N$ i.e., $v_1 = v_N = 1$) with those obtained by executing the Ostrowski's algorithm (Please refer (Ostrowski & Koszelew, 2011), their Table 5 for Ostrowski_CG and Table 7 for Ostrowski_IG) on Real Road Network database with 306 cities of Poland	172
Table 4.4	The Highest Score Collected, Mean of Score Collected, Mean Time to Traverse the Path and % of Time Budget Utilized values obtained by <i>RWS_OP</i> at $\alpha = 0.6$ (keeping $v_1 \neq v_N$ i.e., $v_1 = 1$ and $v_N = 306$) when implemented on a Real Road Network database with 306 cities of Poland	176

Table 4.5	The Highest Score Collected, Mean of Score Collected, Mean Time to Traverse the Path and % of Time Budget Utilized values obtained by RWS_OP at $\alpha = 0.6$ (keeping $v_1 \neq v_N$ i.e., $v_1 = 1$ and $v_N = 160$) when implemented on a Real Road Network database with 160 cities of Poland	177
Table 4.6	The Highest Score Collected, Mean of Score Collected and confidence interval (CI) for Mean of Score Collected obtained by RWS_OP when implemented on a Real Road Network database with 306 cities of Poland for different T_{max} values at $\alpha = 0.6$ (keeping $v_1 = v_N$ i.e., $v_1 = v_N = 1$)	183

PREFACE

Graphs can represent the pair wise relations between the objects from a collection and can be denoted as $G(V, E)$, where V and E signifies the set of vertices and set of edges respectively. Graphs find application in several real life areas like in the GPS system, social networking sites, telecommunication industry, computer science and mathematics etc. In most of the applications, weighted graphs are used, where the weights associated with the edges and the nodes represent the parameters like delay, cost, time, score, distance, capacity etc., which cannot be determined precisely. Therefore, the best way to deal with prevailing uncertainty, is to use fuzzy numbers (instead of crisp numbers) to represent these parameters.

Here, two major graph problems have been considered, namely the orienteering problem and the constrained shortest path problem. Some fuzzy and intuitionistic fuzzy models have been proposed to deal with the problem. Since the orienteering problem is NP-Hard and the constrained shortest path problem is NP-Complete, some heuristics / meta-heuristic have also been proposed to tackle the two problems.

Firstly, a new ranking method has been proposed for the Quasi-Gaussian fuzzy number and applied on the shortest path problem, minimum spanning tree problem and the Steiner tree problem. Few latest ranking methods, available for the trapezoidal fuzzy numbers, have been applied on the constrained shortest path problem with the aim to determine the ranking method, which is appropriate for practical applications. A max-min formulation has been presented for the fuzzy version of the orienteering problem. Also, a parallel formulation has been presented for the fuzzy orienteering problem.

Next, a new type of fuzzy number called the Quasi-Gaussian intuitionistic fuzzy number (QGIFN) has been defined. Also, a centroid based ranking method has been proposed for QGIFN. A centroid based ranking method has been suggested for the trapezoidal intuitionistic fuzzy number (TIFN) that uses a more generalized, eight parameter representation. Another, centroid of centroids method of ranking has been proposed for TIFN. These

intuitionistic fuzzy numbers (IFN) and their respective ranking methods have been applied on the constrained shortest path problem. A max-min formulation has also been stated for the intuitionistic fuzzy version of the orienteering problem. A theoretical analysis, called the work-depth analysis has been performed for the intuitionistic fuzzy orienteering problem (IFOP) to determine its parallelism. A new intuitionistic fuzzy metric space using the concept of intuitionistic fuzzy points has been proposed. Further, the distance metric proposed has been applied on IFOP.

Finally, a roulette wheel selection heuristic (*RWS_OP*) has been proposed for the orienteering problem that can be applied on incomplete as well as complete graphs. A flower pollination meta-heuristic (*FPA_OP*) have been stated that can be applied on complete graphs only and a bidirectional search heuristic for the constrained shortest path problem has been presented.