Chapter 2

The formation of shock wave in a two-dimensional supersonic planar and axisymmetric non-ideal gas flow with magnetic field [*]

"What is mathematics? It is only a systematic
effort of solving puzzles posed by nature"

–Shakuntala Devi

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2.1 Introduction

In the nonlinear system, the wave is considered to be moving surface along which the flow variables and their derivatives possess certain kind of discontinuity that are carried along the surface. These phenomenon occurring in the nature are represented by mathematical system of quasilinear PDEs. Due to the interdisciplinary applications in several physical phenomena, study of non-linear waves has great importance. The shock waves are discontinuities that occur in the derivative of the solution along the characteristics however, the solution itself remains continuous. Therefore, the shock waves in fluids received considerable attention because they are a particular class of discontinuous wave processes, which are studied by some analytical and numerical methods. The occurrence of these type of discontinuities are natural phenomenon in several physical situations like collision of galaxies, space science, supernova explosion, space re-entry vehicles, photo ionization and other astrophysical situations. The process of shock formation and decay in the characteristic plane is one of the most critical aspect of non-linear wave theory in gasdynamics. The problem of the study of flow pattern of nonlinear waves in different material media has gained significant attention over the recent couple of decades due to their application in numerous fields like nuclear science, space science and engineering sciences. The authors [1] and [67] developed a large number of numerical and analytical techniques to determine the behavior of solutions to the hyperbolic system of PDEs governing the wave motion. Also, [7], [68], and [69] have proposed several approaches to discuss the asymptotic behavior of weakly non-linear waves governed by the hyperbolic system of PDEs and derived the transport equations which are utilized in the evolution of discontinuity waves. In the general theory of the evolution of shock wave, in solutions to the non-linear systems and determining the time of the occurrence of shock, there are many theoretical approaches in order, such as wavefront expansion technique by

[67] and [70], parameter expansion technique by [71], the method of wavefront analysis by [72], reductive perturbation method by [73], asymptotic method by [74], and the singular surface method by [75],[76] and [77]. These theoretical techniques are very effective due to the number of independent and dependent variables involved. These methods which govern the evolution of the amplitude of the shock wave, yields a Bernoulli type ordinary differential equation. A concise explanation of the system of wavefront analysis is discussed by [78], [75], [67], and [79]. [80] have discussed the shock formation distance in 2-D magnetogasdynamic flow over a concave corner under the effect of radiative heat transfer. [81] and [82] have examined the evolution of the shock wave and obtained the relation for shock formation distance in 2-D radiating gas flow and radiative non-ideal gas flow by using same technique. Further, [83] have also studied the shock formation distance in 2-D dusty gas. [84] have discussed the propagation of discontinuous waves in one-dimensional radiative non-ideal gas flow. Many authors like [79], [85], [86], [87], [88], [89], [90] etc. have studied the shock wave phenomenon in several regimes by using different analytical approaches. Recently, the propagation of shock wave and its evolutionary process for planar and non-planar dusty gasdynamic flow is also studied by [91, 92].

The study of shock formation and variation in shock formation distance in 2-D steady supersonic non-ideal gas flow under the action of magnetic field have not been discussed by any researchers till now. In this theoretical work, we study the shock wave in 2-D steady supersonic inviscid non-ideal gas flow with magnetic field for planar and axisymmetric cases by using the method of wavefront analysis. Also, it is discussed that how the upstream flow Mach number and magnetic field influence the shock formation distance in non-ideal magnetogasdynamic flow. The investigation of shock wave in non-ideal gas flow have great attention of engineers and scientists due to its important role in the study of several areas like oceanography, astrophysics, hypervelocity impact, atmospheric science, hypersonic flow and aerodynamics. Many interesting results of weak discontinuities in non-ideal gas have been investigated recently by many researches as [93] [94], [95], [96], [87]. Also, the presence of magnetic field in gasdynamics plays significant role in momentum and energy transport and can rapidly release energy in flares. But study of magnetic field in non-ideal gasdynamics is more typical in theoretical investigation. Recently, [89], [97], [98], [99], [87], [100] and [101] have studied the propagation of shock wave and its behavior in non-ideal magnetogasdynamics. Currently, the interaction of discontinuity wave in generalized magnetogasdynamics have been investigated by the authors in [102]. The different proposition of hyperbolic conservative mixture models and the application of the modern solution methods for two-phase compressible flows were employed with the aim to provide reliable results in [103, 104, 105]. Also, the numerical investigation of the non linear wave propagation in composition of two phase flow and turbulent cavitating flows in thermal regime is studied in [106], [107], and [108].

This chapter uses the method of wavefront analysis to study the propagation of shock waves in a 2-D steady supersonic flow of non-ideal gas with a magnetic field for the planar flow and axisymmetric flows. The transport equations are derived, which leads to the determination of shock formation distance and provides the conditions of shock formation. It is also assessed as to how the shock wave formation is affected by the presence of a magnetic field, non-ideal parameter, and upstream flow Mach number $M_0 > 1$.

2.2 Governing equations describing the non-ideal magnetogasdynamic flow

The governing equations describing two-dimensional steady supersonic non-ideal gas flow under the action of the magnetic field may be written as (See [80, 101])

$$u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{nv}{y}\right) = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\left(\frac{\partial p}{\partial x} + \frac{\partial h}{\partial x}\right) = 0, \tag{2.2}$$

$$\rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} + \frac{\partial h}{\partial y} = 0, \tag{2.3}$$

$$u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + 2h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{nv}{y}\right) = 0, \tag{2.4}$$

$$u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} - C^2\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right) = 0, \tag{2.5}$$

where p, ρ, u and v are the pressure, gas density and the components of velocity along the x and y axes, respectively. The parameter $h = \frac{\mu H^2}{2}$ is defined as magnetic pressure with transverse magnetic field H and μ is the magnetic permeability. The parameter n is a constant, where the value $n = 0$ is used for planar flow and the value $n = 1$ is used for axisymmetric flow. The parameter C is known as the speed of sound in non ideal gas defined by

$$C^2 = \frac{a^2}{(1 - b\rho)},$$

where a is speed of sound with γ as adiabatic index defined as $a^2 = \frac{\gamma p}{\rho}$, and b represents non-ideal parameter as $0 < b << 1$.

2.3 Characteristic formulation

The Equation (2.1 − 2.5) can be rewritten as

$$\begin{cases} \rho_x + \frac{v}{u}\rho_y - \frac{\rho v}{(u^2-d^2)}u_y + \frac{\rho v}{(u^2-d^2)}v_y + \frac{v}{u(u^2-d^2)}h_y + \frac{v}{u(u^2-d^2)}p_y + \frac{\rho n v u}{y(u^2-d^2)} = 0, \\ u_x + \frac{uv}{(u^2-d^2)}u_y - \frac{d^2}{(u^2-d^2)}v_y - \frac{v}{\rho(u^2-d^2)}h_y - \frac{v}{\rho(u^2-d^2)}p_y - \frac{nvd^2}{y(u^2-d^2)} = 0, \\ v_x + \frac{v}{u}v_y + \frac{1}{\rho u}h_y + \frac{1}{\rho u}p_y = 0, \\ h_x - \frac{\rho e^2 v}{(u^2-d^2)}u_y + \frac{\rho e^2 u}{(u^2-d^2)}v_y + \frac{v\left(u^2-C^2\right)}{u(u^2-d^2)}h_y - \frac{ve^2}{u(u^2-d^2)}p_y + \frac{\rho e^2 n v u}{y(u^2-d^2)} = 0, \\ p_x - \frac{\rho C^2 v}{(u^2-d^2)}u_y + \frac{\rho C^2 u}{(u^2-d^2)}v_y + \frac{vC^2}{u(u^2-d^2)}h_y + \frac{v\left(u^2-e^2\right)}{u(u^2-d^2)}p_y + \frac{\rho n v u C^2}{y(u^2-d^2)} = 0, \end{cases}$$

where $q^2 = u^2 + v^2$, and the parameter $d = \left(C^2 + e^2\right)^{1/2}$ is magneto-sonic speed with $e^2 = 2h/\rho$. The parameter e is Alfvén speed and $\epsilon = 1 + \frac{e^2}{C^2}$ is Alfvén number. The matrix form of above reduced equations can be written as

$$\frac{\partial W}{\partial x} + A(W)\frac{\partial W}{\partial y} + F = 0, \tag{2.6}$$

where A is the matrix of 5 × 5, and column vectors W and F are defined by

$$W = \begin{bmatrix} \rho \\ u \\ v \\ h \\ p \end{bmatrix}, A = \begin{bmatrix} \frac{v}{u} & -\frac{\rho v}{(u^2-d^2)} & \frac{\rho v}{(u^2-d^2)} & \frac{v}{u(u^2-d^2)} & \frac{v}{u(u^2-d^2)} \\ 0 & \frac{uv}{(u^2-d^2)} & -\frac{d^2}{(u^2-d^2)} & -\frac{v}{\rho(u^2-d^2)} & -\frac{v}{\rho(u^2-d^2)} \\ 0 & 0 & \frac{v}{u} & \frac{1}{\rho u} & \frac{1}{\rho u} \\ 0 & -\frac{\rho e^2 v}{(u^2-d^2)} & \frac{\rho e^2 u}{(u^2-d^2)} & \frac{v\left(u^2-C^2\right)}{u(u^2-d^2)} & -\frac{ve^2}{u(u^2-d^2)} \\ 0 & -\frac{\rho C^2 v}{(u^2-d^2)} & \frac{\rho C^2 u}{(u^2-d^2)} & \frac{vC^2}{u(u^2-d^2)} & \frac{v\left(u^2-e^2\right)}{u(u^2-d^2)} \end{bmatrix}, \text{and } F = \begin{bmatrix} \frac{\rho n v u}{y(u^2-d^2)} \\ -\frac{nvd^2}{y(u^2-d^2)} \\ 0 \\ \frac{\rho e^2 n v u}{y(u^2-d^2)} \\ \frac{\rho n v u C^2}{y(u^2-d^2)}. \end{bmatrix}. \tag{2.7}$$

The eigenvalues of matrix $A(W)$ are represented by $\lambda^{(i)}$ for $i = 1, 2, 3, 4, 5$, obtained as

$$\lambda^{(1,2,3)} = \frac{v}{u}, \qquad \lambda^{(4,5)} = \frac{uv \pm d^2\left(\frac{M^2(1-b\rho)}{\epsilon} - 1\right)^{1/2}}{(u^2-d^2)}, \tag{2.8}$$

and left eigenvectors corresponding to these eigenvalues are

$$\begin{cases} L^{(1)} = \begin{pmatrix} 1 & 0 & 0 & 0 & -\frac{1}{C^2} \end{pmatrix}, \\ L^{(2)} = \begin{pmatrix} 0 & 1 & \frac{v}{u} & 0 & \frac{\epsilon}{\rho u} \end{pmatrix}, \\ L^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1-\epsilon \end{pmatrix}, \\ L^{(4)} = \begin{pmatrix} 0 & 1 & -\frac{u}{v} & 0 & -\frac{1}{\rho v}\left(\frac{M^2(1-b\rho)}{\epsilon} - 1\right)^{1/2} \end{pmatrix}, \\ L^{(5)} = \begin{pmatrix} 0 & 1 & -\frac{u}{v} & 0 & \frac{1}{\rho v}\left(\frac{M^2(1-b\rho)}{\epsilon} - 1\right)^{1/2} \end{pmatrix}, \end{cases} \tag{2.9}$$

where M, defined as $M^2 = \frac{q^2}{a^2}$, is called upstream flow Mach number.

From equation (2.8 – 2.9), it is clear that the system (2.6) has five real eigenvalues for super sonic flow ($M > 1$). Therefore, the system (2.6) is hyperbolic in nature. There are two families of characteristics curves along $\frac{dy}{dx} = \lambda^{(4,5)}$. Therefore, the waves propagate along the characteristic curve $\frac{dy}{dx} = \lambda^{(4,5)}$ in opposite directions with speeds $\lambda^{(4,5)}$, respectively.

2.4 Transport equation for shock wave

In this section, we drive the transport equations for shock wave in W as they move along the wavefront $\zeta = 0$. On introducing a new curvilinear co-ordinates ζ, y' which are defined by [3]

$$\begin{aligned} &\zeta_x + \lambda^{(4)}\zeta_y = 0, \\ &\zeta\left(x, y_0\right) = x - x_0, \end{aligned} \tag{2.10}$$

and $y = y'$. Then ζ has the required co-ordinate property that ζ is positive (negative) ahead (behind) of the wavefront $\zeta = 0$.

On pre-multiplying by $L^{(i)}$, system (2.6) can be written in terms of these new co-ordinates as

$$\begin{aligned} & L^{(i)}W_x + L^{(i)}A(W)W_y + L^{(i)}F = 0, \\ \text{or} \quad & L^{(i)}W_\zeta\zeta_x + L^{(i)}A(W)\left(W_\zeta\left(-\frac{\zeta_x}{\lambda^4}\right) + W_{y'}\right) + L^{(i)}F = 0, \\ \text{or} \quad & L^{(i)}W_\zeta + \frac{\lambda^{(i)}\lambda^{(4)}}{\lambda^{(4)} - \lambda^{(i)}}\frac{1}{\zeta_x}L^{(i)}W_{y'} + \frac{\lambda^{(4)}}{\lambda^{(4)} - \lambda^{(i)}}\frac{1}{\zeta_x}L^{(i)}F = 0, \\ \text{or} \quad & L^{(i)}W_\zeta + \frac{\lambda^{(i)}\lambda^{(4)}}{\lambda^{(4)} - \lambda^{(i)}}x_\zeta L^{(i)}W_{y'} + \frac{\lambda^{(4)}}{\lambda^{(4)} - \lambda^{(i)}}x_\zeta L^{(i)}F = 0, \end{aligned} \tag{2.11}$$

where x_ζ is the Jacobian of the transformation that is $x_\zeta = \frac{1}{\zeta_x}$.

The vectors W and $W_{y'}$ are continuous across the wavefront $\zeta = 0$ and have their subscripts-0 valued whilst W_ζ and x_ζ are not continuous. On computing (2.11) at the back side of the wavefront $\zeta = 0$ for $i = 1, 2, 3, 5$, yields

$$\rho_\zeta = \frac{1}{C_0^2}p_\zeta, \tag{2.12}$$

$$u_\zeta = -\frac{\epsilon_0}{\rho_0 u_0}p_\zeta, \tag{2.13}$$

$$h_\zeta = \left(\epsilon_0 - 1\right)p_\zeta, \tag{2.14}$$

$$v_\zeta = \frac{1}{\rho_0 u_0}\left(\frac{M_0^2\left(1 - \bar{b}\right)}{\epsilon_0} - 1\right)^{1/2} p_\zeta. \tag{2.15}$$

Where $\bar{b} = b\rho$. Now, we set $i = 4$ in (2.11) and differentiate the producing equation w.r.t. ζ, then evaluate it at the back side of wavefront $\zeta = 0$, we get

$$d_0 a_0 \epsilon_0^{(1/2)} \left(\frac{\left(1-\bar{b}\right) M_0^2}{\epsilon_0} - 1 \right)^{1/2} p_{\zeta y'} + d_0 a_0 \epsilon_0^{(1/2)} u_0 \rho_0 v_{\zeta y'} + \frac{\rho_0 n u_0 C_0^2}{y'} \left(1 - \bar{b}\right)^{1/2} v_\zeta = 0. \tag{2.16}$$

Using (2.15) in (2.16), we have

$$p_{\zeta y'} + \frac{n}{2y'\epsilon_0} p_\zeta = 0. \tag{2.17}$$

As well, along $\zeta =$ constant, we have

$$x_{y'} = \frac{u^2 - d^2}{uv + d^2 \left(M_0^2 \left(1 - \bar{b}\right)/\epsilon_0 - 1\right)^{1/2}}. \tag{2.18}$$

Differentiating (2.18) w.r.t. ζ and evaluate at the back side of $\zeta = 0$, yields

$$x_{\zeta y'} = \frac{2\epsilon_0 \left(1 - \epsilon_0\right) - \left(\gamma + \epsilon_0 + \bar{b}\left(1 - \epsilon_0\right)\right) M_0^2}{2\rho_0 \epsilon_0 d_0^2 \left(\frac{M_0^2 \left(1-\bar{b}\right)}{\epsilon_0} - 1 \right)^{1/2}} \left(\frac{y_0}{y'}\right)^{(n/2\epsilon_0)}. \tag{2.19}$$

Here, (2.17) and (2.18) are transport equations.

2.5 Steepening of shock wave

On integrating (2.17) w.r.t. y', we get

$$\int \frac{dp_\zeta}{p_\zeta} = -\int \frac{m\epsilon_0^{-1}}{2} \frac{dy'}{y'}$$

$$\text{or} \quad p_\zeta = \left(\frac{y_0}{y'}\right)^{\frac{n}{2\epsilon}} p_{\zeta_0}, \tag{2.20}$$

where $p_{\zeta_0} = \lim\limits_{y' \to\, y_0} p_\zeta$, along with $\zeta = 0$.

Putting the value of p_ζ from (2.20) into (2.19) and on integrating, we obtained

$$\begin{aligned} x_\zeta &= 1 + \frac{2\epsilon_0\left(1-\epsilon_0\right) - M_0^2\left(\gamma + \epsilon_0 + \bar{b}\left(1-\epsilon_0\right)\right)}{2\epsilon_0\rho_0 d_0^2\left(\frac{M_0^2\left(1-\bar{b}\right)}{\epsilon_0} - 1\right)^{1/2}}\left(y_0\right)^{n/2\epsilon_0} p_{\zeta_0}\int_{y_0}^{y} s^{(-n/2\epsilon_0)}ds, \\ \text{or}\;\; x_\zeta &= 1 + \frac{2\epsilon_0\left(1-\epsilon_0\right) - M_0^2\left(\gamma + \epsilon_0 + \bar{b}\left(1-\epsilon_0\right)\right)}{2\epsilon_0\rho_0 d_0^2\left(\frac{M_0^2\left(1-\bar{b}\right)}{\epsilon_0} - 1\right)^{1/2}}\left(y_0\right)^{n/2\epsilon_0} p_{\zeta_0}\frac{y^{1-n/2\epsilon_0} - y_0^{1-n/2\epsilon_0}}{1-n/2\epsilon_0}, \end{aligned} \tag{2.21}$$

where s is a variable and the boundary condition (2.10) and

$$x_{\zeta_0} = x_\zeta\mid_{\zeta=0^-} = x_\zeta\mid_{\zeta=0^+} = 1.$$

Suppose $y = Y\left(x\right)$ is the equation of body contour with tangent at the tip of body edge and parallel to the velocity of the stream line, we have

$$\frac{dy}{dx} = \frac{v}{u}. \tag{2.22}$$

Taking derivative of (2.22) w.r.t. ζ at the rear of $\zeta = 0$, we have

$$v_{\zeta_0} = u_0 Y_0'', \tag{2.23}$$

where Y_0'' is curvature at the tip of the body.

Using (2.15) and (2.23), (2.21) can be written as

$$\begin{aligned} x_\zeta &= 1 + \tfrac{2\epsilon_0(1-\epsilon_0) - M_0^2\left(\gamma+\epsilon_0+\bar{b}(1-\epsilon_0)\right)}{2\epsilon_0\left(M_0^2\left(1-\bar{b}\right)-\epsilon_0\right)}\left(y_0\right)^{n/2\epsilon_0} M_0^2\left(1-\bar{b}\right)Y_0''\textstyle\int_{y_0}^{y} s^{(-n/2\epsilon_0)}ds, \\ \text{or}\;\; x_\zeta &= 1 + \tfrac{2\epsilon_0(1-\epsilon_0) - M_0^2\left(\gamma+\epsilon_0+\bar{b}(1-\epsilon_0)\right)}{2\epsilon_0\left(M_0^2\left(1-\bar{b}\right)-\epsilon_0\right)}\left(y_0\right)^{n/2\epsilon_0} M_0^2\left(1-\bar{b}\right)Y_0'' \times \tfrac{y^{1-n/2\epsilon_0} - y_0^{1-n/2\epsilon_0}}{1-n/2\epsilon_0}. \end{aligned} \tag{2.24}$$

If the Jacobian x_ζ vanishes for some $y = y_t$, on the wavefront $\zeta = 0$ the characteristics of the family of neighbouring $\zeta =$ constant must intersect on the wavefront $\zeta = 0$ and results in a discontinuity which is known as shock wave, occur in the solution vector W. If, we assume W_ζ is finite at $y = y_t$ as $x_\zeta = 0$, for then, just at the back of the wave front $\zeta = 0$, $W_x = \frac{W_\zeta}{x_\zeta}$ becomes infinite. This phenomenon is known as the steepening of the wavefront. A brief discussion of result (2.24) for planar $(n = 0)$ and axisymmetric $(n = 1)$ cases is discussed in next section.

2.6 Results and discussion

Now, we will discuss the results obtained in this study for a planar and axisymmetric non-ideal magnetogasdynamic flow. As described above, we discuss supersonic flow for these two cases; one is planar $(n = 0)$ and another is a axisymmetric case $(n = 1)$.

2.6.1 Planar case $(n = 0)$

For the planar case, putting $n = 0$ in (2.24) and assuming the body contour $y = Y_b(x)$, (plane beak) , the initial disturbance is assumed to be released from a sharp edge of the contour with a vanishing small initial tangent. Then (2.24) can be written as

$$x_\zeta = 1 - \frac{Y_b''(0)\,(y - y_0)}{\phi}, \tag{2.25}$$

where

$$\phi = 2\left(M_0^2\left(1 - \bar{b}\right) - \epsilon_0\right)\left(\left(\frac{M_0^2}{\epsilon_0}\left(\gamma + \epsilon_0 + \bar{b}\,(1 - \epsilon_0)\right) - 2\,(1 - \epsilon_0)\right)\left(M_0^2\left(1 - \bar{b}\right)\right)\right)^{-1}.$$

Here $\phi > 0$ and $Y_b''(0)$ is the value of radius of curvature at which the body contour initiates to bend.

As discussed previously, the vanishing of x_ζ describes the occurance of a shock wave. We see that in (2.25), the Jacobian will vanish on the leading wavefront for $y_0 < y$ only when $Y_b''(0) > 0$ with $Y_b''(0) > \phi$. Now, for $Y_b''(0) \leq \phi$, the Jacobian remains positive for $y_0 < y$ (finite) and there will not form any shock on the ahead of a wavefront. Thus, ϕ represents a critical level such that when this level is exceeded by $Y_b''(0)$, a discontinuity will form at a finite distance away from the body. It may be noticed that at $\zeta = 0$, v_x and v_ζ are related to each other by the relation $v_x = v_\zeta / x_\zeta$; therefore, it is inconsequential, and we get an expression for v_x and v_ζ at $\zeta = 0$. Since v_x has slightly more physical explanation, we shall choose to work in terms of that quantity for planar case, we have

$$v_x = \frac{d_0 M_0 \left(1 - \bar{b}\right)^{1/2} Y_b''(0)}{\epsilon_0^{1/2} \left(1 - \frac{Y_b''(0)}{\phi} (y - y_0)\right)}. \tag{2.26}$$

From the above equation, the evolutionary process of shock wave can be derived and it is evident from (2.26) when $Y_b''(0)$ is positive, and the modulus value of $Y_b''(0)$ is greater than ϕ, that is, $\mid Y_b''(0) \mid > \phi$, then the wave terminates into the shock and the corresponding equation of shock formation distance can be written as

$$y_t = y_0 + \frac{\phi}{Y_b''(0)}. \tag{2.27}$$

The equation (2.27) is derived from (2.26) when numerator remains finite, and its denominator is zero. that is at the wavefront $\zeta = 0$, the velocity gradient will be unbounded at $y = y_t$. The coincidence of this behavior with the vanishing of x_ζ is

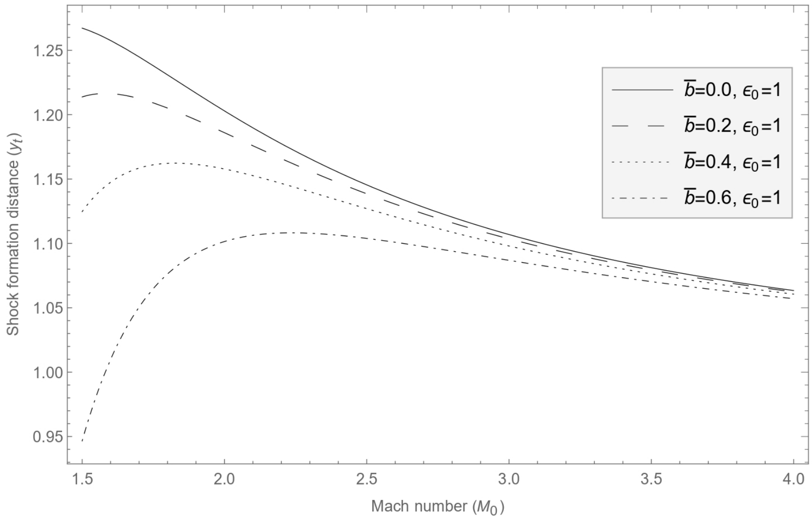

FIGURE 2.1: Effect of non-ideal parameter and Mach number M_0 on shock formation distance y_t for planar non-magnetic case $(n = 0)$ with $\gamma = 1.67$.

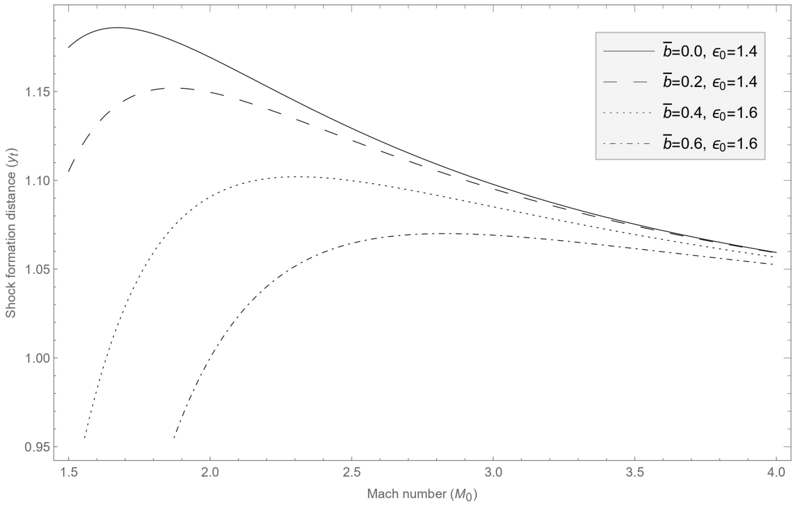

FIGURE 2.2: Effect of non-ideal parameter and Mach number M_0 on shock formation distance y_t for planar magnetic case $(n = 0)$ with $\gamma = 1.67$.

clear from (2.27) which causes the steepening of waves into the shock wave. Further, for the case $Y_b''(0) \leq \phi$, steepening of the velocity gradient does not occur while wave is still compressive. On the contrary we can say that either v_ζ decreases along the wave head or remains constant if $Y_b''(0) < \phi$ or $Y_b''(0) = \phi$.

From Fig.2.1, we can see that an increase in the value of upstream flow Mach number M_0 causes to decrease the shock formation distance y_t which means an increase in

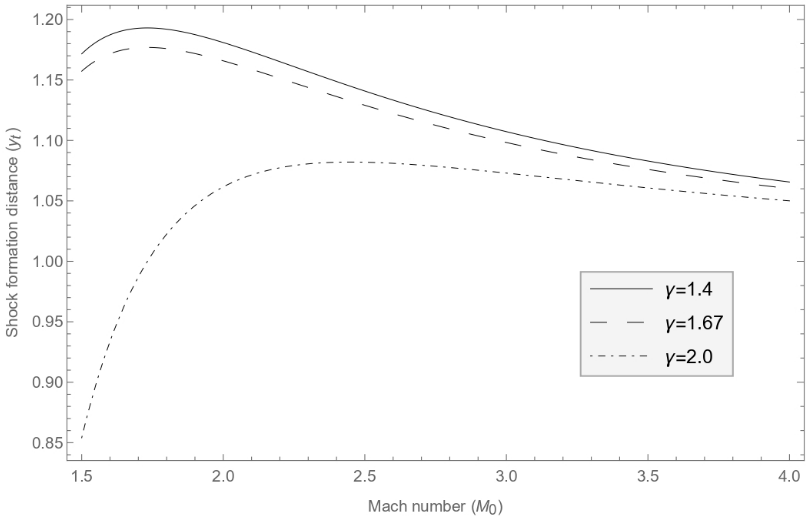

Figure 2.3: Effect of γ on shock formation distance for planar non-ideal gas flow with magnetic field.

the value of upstream flow Mach number M_0 causes early shock formation. The parameter $\bar{b} = 0$ indicates the ideal gas and $\epsilon_0 = 1$ is used for non-magnetic case. Therefore, it is noticeable that an increase in the value of non-ideal parameter $\bar{b}$ decreases the shock formation distance in absence of magnetic field. From Fig.2.2, it is obtained that in the presence of magnetic field, the shock formation distance decreases with an increase in the value of non-ideal parameter $\bar{b}$ and magnetic field parameter ϵ_0 for planar flow case. From Fig.2.1 and Fig.2.2, we observe that the shock formation distance for non-magnetic case decreases more rapidly compared to magnetic case and decay rate of y_t becomes slow as we increase the value of magnetic parameter. Also, the presence of non-idealness in fluid slow down the decay rate of y_t in both magnetic and non-magnetic cases. It means, shock occurs early in non-ideal magnetogasdynamic flow as compared to ideal gasdynamic flow. It is observed that y_t also decreases on increasing the value of specific heat ratio ($\gamma = 1.4, 1.67, 2.0$) for planar non-ideal gas flow under the action of magnetic field, shown in Fig.2.3.

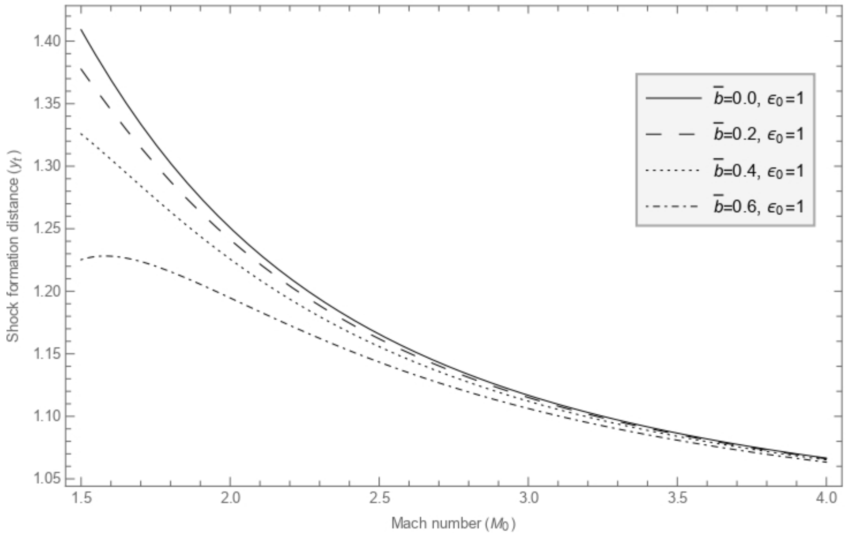

FIGURE 2.4: Effect of non-ideal parameter and Mach number M_0 on shock formation distance y_t for axisymmetric non-magnetic case ($n = 1$) with $\gamma = 1.67$.

2.6.2 Axisymmetric case ($n = 1$)

In axisymmetric case, we assume that $y = Y_t(x)$ which represents ring-shaped body with sharp-edged inlet releasing the initial discontinuity, which runs outwards and inwards both along characteristics lines. For $y_0 < y$, similar process is seen as in planar flow. Now putting $n = 1$ into (2.24), we obtain

$$x_\zeta = 1 - J_0 Y_t''(0)\left(y^{\left(\frac{2\epsilon_0-1}{2\epsilon_0}\right)} - y_0^{\left(\frac{2\epsilon_0-1}{2\epsilon_0}\right)}\right), \tag{2.28}$$

where

$$J_0 = \frac{\left(M_0^2\left(\gamma + \epsilon_0 + \bar{b}\left(1-\epsilon_0\right)\right) - 2\epsilon_0\left(1-\epsilon_0\right)\right)}{2\epsilon_0\left(M_0^2\left(1-\bar{b}\right) - \epsilon_0\right)} y_0^{1/2\epsilon_0} M_0^2\left(1-\bar{b}\right)\left(\frac{2\epsilon_0}{2\epsilon_0 - 1}\right). \tag{2.29}$$

In view of (2.28), we observe that for $y_0 < y$, the right hand side of (2.28) lies between zero and one. Thus, the Jacobian x_ζ will vanish, which leads to the occurrence of a

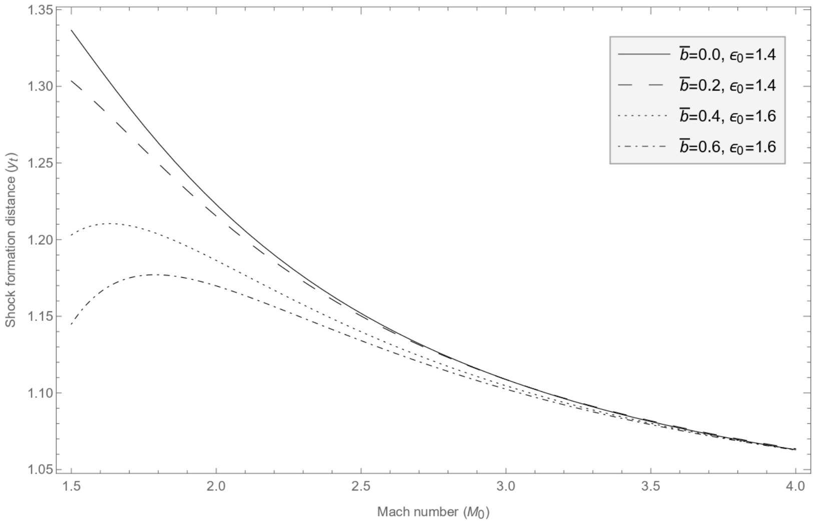

FIGURE 2.5: Effect of non-ideal parameter and Mach number M_0 on shock formation distance y_t for axisymmetric magnetic case ($n = 1$) with $\gamma = 1.67$.

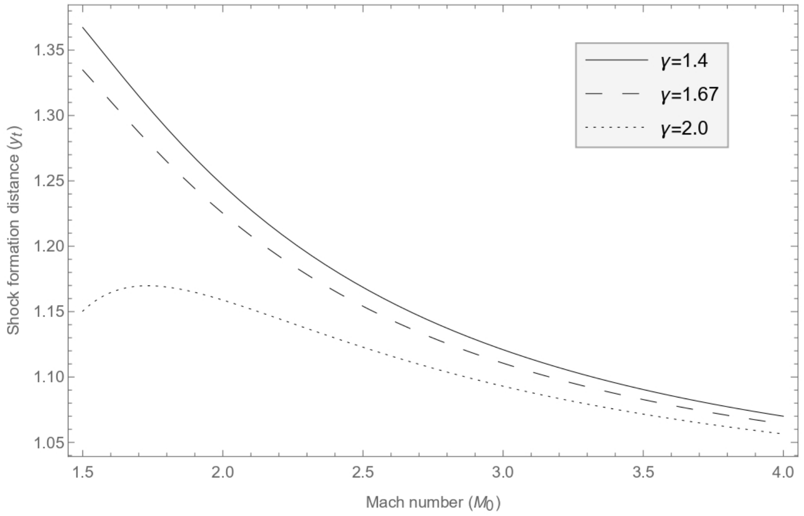

FIGURE 2.6: Effect of γ on shock formation distance for axisymmetric non-ideal gas flow with magnetic field.

shock wave, provided $Y_t''(0) > 0$ and $Y_t''(0) > J_0^{-1}$. If $Y_t''(0) \leq J_0^{-1}$, $x_\zeta > 0$, there will not occur any shock wave on the leading wavefront. Hence, one can observe that the formation of a shock will take place only when $Y_s''(0) > J_0^{-1}$, so for the ordinate $y = y_t$, the equation for the shock formation distance when the first shock

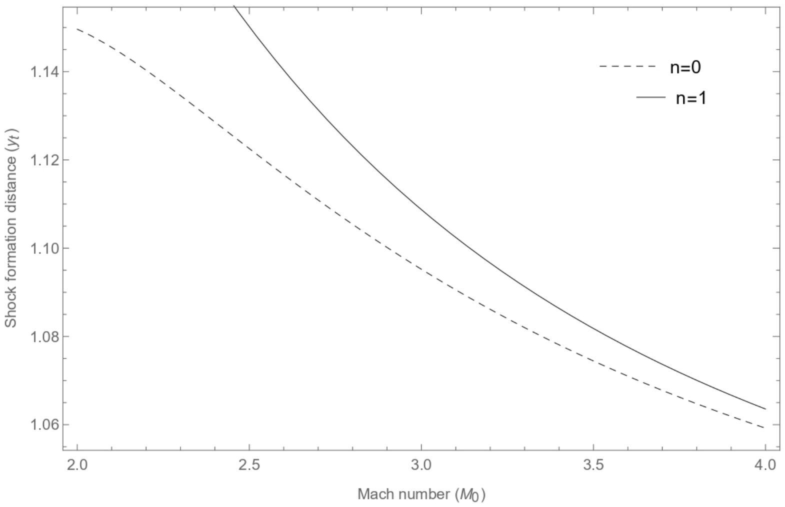

FIGURE 2.7: Comparison of decay rate of shock formation distance for the cases $n = 0$ and $n = 1$ in non-ideal gas flow with magnetic field.

builts, can be written as

$$y_t = \left(y^\alpha + \frac{1}{J_0 Y_t''(0)} \right)^{1/\alpha}, \tag{2.30}$$

where $\alpha = \frac{2\epsilon_0 - 1}{2\epsilon_0}$. From (2.28) and (2.30), it is clear that y_t depends on M_0 and the initial body curvature which is either $\mid Y_b''(0) \mid^{-1}$ or $\mid Y_t''(0) \mid^{-1}$. The increase in the value of M_0 is a reason to decrease the value of y_t in the region, which can be seen from (2.27).

For axisymmetric case, we study the following results related to the effect of non-ideal and magnetic field parameters. From Fig.2.4 − 2.6, it is clear that y_t is a decreasing function of M_0 and decreases the value of y_t on increasing the value of M_0. From Fig.2.4, it is obtained that the presence of non-ideal parameter in the fluid decreases y_t in absence of magnetic field. While it has same behavior in the presence of magnetic field which is shown in Fig.2.5. Also, from Fig.2.4 and Fig.2.5, it is noticeable that y_t for non-magnetic case decreases more rapidly as compared to magnetic case and decay rate of y_t becomes slow if we increase the value of magnetic parameter for axisymmetric flow case. Ultimately, it is observed that y_t decreases with an increase in the value of non-ideal parameter $\bar{b}$ and magnetic field parameter ϵ_0 for axisymmetric flow case which is same as in planar flow. On increasing the value of γ and M_0, we also observe the decreasing trend in y_t which is shown in Fig.2.6.

In Fig.2.7, we analyze the difference of the shock formation for both planar $(n = 0)$ and axisymmetric $(n = 1)$ cases in non-ideal magnetogasdynamic flow. We observed that an increase in the value of M_0 have effect to decrease the value of y_t for both cases together with fixed value of all parameters of non-ideal magnetogasdynamic flow. From Fig.2.7, it is clear that in case of axisymmetric flow, shock will occur early in comparison to planar flow case.

2.7 Conclusion

In this study, the conditions of the occurrence of shock wave and the relation for shock formation distance in two-dimensional supersonic planar and axisymmetric flows of non-ideal gas under the influence of magnetic field, are investigated.The transport equations for the governing equations which insure that there would not evolve any shock on the wavefront is obtained. Further, the effect of upstream flow Mach number, non-ideal parameter, specific heat ratio and magnetic field parameter on shock formation distance is obtained for both planar and axisymmetric cases. It is obtained that an increase in the value of the magnetic field parameter and non-ideal parameter causes to decrease y_t. The analysis shows that an increase in the value of M_0 causes to decrease the value of y_t for both planar and axisymmetric flows. Also, it is observed that in axisymmetric case, there is an early shock formation as compared to planar case on increasing the value of M_0. The results obtained in this study are in close agreement with the results reported by the authors in [80, 82, 83].