Chapter 6

Summary and Future scope

6.1 Summary

This thesis focuses on the investigation of non-linear wave propagation and its characteristics across different gas regimes. It examines the fundamental aspects of waves traveling under a quasi-linear system of hyperbolic partial differential equations (PDEs) through various analytical techniques applied to different material media. The impact of various differential constraints on the behavior of the propagating waves is thoroughly analyzed. The key highlights of the thesis are as follows:

- Investigation of shock wave solution for planar, cylindrically, and spherically symmetric flows of non-ideal flows of relaxing gas.
- The study of evolutionary behavior of weak discontinuity waves in non-ideal interstellar environments.
- Examination of the behavior of shock wave in two dimensional planar and axisymmetric non-ideal dusty gas flow under the influence of magnetic field.

- Investigation of self-similar solutions for imploding shock waves in non-ideal gravitational field with equation of state as Mie-Gruneisen type using numerical description of flow field.

The initial chapter serves as an introduction, providing a comprehensive overview of research conducted in the area of non-linear wave propagation, along with its applications and methodologies. It also offers a concise discussion of the underlying mathematical theories and essential properties. The chapter outlines the key characteristics of non-linear waves and their propagation. Additionally, it includes a brief review of the physical properties of non-ideal gases, dusty gases, and magnetogasdynamics gases relevant to this study. Furthermore, certain results that will be necessary for the following chapters are presented in this section.

In the second chapter, we have studied the impact of various parameters on the propagation of shock waves through the method of characteristics. We derived analytical solutions for the quasi-linear system of hyperbolic partial differential equations and analyzed the evolutionary behavior of shocks within the characteristic plane. A comprehensive examination of the solutions to the Bernoulli equation, which governs the evolution of shock waves in a non-linear context, is presented. Additionally, we address the formation and distortion of compressive and expansive discontinuities in planar, cylindrical, and spherical symmetric flows of non-ideal relaxing gases. The effects of non-ideal characteristics on the comparison between planar and non-planar flows are also investigated.

The third chapter presents the study of the evolutionary behavior of planar and non planar weak discontinuity waves along the characteristic path in non-ideal interstellar environments. Analytical solutions of the quasi-linear system of hyperbolic PDEs are obtained and the transport equation leading to the evolution of acceleration waves is determined, which provides the relation for the occurrence of shock. The general behavior of solutions to the Bernoulli equation, which determines the evolution of shock wave in a non-linear system, is studied in detail. The impact of non-idealness and self gravitating parameter are discussed.

In the fourth chapter, we investigate the impact of dust particles on the propagation of shock waves within a magnetic field, using the wavefront analysis method. Our focus is on the behavior of waves traveling through a two-dimensional (2-D) steady supersonic magnetogasdynamic flow of non-ideal dusty gas. We explore the effects of non-idealness with dust particles and their implications under magnetic influence, analyzing how the disturbance flow patterns shift in response to changes in flow parameters. Additionally, we assess the relationship between the solution characteristics and the Mach number, specifically examining how variations in the Mach number affect the distance at which shock formation occurs.

In the fifth chapter, we explored self-similar solutions of the second kind for imploding shock waves in unsteady one-dimensional flow occurring behind strong shock waves in a non-ideal gas influenced by a gravitational field. We adopted the Mie-Gruneisen equation of state for the medium. A numerical approach was employed to ascertain the similarity exponent. We provided a numerical characterization of the flow field within the context of a non-ideal gravitational field. The findings were compared with the numerical solutions obtained through the CCW approximation method. Detailed analyses were performed for two distinct non-ideal media that are physically significant in the presence of a gravitational field.

6.2 Future Scope

In this thesis, we addressed several issues related to non-linear wave propagation as described by the Euler system of partial differential equations (PDEs). A significant contribution of our research is the examination of wave behavior across different gas regimes, along with an analysis of how various parameters influence the solutions derived through analytical methods. This section outlines future work to further enhance the findings presented in this thesis. Our investigation is limited to a one-dimensional framework of non-linear partial differential equations within the field of gas dynamics. Nevertheless, this analysis can be expanded to encompass two-dimensional or higher-dimensional non-linear partial differential equations in gas dynamics. Key areas for future research are highlighted here, including a more in-depth examination of specific mechanisms, the introduction of new methodologies, or simply the pursuit of knowledge driven by curiosity. Below are some key points regarding the proposed extensions of the work conducted in this thesis:

- In this thesis, we concentrated exclusively on one-dimensional wave propagation across different gas regimes and examined how various parameters influence wave behavior. Additionally, one could investigate wave interaction issues within diverse gas regimes, which are governed by a quasilinear hyperbolic system of partial differential equations.

- In this thesis, we examined the propagation of waves in dusty gases, assuming that the velocity of the dust particles is negligible. The particles were selected to be of uniform size, and interactions between the dust particles were disregarded. This study can be extended to encompass two-phase flow, where we also take into account the density of the particles.

- Throughout the thesis, we have employed various analytical methods to investigate wave propagation phenomena. Numerous numerical techniques exist that allow for the examination of this phenomenon across different systems of hyperbolic partial differential equations (PDEs).

- Study of the Riemann problem for Chaplygin gas.

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