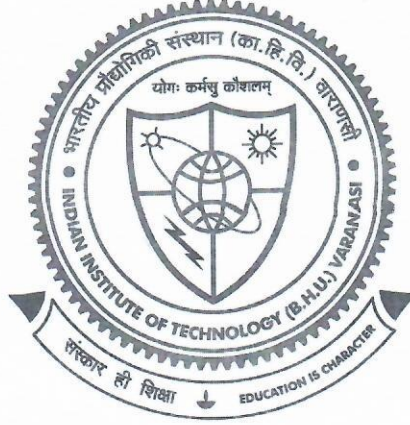

SOME INVESTIGATIONS INVOLVING NON-LINEAR
WAVE PROPAGATION PHENOMENON IN GAS
DYNAMICS



*The thesis submitted in partial fulfillment
for the Award of Degree*

DOCTOR OF PHILOSOPHY

by

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CERTIFICATE

It is certified that the work contained in this thesis titled "**SOME INVESTIGATIONS INVOLVING NON-LINEAR WAVE PROPAGATION PHENOMENON IN GAS DYNAMICS**" by **Shweta** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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I, **Shweta**, certify that the work embodied in this thesis is my own bonafide work and carried out by me, under the supervision of **Dr. L. P. Singh** from **July, 2019** to **May, 2025** at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, thesis, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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*This thesis is dedicated
with love and deepest gratitude*

To my lovable
Mummy and Papa

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List of Figures

2.1	Effect of relaxing gas parameter R_0 for $\delta < 0$ with $m = 0, \gamma = 1.67, \delta = -0.5, \bar{b} = 0.2$	39
2.2	Effect of relaxing gas parameter R_0 for $\delta > 0$ with $m = 0, \gamma = 1.67, \delta = 0.5, \bar{b} = 0.2$	39
2.3	Impact of non-ideal parameter \bar{b} for $\delta < 0$ in planar relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$	40
2.4	Impact of non-ideal parameter \bar{b} for $\delta > 0$ in planar relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$	41
2.5	Impact of non-ideal parameter \bar{b} for $\delta < 0$ in cylindrically symmetric flow of relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$	42
2.6	Impact of non-ideal parameter \bar{b} for $\delta > 0$ in cylindrically symmetric flow of relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$	42
2.7	Impact of non-ideal parameter \bar{b} for $\delta < 0$ in spherically symmetric flow of relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$	43
2.8	Impact of non-ideal parameter \bar{b} for $\delta > 0$ in spherically symmetric flow of relaxing gas ($R_0 = 1.0$) and non-relaxing gas ($R_0 = 0.0$) with $\gamma = 1.67$	43
2.9	Comparative analysis of the geometry of non-ideal relaxing gas flow for $\gamma = 1.67, \delta = -1, \bar{b} = 0.2$	44
3.1	Effect of self gravitating parameter \bar{g}_0 for $\delta < 0$ with $\gamma = 1.67, \delta = -1.5, m = 0$ (<i>planar case</i>), $\bar{b} = 0.3$	58
3.2	Effect of self gravitating parameter \bar{g}_0 for $\delta > 0$ with $\gamma = 1.67, \delta = 0.5, m = 0$ (<i>planar case</i>), $\bar{b} = 0.3$	60
3.3	Effect of self gravitating parameter \bar{g}_0 for $\delta < 0$ with $\gamma = 1.67, \delta = -1.5, m = 1$ (<i>cylindrical case</i>), $\bar{b} = 0.3$	61
3.4	Effect of self gravitating field \bar{g}_0 for $\delta > 0$ with $\gamma = 1.67, \delta = 0.5, m = 1$ (<i>cylindrical case</i>), $\bar{b} = 0.2$	63
3.5	Effect of net volumetric cooling rate \bar{L}_0 for $\delta < 0$ with $\delta = -0.5, \gamma = 1.67, m = 1$ (<i>cylindrical case</i>), $\bar{b} = 0.3$	63

3.6	Effect of net volumetric cooling rate \bar{L}_0 for $\delta > 0$ with $\delta = 0.5, \gamma = 1.67, m = 1$ (<i>cylindrical case</i>), $\bar{b} = 0.3$	64
3.7	Effect of non-ideal parameter \bar{b} for $\delta < 0$ with $\bar{L}_0=0.4, \bar{g}_0=0.2, m=1$ (<i>cylindrical case</i>), $\delta=-0.5$	65
3.8	Effect of non-ideal parameter \bar{b} for $\delta > 0$ $\bar{L}_0=0.2, \bar{g}_0=0.8, m=1$ (<i>cylindrical case</i>), $\delta=0.5$	65
3.9	Effect of self gravitating parameter \bar{g}_0 for $\delta < 0$ with $\gamma = 1.67, \delta = -1.5, m = 2$ (<i>spherical case</i>), $\bar{b} = 0.3$	67
3.10	Effect of self gravitating parameter \bar{g}_0 for $\delta > 0$ with $\gamma = 1.67, \delta = 0.5, m = 2$ (<i>spherical case</i>), $\bar{b} = 0.3$	67
3.11	Comparative study of the geometry of non-ideal interstellar gas cloud for $\gamma = 1.67, \delta = -1.5, \bar{b} = 0.3, \bar{g}_0 = 0.5$	68
3.12	Comparative study of the geometry of non-ideal interstellar gas cloud for $\gamma = 1.67, \delta = 0.5, \bar{b} = 0.3, \bar{g}_0 = 0.5$	68
4.1	Convergence of the characteristics for planar and axisymmetric supersonic flow.	82
4.2	Influence of dust particles and magnetic field in non-ideal gas on y_r with $\gamma = 1.67, Z = 0.01, \beta = 0.8$	84
4.3	Influence of dust particles and magnetic field in non-ideal gas on y_r with $\gamma = 1.67, Z = 0.01, \beta = 0.8$	85
4.4	Comparative study of geometry of ideal ($\bar{b} = 0$) dusty magneto-gasdynamics for $\gamma = 1.67, Z = 0.01, \beta = 0.8, \tilde{Y}_s''(0) = 0.7$	87
4.5	Comparative study of geometry of non-ideal ($\bar{b} = 0.5$) dusty magneto-gasdynamics for $\gamma = 1.67, Z = 0.01, \beta = 0.8, \tilde{Y}_s''(0) = 0.7$	87
5.1	Density profile for $K= 1.4, M=5, Z_0 = 0.004, 0.01, 0.02, 0.1$ and $\gamma = 1.4$	102
5.2	Pressure profile for $K= 1.4, M=5, Z_0 = 0.004, 0.01, 0.02, 0.1$ and $\gamma = 1.4$	103
5.3	Velocity profile for $K= 1.4, M=5, Z_0 = 0.004, 0.01, 0.02, 0.1$ and $\gamma = 1.4$	103
5.4	Density profile for $\gamma = 4/3, M = 5, \Theta_0 = 1.78, 2.19, 2.19, 2.19, D_m = 0.5, 0.5, 0.65, 0.80$	106
5.5	Pressure profile for $\gamma = 4/3, M = 5, \Theta_0 = 1.78, 2.19, 2.19, 2.19, D_m = 0.5, 0.5, 0.65, 0.80$	106
5.6	Velocity profile for $\gamma = 4/3, M = 5, \Theta_0 = 1.78, 2.19, 2.19, 2.19, D_m = 0.5, 0.5, 0.65, 0.80$	107