Chapter 3

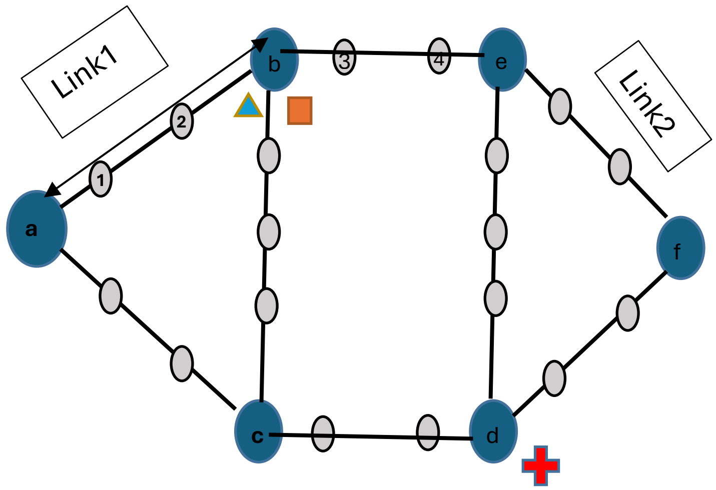
Vulnerability Analysis and Evaluation of Relative Importance of Links in a rail network.

3.1. Introduction and Problem Description :

In this chapter, we examine the network's susceptibility to specific types of accidents and underscore the significance of individual links concerning the likelihood of particular types of accident occurrences. The analysis of vulnerability focuses on the repercussions stemming from system component failures when estimating relief demand probabilities becomes impracticable. These repercussions typically pertain to human life, public health, and environmental integrity. When monetized, these consequences can further be articulated as operational costs, network disruptions, and broader impacts on the interconnected system.

The railway network considered in our problem can be represented as an undirected Graph $G = (N, A)$ where N represents the set of all nodes (i.e., stations) in the network and A represents the set of all arcs connecting the stations. Let N represent the set of stations that are normal stations or junctions. Then, we have a series of continuous arcs connecting the two possible nodes in the set N and such a combination of arcs are termed as 'links' in this work. A link thus represents a set of nodes/stations in a segment. A symbolic railway network in our study is as shown in Figure 3.1.

Each link will have unique characteristics in terms of track specifications, the flow of passenger and freight traffic, and vulnerability. So, based on the above factors the importance of link varies in the network.



Legend		
1 Node		ART
a Potential station		AMRV
		Crane

Figure 3.1 Symbolic Diagram of Railway Network.

The challenge of the problem is to identify the relative importance of the track link with respect to other links in the network. In this chapter, we conceptualize network vulnerability in terms of its predisposition to encounter specific railway accidents, contingent upon the type of service it facilitates. This encompasses the transportation of goods, passenger traffic, empty wagons, and other departmental or irregular miscellaneous traffic. A network link serves to transport one or a combination of these traffic types from one node to another. Consequently, the vulnerability of a link assumes paramount importance in its academic characterization and warrants further investigation. The evaluation of importance of a link has been undertaken with the help of combination of popular 'Multi Criteria Decision Making' techniques, namely 'Analytical Hierarchy Process' (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS).

The rest of the chapter is organized as follows. The concept of MCDM techniques AHP is presented in Section 3.2. Section 3.3 presents the concept of 'TOPSIS' . The calculation of importance of the rail link is demonstrated and results of the computational experiments are presented in Section 3.4. Finally, Section 3.5 concludes the chapter.

3.2 Analytical Hierarchy Process :

AHP was developed by (Saaty 1990). It is a MDODM technique used to rank a set of predefined alternatives. It is a particularly useful method when the decision maker is unable to construct a utility function. In this process the decision makers are asked to specify the preferences among the various alternatives on each criteria and the among criterion themselves by comparing the alternatives in pair wise manner. The decision maker is requested to specify his preference over other criterion on a ratio scale. The 'N' point ratio scale is used in AHP where each point represents a level of importance of one factor over another in a pair wise comparison scheme. The end points on a ratio scale represent the equal importance at point 1 and maximum importance at the point n respectively.

For example, assume two alternative objectives A and B, the decision maker may consider A and m times more important than objective B. This implies that the decision maker considers B as 1/m times important than A. In this fashion if there are p objectives, we can get a matrix (M) of $p * p$ order for the responses of the decision maker.

This matrix will have its diagonal elements as 1 since the relative importance of any objective with itself is 1.

Let the comparison factors f_i and f_j on the ration scale be S_{ij}, where $i,j = 1,2,3....p$. Then the elements of this normalized matrix M_{norm} are given by –

$$S'_{ij} = \frac{S_{ij}}{\sum_{i=1}^{p} S_{ij}}, \forall\, j = 1,2, \ldots\ldots p$$

The normalized weights w_i of each objective f_i is given by –

$$\frac{\sum_{j=1}^{p} S'_{ij}}{p}, \forall\, j = 1,2, \ldots\ldots p.$$

Which can be represented by $1\ x\ p$ weight matrix w.

When the matrix is complete, a consistency check may be performed to detect the possibility of contradictions in the responses. In the case of several successive pairwise comparisons are presented to the decision makers, they may contradict each other.

The reason for these contradictions could be vaguely defined problems, insufficient information and uncertainty of the information or lack of concentration of the decision maker. Mentioned below are rules that need to be adhered to while collecting responses from the decision maker. However, the responses must follow the rules for consistency of responses as mentioned below.

Transitivity Rule :-

$S_{ij}=S_{ik}.\ S_{kj}$ Where S_{ij} is comparison of alternatives i with alternative j.

Reciprocity Rule:-

$S_{ij}= 1/\ S_{ji}$

Where i,j, and k are any alternatives of the matrix.

The most commonly used method for consistency check was developed by (Saaty 1990) who proposed a consistency index (CI), which is related to the eigenvalues of the matrix.

$$CI = \frac{\lambda max - n}{n - 1}$$

Where λmax is the maximal eigen value. The consistency ratio is given by –

CR=CR/RI

Where RI is the random index (the average CI of 500 randomly filled matrices). If the CR is less than 10% then the matrix is acceptable.

Table No. 3.1 (Consistency Numbers)

Size of matrix	1	2	3	4	5	6	7	8	9	10
Random Consistency Number	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

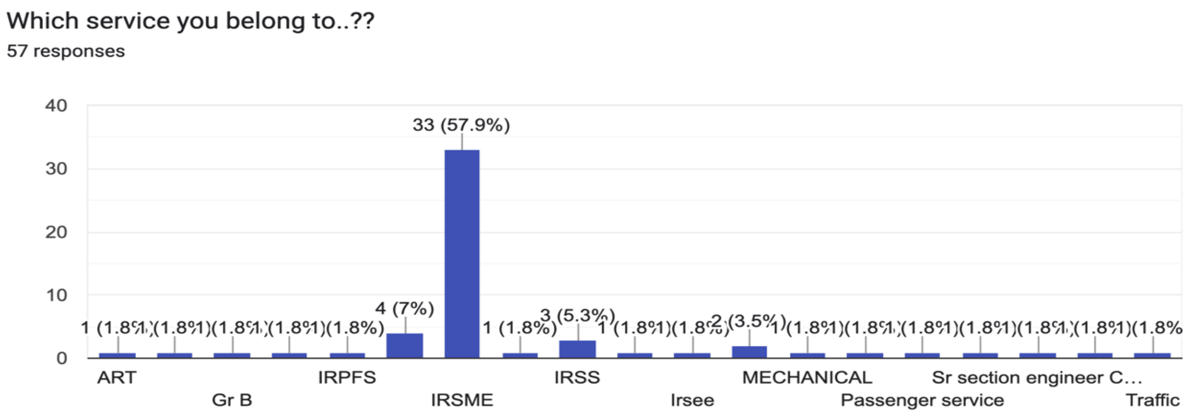
If the inconsistency is very high the decision maker is required to specify the preference again. Once the acceptable level is reached, the preferences are combined to proceed further for evaluation. In this study eigen value method for consistency check is adopted.

3.2.1 Details of the Experts and Sample Questionnaire for Calculating Attributes Weights Using APH.

For calculating the weightages of attributes for link importance, we have taken inputs from the experts. A questionnaire (as given below) has been circulated online using Google form and social media to a group of 250 officers from the Indian Railways.

A total of 55 officers from various spheres of railway management services had participated in the survey and expressed their evaluation score. The participation in the survey was wide and profound. It included the officers working since 1982 (batch of their entry in railways) that is 38 years of service experience to the officers inducted as recently as in 2018 in railways management. The average experience of the officers who contributed to the study is 16.6 years. Department wise participation indicates that maximum participation has been received from the Mechanical Engineering department (39 of 55 officers participated from the Mechanical department). Mechanical Engineering Department of Indian railways is a nodal department for disaster management and coordinates all the activities during crises. They are the officers who majorly decide on the location and allocation of the accident relief facilities in their area of influence. Therefore, their opinion has tremendous weight in decision making on the matter of disaster management. Further, the data used in this study is available in the public domain and the opinions expressed by the experts in the survey are personal opinions of the officers working in the organization. No official document, data or remark has been used in this work. Therefore, there is no breach of any ethics or moral code that took place in the process.

Fig 3.2. Distribution of Department Wise Responses Received in the Survey.



3.2.2 Details of the Questionnaire Circulated Among the Participants :

We are doing research on the problem of locating the accident relief facility (ART, ARMV and Crane) in the context of Indian railways. The objective is to strategically locate the relief facilities so that the response time to an accident site will be within the golden hour considering the vulnerability of the sections and restoration of the traffic as priority. In this research, we are also trying to figure out the relative importance of a railway section/route. An attempt is being made to quantify the importance of a train section with certain parameters, assuming that the attainable speed and other technical parameters are the same for each section. Following four operational criteria are identified as leading parameters for decision on importance of the section.

1. Total No. of passenger trains being operated in the section

2.Total No. of Goods trains being operated in the section

3.Total No. of empty coaching rakes operated in the section

4.Total No. of empty wagons rakes being operated in the section

We need to understand the relative weight of these criteria in order to evaluate the importance of a railway section, where mixed traffic is in operation. For this purpose, you are requested to answer following questions and rate the relative importance of activity.

The ratings are to be given as per the following scale.

Rating (on a scale of 10)	**Description**
Equal Importance	1
Moderate importance of one over other	2 to 4
Demonstrated Importance	5 to 7
Absolute and extreme Importance	7 to 10

For example, if you think that the passenger train operation is ABSOLUTELY IMPORTANT as compared to a goods train the rating will be between 7 to 10. The questions are -

1. Compare a section with more passenger train operation viz a viz a section with more Goods train operation from severity of accident and urgency of rescue point of view? On a scale of 10, a score 1 should be given for equal rating and if a section with passenger trains has higher weight, it should be given towards 10 on the scale.

2. Compare a section with more passenger train operation viz a viz a section with more Empty rake operation from severity of accident and urgency of rescue point of view? On a scale of 10, a score 1 should be given for equal rating and if a section with passenger trains has higher weight, it should be given towards 10 on the scale.

3. Compare a section with more passenger train operation viz a viz a section with more Empty wagon rake operation from severity of accident and urgency of rescue point of view?

On a scale of 10, a score 1 should be given for equal rating and if a section with passenger trains has higher weight it should be given towards 10 on the scale.

4. Compare a section with more Goods train operation viz a viz a section with more Empty coaching rake operation from severity of accident and urgency of rescue point of view?

On a scale of 10, a score 1 should be given for equal rating and if a section with passenger trains has higher weight it should be given towards 10 on the scale.

5. Compare a section with more Goods train operation viz a viz a section with more Empty wagon rake operation from severity of accident and urgency of rescue point of view?

On a scale of 10, a score 1 should be given for equal rating and if a section with more Goods trains has higher weight it should be given towards 10 on the scale.

6. Compare a section with more Empty coaching rake operation viz a viz a section with more Empty wagon rake operation from severity of accident and urgency of rescue point of view?

On a scale of 10, a score 1 should be given for equal rating and if a section with more Goods trains has higher weight it should be given towards 10 on the scale. Details of the responses are appended in Annexure-A.

3.3 TOPSIS – The Technique for Preference by Similarity to Ideal Solution :

The TOPSIS technique is based on the concept that there exist ideal alternatives for objective to choose. The geometric distance between the possible alternatives from ideal solutions is considered with the assumption that the chosen alternative should have shortest distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS). This technique was originally developed by (Hwang &Yoon 1981) for solving multiple attribute decision making (MADM) problems and extended to solve multiple objective problems by (Lai, Liu, and Hwang 1994).

The methodology for Topsis considering a MADM problem is as follows:

$$max/\min[f_1(x), f_2(x), \ldots\ldots f_k(x)]$$

Such that $x \in X = \{x | g_i(x)\{\geq, =, \leq\}, i = 1,2 \ldots.m\}$

Where $$f_j(x) = Benefit\ Objective\ f\ or\ maximization, j \in J$$

$$f_i(x) = Cost\ Objective\ f\quad or\ minimization, i \in I$$

The steps for calculation are as follows:

1. Obtain PIS(f^*) and NIS(f^-), which are given by

$$f^* = \{\max_{x \in X}(or\ \min) f_j(x)(or\ f_i(x), \forall\ j(and\ i)\}$$

$$f^- = \{\max_{x \in X}(or\ \min) f_j(x)(or\ f_i(x), \forall\ j(and\ i)\}$$

2. Using the PIS and NIS , the following distance functions are obtained:

$$d_p^{PIS} = \left\{ \sum_{j \in J} w_j^p * \left[\frac{\left(f_j^* - f_j(x)\right)}{(f_j^* - f_j^-)} \right]^p + \sum_{i \in I} w_i^p * \left[\frac{\left(f_i^* - f_i(x)\right)}{(f_i^- - f_i^*)} \right]^p \right\}^{\frac{1}{p}}$$

$$d_p^{NIS} = \left\{ \sum_{j \in J} w_j^p * \left[\frac{\left(f_j^* - f_j(x)\right)}{(f_j^* - f_j^-)} \right]^p + \sum_{i \in I} w_i^p * \left[\frac{\left(f_i^* - f_i(x)\right)}{(f_i^- - f_i^*)} \right]^p \right\}^{\frac{1}{p}}$$

Where w_t t=1,2,3……k are the relative weights (or the relative importance) of the objective functions, obtained by the Analytic Hierarchy Process (AHP) described in section 3.2 above. P is the distance parameter.

3. The problem $\min d_p^{PIS}(x)\ such\ that\ x \epsilon X$ is solved to obtain the solution $x^p\ and\ \left(d_p^{PIS}\right)^* = \min d_p^{PIS}(x)$.
4. The problem $\max d_p^{NIS}(x)\ such\ that\ x \epsilon X$ is solved to obtain the solution $x^N\ and\ \left(d_p^{NIS}\right)^* = \max d_p^{NIS}(x)$.
5. Thus, following is calculated: obtain $\left(d_p^{PIS}\right)^- = d_p^{PIS}(x^N)\ and\ \left(d_p^{NIS}\right)^- = d_p^{NIS}(x^p)$.
6. The following problem is solved to obtain the solution of the ODM problem.

$\max \alpha,$

$$\frac{d_p^{PIS}(x) - \left(d_p^{PIS}\right)^*}{\left(d_p^{PIS}\right)' - \left(d_p^{PIS}\right)^*} \geq \alpha$$
$$\frac{\left(d_p^{NIS}\right)^* - d_p^{NIS}(x)}{\left(d_p^{NIS}\right)^* - \left(d_p^{NIS}\right)'} \geq \alpha$$
$$x \epsilon X$$

3.4 Estimating Importance of a Link in a Rail Network :

The overall procedure for calculating the link importance is shown in Figure 1. In this study, the importance of a link is calculated in terms of a set of operational attributes reflecting the criticality of the link in the network. These attributes are gathered from the literature and further refined to fit into the Indian context. Bababeik et al.(Bababeik, Khademi, and Chen 2018) proposed monthly passenger and freight traffic, export, and serviceability to the industry as the major attributes to calculate the link importance based on the survey of the extant literature. For the applicability of these attributes for the Indian railway network, we consulted officers working in the Indian railways, who have several years of experience in decision making regarding the operation, placement of these accident relief trains, along with a vast experience of managing accident sites. They were the officers who understand the train operation and have been associated with the implementation of the disaster management plan of the Government of India. After deliberations, we have considered the following attributes to calculate the importance of links in the network.

A1. Total Number of passengers carrying trains operating in a section.

A2. Total Number of loaded wagon goods trains operated in a section.

A3. Total number of empty wagons moved through a section.

A4. Other miscellaneous traffic such as departmental material train (DMT), over dimension consignment (ODC), empty coaches, movement of ART, and light engine over the section. These factors are deliberately chosen against the derived parameters specifically used by railways for statistical analysis like Gross ton-kilometres (GTKM) or net tons kilometres (NTKM). These parameters indicate the capacity of the link and percentage of the capacity utilized by the link, but they do not give a real picture about the vulnerability or importance of the link from an accident point of view. It should also be noted that some of the factors such as the movement of empty coaches are not taken as a separate entity because passenger trains are operated end to end units on the same rake. However, there are a limited number of empty coaches that are moved in the case of special bookings and special occasions such as festivals and *Mela*.

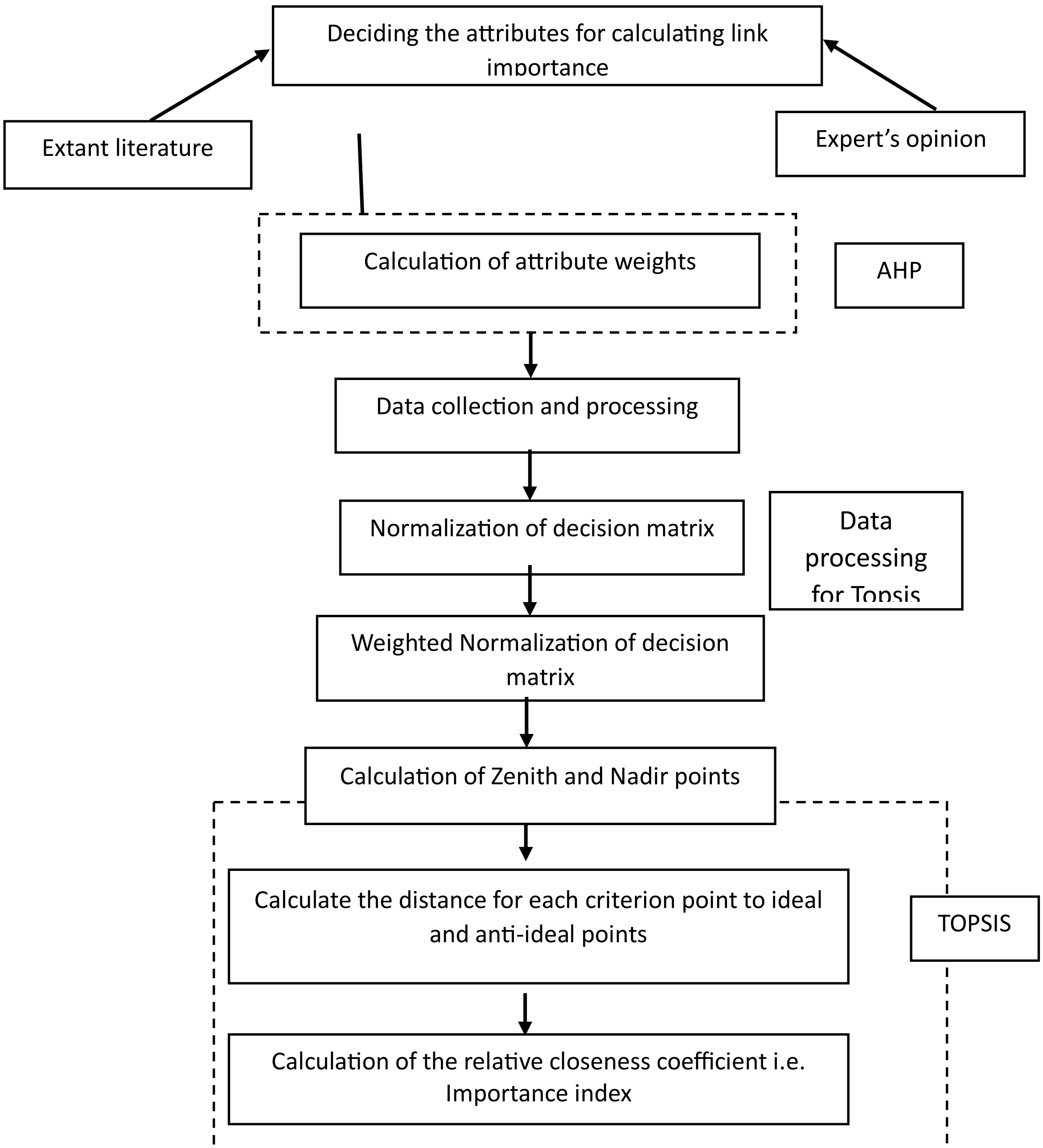


Figure 3.3: Schematic Representation of Procedure to Calculate the Link Importance.

The attributes listed above are used to calculate the importance of each link in the network. For this purpose, we first calculate the weights of the attributes using the Analytical Hierarchy Process (AHP). For the attribute weights, the inputs (for relative comparison of the attributes) are taken from the experts in the Indian railways through a questionnaire-based survey. The details about the experts and the questionnaire are given in section 3.3. To calculate the weights of these attributes, the subjective judgments of these officers are quantified using a linear scale suggested by (Saaty 1977) and a comparison matrix is drawn for each feedback. We have used the eigenvalue method of AHP for calculating the individual weights given by the experts and then these individual weights are aggregated to obtain final normalized weights for attributes in the question. A consistency check (consistency ratio) is applied to the comparison matrix received through the survey and only a consistent matrix is considered for inclusion in weight calculation.

The weights obtained for criteria using AHP are:

A1. Total Number of passengers carrying trains operating in a section.- **0.6315.**

A2. Total Number of loaded wagon goods trains operated in a section.-**0.1873.**

A3. Total number of empty wagons moved through a section.- **0.096.**

A4. Other miscellaneous traffic such as departmental material train (DMT), over dimension consignment (ODC), empty coaches, movement of ART, and light engine over the section.- **0.088.**

The next step is the collection of data, normalization, and weighted normalization of the decision matrix. The data pertaining to all these attributes collected for various links in the network under study and was organized in a matrix form, where a row represents the values of various attributes for a link and a column represents an attribute for various links. This raw data has different units of measurement and hence it needs to be normalized. Among several normalization methods available in the literature, we have used a distributive normalization method for this study. For distributive normalization, the performance parameters are divided by the square root of the sum of each squared element in a column of the decision matrix. Each element of the normalized decision matrix is multiplied by the weight of criteria obtained using AHP to obtain the weighted normalized decision matrix.

Having calculated the weighted normalized decision matrix, the most important link in the network, as well as the relative importance of each link in the network needs to be calculated. For this purpose, we use a multi-attribute decision-making technique - 'Techniques for Order of Preference Similarity to the Ideal Solution' abbreviated as TOPSIS originally proposed by (Lai et al. 1994).

The steps of the TOPSIS method are described as calculation of the Zenith (an Ideal point) and Nadir (anti-ideal point) points for each criterion, calculating the distance for each criterion point to Zenith and Nadir points and Calculation of the relative closeness coefficient i.e. Importance indexes are followed as reported in the literature (Lai et al., 1994).

The scores of link importance obtained using TOPSIS for the considered network are presented in the table no 3.2 below.

Table No. 3.2 Link Importance Obtained Using TOPSIS.

Link	Route length(km)	Number of passengers' trains	Number of goods trains	Number of empty wagons	Miscellaneous traffic	Link importance
DDU-ALD	148.26	92.80	60.20	30.60	2.00	**0.72**
ALD-CNB	194.21	85.20	66.50	34.30	1.00	**0.66**
CNB-TDL	228.76	127.40	61.80	32.70	0.50	**0.86**
TDL-GZB	179.99	118.80	34.70	17.70	0.05	**0.78**
NYN-MKP-BANSAPAHAR	94.76	29.10	9.90	4.30	0.02	**0.21**
COI LINK JN.	0.75	31.30	15.40	7.20	0.03	**0.23**
CNB-LKO	1.14	101.20	25.60	15.20	0.04	**0.70**
MANIKPUR-JHS	0.60	14.00	2.60	1.30	0.05	**0.10**
CNB-JHS	2.51	36.00	20.30	8.70	0.01	**0.27**
CHUNAR-CHOPAN	99.93	11.20	3.90	2.50	0.02	**0.08**
SHIKOHABAD-FKD	104.63	6.20	1.20	0.90	0.01	**0.04**

BRHAN-ETAH	58.51	3.40	0.60	0.30	0.02	**0.02**
HATHRASH-HATRASH QUILLA	8.70	4.00	0.06	0.03	0.15	**0.03**
ALJN-BE	13.91	10.40	9.20	4.60	0.05	**0.08**
KHURJA-MEERUT	0.34	8.60	2.30	1.20	0.05	**0.06**
ALD-FZD	0.82	29.20	2.30	1.60	0.04	**0.21**
ETMAD PUR - MITAWALI	3.00	0.40	8.30	3.80	0.03	**0.03**
CNB-CPA	3.04	57.80	10.40	5.30	0.02	**0.41**
ETAWAH-MAINPURI	55.15	4.00	2.04	2.02	0.01	**0.03**
CNB DIVIRISON	5.90	82.80	8.30	3.80	0.04	**0.57**
AGC-MTJ	53.79	97.72	54.35	26.08	3.00	**0.76**
MTJ-PWL	82.34	129.42	65.76	30.15	2.00	**0.94**
AGC-DHO	54.27	91.70	56.84	25.44	1.00	**0.70**
MTJ-AWR	117.90	8.00	13.45	5.07	0.03	**0.08**
BXN-TDL	85.82	12.84	15.63	9.12	0.02	**0.11**

AF-BKI	146.40	23.14	14.00	7.50	0.01	**0.17**
MTJ-KOTA	1.20	37.72	36.80	17.30	0.02	**0.31**
IDH-AGC	1.53	4.58	3.00	1.50	0.00	**0.03**
AGA-JAB	20.50	8.56	11.49	5.04	0.00	**0.08**
RKM-AGA	2.62	8.56	11.49	5.04	0.00	**0.08**
ETMAD PUR - MITAWALI	1.49	0.58	8.44	5.04	0.00	**0.09**
AH-MTJ	35.60	10.58	11.30	4.18	0.01	**0.01**
AGC-Cantt-(JNP)- Bayana Chord Line	1.37	0.58	1.10	0.85	0.00	**0.02**
BHANDAI-UDIMORE	124.26	2.58	1.93	0.84	0.00	**0.07**
MTJ-VRBD (MG)	11.48	9.70	0.00	0.00	0.00	**0.01**
DHO-TANTPUR (NG)	58.63	2.00	0.00	0.00	0.00	**0.03**
MOHARI-SIRMUTRA (NG)	30.16	4.00	0.00	0.00	0.00	**0.71**
DHO-JHS	161.28	88.60	64.30	30.90	3.00	**0.65**

JHS-BINA	150.72	85.60	55.00	26.20	0.50	**0.20**
JHS-CNB	211.85	26.90	12.61	5.59	0.01	**0.19**
KHAIRAR-MANIKPUR	109.70	24.80	12.04	5.33	0.00	**0.13**
JHS-KHAIRAR	180.20	17.72	8.44	3.96	0.00	**0.06**
BANSAPAHAR-OHAN	1.85	5.46	10.60	5.30	0.00	**0.07**
KHAIRAR-BZN	118.03	9.00	3.12	1.56	0.00	**0.06**
MAHOBA-KHAJURAHO	64.64	8.80	0.60	0.20	0.00	**0.06**
GWL-PANIHAR	0.75	8.00	1.00	0.50	0.00	**0.04**
BIRLANAGAR-ETAWAH	101.10	1.94	9.93	4.77	0.00	**0.07**
AIT-KONCH	13.13	10.00	0.00	0.00	0.00	**0.12**
AGD-MALKHEDRI	6.52	9.40	29.90	16.00	0.03	**0.06**
LAR-KHAJURAHO	164.25	8.00	0.60	0.20	0.01	**0.06**

3.5. Conclusion :

We demonstrated that the concept of MCDM techniques can be effectively used for evaluating the vulnerability of a network toward a specific type of accident in train operation. In this study, the vulnerability of railway networks is conceptualized in relation to their susceptibility to specific types of accidents, depending on the type of service they support, such as transporting goods, passengers, empty wagons, and other miscellaneous traffic. The importance of individual links in the network is evaluated using Multi Criteria Decision Making techniques, specifically the Analytical Hierarchy Process (AHP) and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS).

Overall, this study underscores the critical role of individual network links in determining the vulnerability of transportation systems to accidents and emphasizes the need for comprehensive methodologies to assess and mitigate these vulnerabilities effectively. The importance of the link on the basis of traffic pattern, which influences the possibility of an accident and hence the demand for specific type of equipment becomes a critical parameter in decision locating these equipment over the network. The critical concept will be utilized in further conceptualization of the mathematical model for optimization of location decision.

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