

# Contents

<b>List of Figures</b>	<b>xix</b>
<b>Preface</b>	<b>xxi</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Background . . . . .	2
1.1.1 Linear and Nonlinear Waves . . . . .	2
1.1.2 Hyperbolic PDEs . . . . .	4
1.1.3 Riemann Problem . . . . .	6
1.1.4 Chaplygin Gas . . . . .	10
1.2 Motivation . . . . .	11
1.3 Literature Review . . . . .	14
1.4 Thesis Objectives . . . . .	18
<b>2 On the Existence of Simple Waves for Two-Dimensional Non-ideal Magnetohydrodynamics</b>	<b>21</b>
2.1 Introduction . . . . .	21
2.2 Steady MHD 2D system . . . . .	25
2.3 Pseudo-steady MHD 2D system . . . . .	32
2.4 Application of Characteristic Decomposition - Full MHD 2D system . . . . .	39
2.5 Conclusion . . . . .	41
<b>3 Simple waves for anti-van der Waals modified Chaplygin gas in 2D magnetohydrodynamics</b>	<b>43</b>
3.1 Introduction . . . . .	43
3.2 Methodology . . . . .	46
3.3 Steady 2D MHD system . . . . .	48
3.4 Pseudo-steady 2D MHD system . . . . .	49
3.5 Full 2D MHD system . . . . .	51
3.6 Conclusion . . . . .	53

<b>4</b>	<b>The Application of Differential Constraint Method for the Solution of Non-homogeneous Generalized Riemann problem</b>	<b>55</b>
4.1	Introduction . . . . .	55
4.2	The Differential Constraint Method . . . . .	59
4.3	Governing equation and solution procedures . . . . .	61
4.4	Rarefaction Wave . . . . .	65
4.5	Shock Wave . . . . .	67
4.6	Generalized Riemann problem (GRP) . . . . .	69
4.7	Conclusion . . . . .	72
<b>5</b>	<b>Shock Interaction with a Right-Angled Wedge: Diffraction and Reflection in an Extended Chaplygin Gas</b>	<b>75</b>
5.1	Introduction . . . . .	75
5.2	Fundamental equations and Configuration of shock reflection-diffraction in self-similar flow . . . . .	77
5.3	R-H conditions for the incident and reflected shock and asymptotic expansions . . . . .	82
5.4	Behaviour of asymptotic expansions in the diffracted region $\bar{\Gamma}$ . . . . .	85
5.5	Behaviour of nonlinear expansions on curve ABC . . . . .	87
5.6	Asymptotic expansion for a shock coming off a wedge and shock incident on a screen . . . . .	92
5.6.1	Shock coming off a wedge . . . . .	92
5.6.2	Shock incident on a screen . . . . .	93
5.7	Behaviour of nonlinear expansions near singular point $B(a_0, \pi)$ . . . . .	94
5.8	Numerical Results and Discussion . . . . .	97
5.8.1	Diffraction and Expansion wave Profile . . . . .	98
5.8.2	Shock Position or speed of the diffracted shock . . . . .	101
5.9	Conclusions . . . . .	102
<b>6</b>	<b>Two-Dimensional Analysis of Weak Shock Diffraction and Reflection in Extended Chaplygin Gas</b>	<b>105</b>
6.1	Introduction . . . . .	105
6.2	Fundamental equations and Configuration of shock reflection-diffraction in self-similar flow . . . . .	108
6.3	R-H conditions for the incident and reflected shock and asymptotic expansions . . . . .	111
6.4	Asymptotic expansions in the diffracted region $\bar{\Gamma}$ . . . . .	112
6.5	Nonlinear expansions on curve RQS . . . . .	113
6.6	Nonlinear expansions near singular point $Q$ . . . . .	115
6.7	Numerical Results and Discussion . . . . .	116
6.7.1	Diffraction Shock and Expansion wave Profile . . . . .	116
6.7.2	Shock Position or speed of the diffracted shock . . . . .	119

---

6.8	Conclusions . . . . .	120
<b>7</b>	<b>Conclusion and Future Scope</b>	<b>123</b>
7.1	Conclusions . . . . .	123
7.2	Future Scope . . . . .	125
	 <b>Bibliography</b>	 <b>129</b>



# List of Figures

2.1	Flow adjacent to a constant state is simple. . . . .	38
4.1	The $(\rho, \nu)$ phase plane for the model (4.1.1). . . . .	68
5.1	Incident shock $C_0$ hits the right - angle wedge (a) at $t = 0$ moving along the X - axis, (b) at $t > 0$ , the diffraction - reflection phenomena in a self-similar plane $(\xi, \eta)$ . $C_1$ : diffracted shock, $C_2$ : reflected shock, $C_3$ : rarefaction wave. . . . .	78
5.2	Plots of diffracted shock wave profile equation (5.5.8) with $n = 1.4, B' = 0.5, \alpha = 0.5, \eta = 5\pi/6, \rho_2^{(j)} = 0$ . Solid lines $h_1s_1 \cup g_1h_3, h_1s_2 \cup g_2h_3$ and $h_2s_3 \cup g_3h_3$ represent shock wave profiles for $b' = 0, 0.3$ and $b' = 0.4$ , respectively. Segments $s_1g_1, s_2g_2$ , and $s_3g_3$ are jumps in shock profiles corresponding to $b' = 0, 0.3$ and $b' = 0.4$ , respectively. Here, $g_1, g_3$ and $g_3$ represent shock positions for $b' = 0, 0.3$ and $b' = 0.4$ , respectively. . . . .	98
5.3	Plots of diffracted shock wave profile equation (5.5.8) with (a) $n = 1.4, b' = 0.2, \alpha = 0.5, \eta = 5\pi/6, \rho_2^{(j)} = 0$ , (b) $n = 1.4, b' = 0.2, B' = 0.6, \eta = 5\pi/6, \rho_2^{(j)} = 0$ . $s_1g_1$ and $s_2g_2$ are jumps in shock profiles for $B' = 0.1$ and $0.6$ , respectively. Here, $g_1$ and $g_2$ are shock positions corresponding to $B' = 0.1$ and $0.6$ , respectively. . . . .	99
5.4	Plots of expansion wave profile, $\tilde{\rho}$ vs. $\omega$ : equation (5.5.8) with $n = 1.4, B' = 0.5, \alpha = 0.5, \eta = 4\pi/3$ . Solid lines $H_1s_1e_1, H_2s_2e_2$ and $H_2s_3e_3$ represent expansion wave profiles for $b' = 0, 0.3$ and $b' = 0.4$ , respectively. . . . .	99
5.5	Plots of expansion wave profile, $\tilde{\rho}$ vs. $\omega$ : equation (5.5.8) with (a) $n = 1.4, b' = 0.2, \alpha = 0.5, \eta = 4\pi/3$ (b) $n = 1.4, b' = 0.2, B' = 0.6, \eta = 4\pi/3$ . . . . .	100
5.6	Plots of speed of diffracted shock $\omega_s$ vs. $b'$ : equation (6.5.3) with parameters (a) $n = 1.4, \alpha = 0.1, \eta = 5\pi/6$ (b) $n = 1.4, B' = 0.9, \eta = 5\pi/6$ . . . . .	101
6.1	Incident shock $C_0$ hits the wedge inclined at angle $\theta$ at point $P$ . The reflection-diffraction phenomena in a self-similar plane $(\xi, \eta)$ . $C_1$ : diffracted shock, $C_2$ : reflected shock, $C_3$ : rarefaction wave. . . . .	108

- 6.2 Plots of diffracted shock wave profile equation (5.5.8) with  $n = 1.4, B' = 0.5, \alpha = 0.5, \eta = 2\pi/3, \theta = \pi/6, \rho_i^{(2)} = 0$ . Solid lines  $h_1s_1 \cup g_1h_3, h_1s_2 \cup g_2h_3$  and  $h_2s_3 \cup g_3h_3$  represent shock wave profiles for  $b' = 0, 0.3$  and  $b' = 0.4$ , respectively.  $s_1g_1, s_2g_2$  and  $s_3g_3$  are jumps in shock profiles.  $g_1, g_2$  and  $g_3$  represent shock positions. . . . . 117
- 6.3 Plots of diffracted shock wave profile equation (5.5.8) with (a)  $n = 1.4, b' = 0.2, \alpha = 0.5, \eta = 2\pi/3, \theta = \pi/6$  (b)  $n = 1.4, b' = 0.2, B' = 0.6, \eta = 2\pi/3, \theta = \pi/6$ . . . . . 117
- 6.4 Plots of expansion Wave profile,  $\tilde{\rho}$  vs.  $\omega$ : equation (5.5.8) with  $n = 1.4, B' = 0.5, \alpha = 0.5, \eta = \pi/4, \theta = \pi/6, \rho_i^{(2)} = 0$ . Solid lines  $H_1s_1e_1, H_2s_2e_2$  and  $H_2s_3e_3$  represent expansion wave profiles for  $b' = 0, 0.3$  and  $b' = 0.4$ , respectively. . . . . 118
- 6.5 Plots of expansion Wave profile,  $\tilde{\rho}$  vs.  $\omega$ : equation (5.5.8) with (a)  $n = 1.4, b' = 0.2, \alpha = 0.5, \eta = \pi/4, \theta = \pi/6$  (b)  $n = 1.4, b' = 0.2, B' = 0.6, \eta = \pi/4, \theta = \pi/6$ . . . . . 118
- 6.6 Plots of speed of diffracted shock  $\omega_s$  vs.  $b'$ : equation (5.5.8) with (a)  $n = 1.4, b' = 0.2, \alpha = 0.1, \eta = 2\pi/3, \theta = \pi/6$  (b)  $n = 1.4, b' = 0.2, B' = 0.9, \eta = 2\pi/3, \theta = \pi/6$ . . . . . 119