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A Robustness Consideration in Continuous Time $[\mathcal{K}, \mathcal{KL}]$ Sector for Nonlinear System

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ABSTRACT In this paper, we concerned to achieve an asymptotic stabilization of a generalized nonlinear system with significant uncertainties by rendering a relatively lesser amount of control. Here, a combined control consist of *Hands-Off* control and an adaptive sliding mode control is allowed to adjust the movement of the system state to the interior of $[\mathcal{K}, \mathcal{KL}]$ sector and compensate the unknown disturbance, respectively. The adaptation methodology consists of a low-pass filter to filter-out the high-frequency component and searches for minimum possible value to cancel the effect of the unknown disturbance throughout the dynamical system. Finally, a design example is shown with the simulation results for $[\mathcal{K}, \mathcal{KL}]$ sector design.

INDEX TERMS $[\mathcal{K}, \mathcal{KL}]$ sector, *Hands-Off* control, adaptive sliding mode control.

I. INTRODUCTION

Consider a nonlinear-affine system represented by a differential equation (1) as

$$\begin{aligned} \dot{\zeta} &= F(\zeta, \varphi(\zeta, t)) + G(\zeta, \varphi(\zeta, t))\nu \\ y &= h(\zeta) \end{aligned} \tag{1}$$

where $F(\zeta, \varphi(\zeta, t))$ and $G(\zeta, \varphi(\zeta, t))$ are unknown bounded nonlinearities with system state $\zeta \in \mathcal{D} \subset \mathbb{R}^n$ and $\varphi(\zeta, t)$ is an unknown exogenous/model uncertainty. The control input is piecewise continuous and bounded function of t, $\forall t \geq 0$ for nonlinear system described as $\nu \in \mathcal{V} \subset \mathbb{R}^m$ $(m \leq n)$ and system output is $y : \mathbb{R}^n \to \mathbb{R}^p$. The most important objective of control theory is to provide a control to obtain a stable nonlinear system, despite of uncertainties. To design such a control law, it is suitable to transfer the system (1) to the normal form (Byrnes-Isodori) [1] as

$$\begin{aligned} \dot{\eta} &= f_0(\eta, x, d(x, t)) \\ \dot{x} &= f(x) + g(x)\nu + d(x, t) \end{aligned} \tag{2}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^n$ are smooth functions. The system (2), $\dot{\eta} = f_0(\eta, x, d(x, t))$, represents the zero-solution of the system and $d(x, t)$ shows the uncertainties that may be vanishing/non-vanishing. For nonvanishing uncertainty, a continuous memoryless controller is not able to stabilize the nonlinear system, but *Input-to-State Stability* (ISS) [2] with respect to uncertainties $d(x, t)$ can be achieved. However, a control algorithm based on advance controllers such as sliding mode control (SMC) [3], [4], model predictive control (MPC) [5], [6], repetitive control [7], [8], active disturbance rejection control [9], [10] and so on to have the capability to fully alleviate the unknown exogenous/model uncertainty and stabilizes the nonlinear system. The above discussed controllers require the information of full states to achieve the desired tracking/regulation. The major disadvantage of all these controllers are that the use of superfluous control effort because of control action to remain online, even after reaching the stable/good region. Therefore, some type of switching controller is designed such that control action remains active only till the stabilization is ensured and subsequently reaches to zero.

For $d(x, t) = 0$, consider a fresh nominal system $\dot{x} = \tilde{f}(x)$ which is classified as being defined on good region. Therefore, the remaining unforced function of system (2) i.e., $\dot{x} = f(x) - \tilde{f}(x)$ is categorized as lying in the unstable/bad region. This peculiar strategy for partitioning the system is done within the framework of Matrosov's theorem [11]. The boundary to decide the separation between two regions is named as a sector which allows the good region to be inside the sector while the bad region to be outside. Here, the requirement of the control action is to bring the system states into a good region which are initially lying in the bad region. Subsequently, the control action is turned-off for further convergence to achieve equilibrium point. This control algorithm is termed as Lazy or *Hands-Off* control [12] whose basic nature resembles a switching control followed by an *ON-OFF* (relay) logic.

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Based on the characteristics of Matrosov's theorem [11], the design for sector formation is firstly done in [13] by deciding the boundary of a sector with the derivative of Lyapunov function for the linear system. Further, the detailed analysis for stabilization of continuous and discrete-time system is done with the design of linear sector and a switching control based on *ON-OFF* (relay) logic [14]. Furthermore, based on a state-dependent time-varying system, a nonlinear sector is designed such that the asymptotic stability is ensured with the help of various nonlinear controls in [16]. Here, a forward integration is done for solving a differential Ricatti equation (DRE) while designing the sector. Here in [17], a nonlinear inverted pendulum is illustrated for the design of nonlinear sector to show the stabilization with the limited amount of control. However, all the above discussed methodologies for designing a sector involves the Ricatti equation, which can be replaced by the introduction of a control-Lyapunov function. As the idea of control-Lyapunov function is well established in the literature [20] and [21] and the choice of smooth and positive-definite Lyapunov function implies the system to have asymptotic stabilizability. Nevertheless, there is no standard procedure for construction of a Lyapunov function for generalized nonlinear system and can be intuitively exhibited by trial and error method to compute a basin of attractive region by Zubov's method [15]. In order to refine the study of generalized nonlinear system for stability analysis, we allow the Lyapunov function to be generated by class $\mathcal{K}_\infty$ function and the convergence nature is satisfied with $\mathcal{KL}-$ stability property. These functions are commonly named as comparison functions [18], [19]. Recently, a new nonlinear sector [22] is designed in perspective of comparison function with a *Hands-Off* control to avoid the superfluous control effort. This elegant sector is termed as [$\mathcal{K}$, $\mathcal{KL}$] sector and it smoothly achieves the stabilization property for a disturbance-free system.

However, in a prototypical system with significant uncertainties (2), the sliding mode control [3] is a popular strategy due to its finite-time convergence and insensitive nature. But this convergence comes at the cost of numerical chattering. To compensate these chattering, one often the higher-order sliding mode control techniques by artificially increasing the relative degree and capable of providing continuous control signals. These higher-order sliding mode controllers involve the nested-structures of the signum functions [23], [24] and these signum functions are pre-multiplied with some constant gain magnitudes. But, even by increasing the relative degree cannot totally eliminate the numerical chattering. So an alternative approach is often allows by an idea of adaptation law in sliding mode control [25]–[29] for designing the system exhibiting the dynamic properties by utilization of current information. As the chattering amplitude is proportional to the magnitude of control, the gain magnitude can be adjusted to a minimum admissible value for sliding mode to exist.

Majorly, the adaptation law under sliding mode may be discussed by two ways:

- The adjustment of gain magnitude withing reaching phase and terminates afterward [25], [26].
- The estimation of equivalent control obtained by a first-order low-pass filter and the adaptation exists throughout the process [27]–[29].

A. MOTIVATION AND MAIN CONTRIBUTION

The motivation of this paper is to form a nonlinear sector in the case of exogenous/model uncertainty such that the concept of a nonlinear sector for nominal case is extended to a perturbed one. The design of nonlinear sector is done in the framework of comparison function to deal with the dependency of time so as to achieve asymptotic stability. In [$\mathcal{K}$, $\mathcal{KL}$] sector [22], a switching control based on *ON-OFF* logic is procured to drag the system states to the interior of the sector and becomes off-line for further movement towards equilibrium point. The adaptation law based on sliding mode control [27], [28] is designed to reject the exogenous/model uncertainties and adjust the chattering amplitude to circumvent the high-frequency switching. So far, the design of [$\mathcal{K}$, $\mathcal{KL}$] sector is never done for a perturbed nonlinear system. Therefore, the *Hands-Off* control is combined with the adaptive sliding mode control to obtain the advantage of both controls in a single algorithm and according to the best of author's knowledge, such an algorithm is never used in literature. This combined algorithm allows the asymptotic stabilizability in case of nonlinear system subjected to exogenous/model uncertainty.

II. PROBLEM FORMULATION

In this paper, a nonlinear system is generalized in the form of an input-affine dynamical system

$$\dot{x}(t) = f(x(t)) + g(x(t))\Big(w(t) + d(t, x(t))\Big) \tag{3}$$

where $x(t) \in \mathbb{R}^n$ and $w(t) \in \mathbb{R}$ refers to state and control vectors respectively, $f, g : \mathcal{X} \to \mathbb{R}^n$ is Lebesgue measurable and locally uniformly bounded in closed set $\mathcal{X} \subset \mathbb{R}^n$ where $\|\mathcal{X}\|$ denotes the $\sup\{\|x\| : x \in \mathcal{X}\}$, and $d(t, x(t))$ is the uncertainty which involves matched exogenous and model disturbances.

Assumption 1: The uncertainty $d(x, t)$ is bounded as $d(x, t) \leq D$, where D is the upper bound of exogenous/model uncertainty.

Assumption 2: The rate of change of uncertainty $d(x, t)$ is bounded as $|\dot{d}(x, t)| \leq A$, where A is a known bounded for the time derivative of uncertainty.

Let, the stability of the closed-loop system (3) is examined with an existing Lyapunov theory as described in [30]. Here, Lyapunov-like storage function $V : \mathcal{X} \to \mathbb{R}^n$ to be continuously differentiable on the domain $\mathcal{X} \subset \mathbb{R}^n$ containing the origin is defined for class $\mathcal{K}$ and $\mathcal{KL}$ function over the region $[0, \infty)$ and provided in the corollary given below.

Corollary 1 [31]: For the nonlinear system (3), let $\mathcal{X} \subset \mathbb{R}^n$ be closed and bounded set that contains origin. Let $V : \mathcal{X} \to \mathbb{R}^n$ is bounded between Lipschitz class $\mathcal{K}$ function i.e., $\alpha_1, \alpha_2 \in \mathcal{K}$ such that

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \quad \forall x \in \mathcal{X} \tag{4}$$
$$\dot{V}(x) \leq -\hat{\alpha}_3(\|x\|) \tag{5}$$

where $\hat{\alpha}_3$ is Lipschitz class $\mathcal{K}$ function on $\mathcal{X}$, choose $r > 0$ such that $B_r = \{x \in \mathbb{R}^n | \ \|x\| \leq r\} \subset \mathcal{X}$ and let $c = \min_{\|x\|=r} V(x)$ with $c > 0$ and $x(t)$ is a solution to (3) which is bounded and satisfies

$$V(x) \to 0 \quad \text{as } t \to \infty. \tag{6}$$

III. REVIEW OF [$\mathcal{K}, \mathcal{KL}$] SECTOR

The design of [$\mathcal{K}, \mathcal{KL}$] sector is summarized in this section for the disturbance-free nonlinear system. The detailed analysis for sector design is discussed in [22]. The idea of [$\mathcal{K}, \mathcal{KL}$] sector is based on suggested methods of Furuta and Pan [14], Young and Özgüner [32].

The [$\mathcal{K}, \mathcal{KL}$] sector for a nonlinear system (3) is defined as a subset of n-dimensional space where a Lyapunov-like storage function (4) decreases automatically to attain asymptotic stability. The partition of nonlinear system is done within the framework of Matrosov's theorem [11]. The preceeding inequality (5) for some states $x \in \mathbb{R}^n$ is classified as defined on good region. While the contrast in (5) i.e., $\dot{V} > -\hat{\alpha}_3(\|x\|)$ for remaining states $x \in \mathbb{R}^n$ is categorized as lying in bad region. Henceforth, the good region is pretend to be as [$\mathcal{K}, \mathcal{KL}$] sector.

Definition 1: The [$\mathcal{K}, \mathcal{KL}$] sector $\wp$ is a subset of $\mathbb{R}^n$ for disturbance-free nonlinear system is defined as

$$\wp = \{x | \ \varrho^2(x) \leq \xi^2(x), \forall x \in \mathbb{R}^n\} \tag{7}$$

where the invariant surface $\varrho(x)$ is used to determine the boundaries of [$\mathcal{K}, \mathcal{KL}$] sector and $\xi(x)$ as a square root for the derivative of Lyapunov-like storage function for without and with control action are respectively represented as

$$\varrho(x) = \mathcal{S}, \quad \mathcal{S} : \mathbb{R}^n \to \mathbb{R} \tag{8}$$
$$\xi(x) = \sqrt{\alpha_3(\|x\|) - \hat{\alpha}_3(\|x\|)} \tag{9}$$

where $\hat{\alpha}_3$ and α_3 are Lipschitz class $\mathcal{K}$ and class $\mathcal{K}_\infty$ functions respectively.

For the nonlinear system, a [$\mathcal{K}, \mathcal{KL}$] sector is designed with the concept of control-Lyapunov function [20]. The solution of nonlinear system (3) should be lies inside the [$\mathcal{K}, \mathcal{KL}$] sector, then the derivative of Lyapunov-like storage function should be negative and system states will converge to the origin asymptotically and determined as

$$\begin{aligned}\dot{V}(x) &= \frac{\partial V}{\partial x} f(x) = \varrho^2(x) - \xi^2(x) - \hat{\alpha}_3(\|x\|) \\ &\leq -\hat{\alpha}_3(\|x\|), \quad \forall x \in \wp \end{aligned}\tag{10}$$

However, if the system states lie outside the [$\mathcal{K}, \mathcal{KL}$] sector, then we need some external force to change the inequality relation for the derivative of Lyapunov-like storage function. So that, the system state approaches to the interior of [$\mathcal{K}, \mathcal{KL}$] sector and further converges to the origin asymptotically is written as

$$\dot{V}(x) = \frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x} g(x) u_{hc} \leq -\alpha_3(\|x\|) \tag{11}$$

In order to procure the invariant surface $\varrho(x)$, we need to calculate the instant where an unstable system is transformed to a stable system be reformulating the above equation (11) in such a manner as

$$\dot{V}(x) = \frac{\partial V}{\partial x} f(x) \leq -\frac{\partial V}{\partial x} g(x) u_{hc} - \alpha_3(\|x\|) \tag{12}$$

Here, the control action is replaced with some dummy control-Lyapunov function i.e., $u_{hc} = -\frac{1}{4}\left(\frac{\partial V}{\partial x} g(x)\right)^\top$, then (12) can be able to render a constructive choice of invariant surface $\varrho(x)$ in the aftergoing inequality.

$$\begin{aligned}\dot{V}(x) &\leq -\frac{\partial V}{\partial x} g(x)\left(-\frac{1}{4}\frac{\partial V}{\partial x} g(x)\right)^\top - \alpha_3(\|x\|) \\ &= \left(\frac{1}{2}\frac{\partial V}{\partial x} g(x)\right)\left(\frac{1}{2}\frac{\partial V}{\partial x} g(x)\right)^\top - \alpha_3(\|x\|)\end{aligned} \tag{13}$$

With this explanation, the design of the invariant surface on the basis of Lyapunov analysis is discussed in below Lemma.

Lemma 1: Consider the [$\mathcal{K}, \mathcal{KL}$] sector (7) for any disturbance-free nonlinear system (3) such that $\alpha_3(\|x\|) - \hat{\alpha}_3(\|x\|) = \rho\alpha_3(\|x\|)$ with strictly positive constant $\rho(0 \leq \rho < 1)$, then [$\mathcal{K}, \mathcal{KL}$] sector defined in (7) can be redesign as

$$\wp = \left\{x | \ |\varrho(x)| \leq \xi(x) \quad \forall x \in \mathbb{R}^n\right\} \tag{14}$$

where

$$\varrho(x) = \frac{1}{2}\frac{\partial V}{\partial x} g(x) \tag{15}$$
$$\xi(x) = \sqrt{\alpha_3(\|x\|) - \hat{\alpha}_3(\|x\|)} = (\rho\alpha_3(\|x\|))^{\frac{1}{2}} \tag{16}$$

Proof 1: For state vectors to occur inside the [$\mathcal{K}, \mathcal{KL}$] sector, the inequality (13) follows

$$\begin{aligned}\dot{V}(x) &\leq \left(\frac{1}{2}\frac{\partial V}{\partial x} g(x)\right)\left(\frac{1}{2}\frac{\partial V}{\partial x} g(x)\right)^\top - \alpha_3(\|x\|) \\ &= \left(\frac{1}{2}\frac{\partial V}{\partial x} g(x)\right)\left(\frac{1}{2}\frac{\partial V}{\partial x} g(x)\right)^\top \\ &\quad - \alpha_3(\|x\|) + \hat{\alpha}_3(\|x\|) - \hat{\alpha}_3(\|x\|) \\ &= \varrho^2(x) - \xi^2(x) - \hat{\alpha}_3(\|x\|) \\ &\leq -\hat{\alpha}_3(\|x\|), \quad \forall x \in \wp.\end{aligned}$$

Therefore, the simplified form of [$\mathcal{K}, \mathcal{KL}$] sector is defined.

Remark 1: For the design of [$\mathcal{K}, \mathcal{KL}$] sector $\wp$ (7), the selection of various functional are as follows

1) Define a Lyapunov-like storage function for nonlinear system.
2) Select a choice of class $\mathcal{K}_\infty$ function ($\alpha_3 \in \mathcal{K}_\infty$) to decide the rate of convergence for the derivative of Lyapunov-like storage function.
3) The class $\mathcal{K}$ function $\hat{\alpha}_3$ and class $\mathcal{K}_\infty$ function α_3 used in Definition 1 to satisfy the relation as follow

$\xi^2(x) = \rho\alpha_3(\|x\|)$ and $\hat{\alpha}_3(\|x\|) = (1-\rho)\alpha_3(\|x\|)$ where ρ is a positive functional $(0 \leq \rho < 1)$

The [$\mathcal{K}, \mathcal{KL}$] sector acknowledged in the above discussion, suggest the division of state-space with the idea of Matrosov's results. As the inequality $|\varrho(x)| \leq \xi(x)$ reported in Theorem 1, the system state appeared to be inside of [$\mathcal{K}, \mathcal{KL}$] sector and converges toward the origin. While the external force is only needed if system state lies outside the [$\mathcal{K}, \mathcal{KL}$] sector. Thus, a switching controller is designed to circumvent the superfluous control effort.

Lemma 2: The *Hands-Off* control for disturbance-free nonlinear system (3) is designed as

$$u = -\chi(\varrho(x), \xi(x)) \left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right)^{-1} \left(\frac{\partial \mathcal{S}}{\partial x} f(x) + K\varrho(x)\right) \quad (17)$$

with the switching logic $\chi(\varrho(x), \xi(x))$ depend on $\varrho(x)$ and $\xi(x)$ and designed as

$$\chi(\varrho(x), \xi(x)) = \begin{cases} 1 & \text{when } x \in \mathbb{R}^n \backslash \wp \\ 0 & \text{when } x \in \wp \end{cases}$$

which enable the movement of system state to the interior of [$\mathcal{K}, \mathcal{KL}$] sector and decreasing of Lyapunov-like storage function at every time instant $t \geq t_0$ to achieve a globally asymptotically stable results for sufficiently large positive constant K that satisfies the inequality.

$$2K\rho\alpha_3(\|x\|) + \left(\frac{\partial \mathcal{S}}{\partial x} f(x)\right)^{\top} \varrho(x) + \varrho^{\top} \left(\frac{\partial \mathcal{S}}{\partial x} f(x)\right) > 0 \quad (18)$$

where K holds the restriction that controller gain is large enough to show the condition that $K > \left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right)/2$.

Proof 2: One can initially assume that the state vectors to lie outside the [$\mathcal{K}, \mathcal{KL}$] sector and follows the constraints $|\varrho(x)| > \xi(x)$. Therefore, the control signal is turned on with a switch $\chi(\varrho(x), \xi(x))$ to be 1, such that

$$\begin{aligned} \dot{V}(x) &= \frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x} g(x) u \\ &\leq \varrho^2(x) - \xi^2(x) - \hat{\alpha}_3(\|x\|) - 2\varrho(x) \\ &\quad \times \left[\left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right)^{-1} \left(\frac{\partial \mathcal{S}}{\partial x} f(x) + K\varrho(x)\right) \right] \\ &= -\left(2\left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right)^{-1} K - 1\right) \varrho^2(x) - \xi^2(x) \\ &\quad -2\varrho(x) \left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right)^{-1} \frac{\partial \mathcal{S}}{\partial x} f(x) - \hat{\alpha}_3(\|x\|) \\ &< -\left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right)^{-1} \left(2K\rho\alpha_3(\|x\|) + \varrho^{\top}(x) \frac{\partial \mathcal{S}}{\partial x} f(x) \right. \\ &\quad \left. + \left(\frac{\partial \mathcal{S}}{\partial x} f(x)\right)^{\top} \varrho(x)\right) - \hat{\alpha}_3(\|x\|) \end{aligned}$$

In order to assume K to be sufficiently large to satisfy the inequality (18), then

$$\dot{V} < -\hat{\alpha}_3(\|x\|), \quad \forall x \in \mathbb{R}^n \backslash \wp. \quad (19)$$

Once the state vectors enter inside the [$\mathcal{K}, \mathcal{KL}$] sector, the inequality constraint is modified to $|\varrho(x)| \leq \xi(x)$ and control signal is turned off with a switch $\chi(\varrho(x), \xi(x))$ to be 0, such that

$$\begin{aligned} \dot{V}(x) &= \frac{\partial V}{\partial x} f(x) = \varrho^2(x) - \xi^2(x) - \hat{\alpha}_3(\|x\|) \\ &\leq -\hat{\alpha}_3(\|x\|), \quad \forall x \in \wp \end{aligned} \quad (20)$$

Therefore, the energy of the overall system keeps decreasing and the below inequality,

$$\dot{V}(x) \leq -\hat{\alpha}_3(\|x\|), \quad \forall x \in \mathbb{R}^n \quad (21)$$

is asymptotically stable for disturbance-free nonlinear system.

IV. ROBUST CONTROLLER DESIGN FOR [$\mathcal{K}, \mathcal{KL}$] SECTOR

For the prototypical nonlinear-affine system with exogenous/model uncertainty as represented in (3), a combined control strategy is obtained to achieve asymptotic stability with relatively lesser amount of control to circumvent superfluous control effort. The combined control consists of *Hands-Off* control to render an *ON-OFF* logic to drag the system state to inside the sector and adaptive sliding mode control for compensation of unknown disturbance. The detailed discussion for robust control is written as follow.

Consider the dynamics of the system (3) at the boundary of the sector $\varrho(x) = \frac{1}{2}\frac{\partial V}{\partial x} g(x) = \mathcal{S}$. Then, the first order equation of invariant function is represented as

$$\dot{\varrho}(x) = \frac{\partial \mathcal{S}}{\partial x} \big(f(x) + g(x)(w(t) + d(x,t))\big) \quad (22)$$

For ensuring the sliding motion at the boundary of the sector, the condition $\varrho^{\top}(x)\dot{\varrho}(x) < 0$ should be satisfied. Therefore, the combined control consists of a *Hands-Off* control [22] to define the convergence rate for the system trajectories outside the [$\mathcal{K}, \mathcal{KL}$] sector and an adaptive sliding mode control to alleviate the unknown disturbance with minimum admissible value of gain magnitude and the need to design such a control is justified below in Theorem 1 such that

$$\begin{aligned} w(t) = -\left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right)^{-1} &\left(\frac{\partial \mathcal{S}}{\partial x} f(x) + K\varrho(x)\right) \\ &- (k(t) + \eta)\,\mathrm{sgn}(\varrho(x)) \end{aligned} \quad (23)$$

where η is some small positive constant and $k(t)$ is a guided control magnitude which satisfied the adaptation law with condition $k(t) > |d(x,t)|$. While the parameters depending on *Hands-Off* control is discussed in previous Section.

On substituting the control law (23) in (22), the dynamics of invariant function $\varrho(x)$ becomes

$$\begin{aligned} \dot{\varrho}(x) = -\left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right) &\Big((k(t) + \eta)\,\mathrm{sgn}(\varrho(x)) \\ &- d(x,t)\Big) - K\varrho(x) \end{aligned} \quad (24)$$

On reaching at the boundary of the [$\mathcal{K}, \mathcal{KL}$] sector, $\varrho(x) \equiv 0$, It should be confined that the derivative of invariant function i.e., $\dot{\varrho}(x) = 0$ gives

$$0 = -\left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right) \left((k(t) + \eta) \operatorname{sgn}(\varrho(x)) - d(x,t) \right) \quad (25)$$

that leads to

$$\left((k(t) + \eta) \operatorname{sgn}(\varrho(x)) \right)|_{eq} = -d(x,t) = v_{eq} \quad (26)$$

This shows that the unknown uncertainty is exactly cancelled-out during the sliding motion i.e., $|v_{eq}(t)| = |d(x,t)|$. Although, a close approximation of the discontinuous function $\operatorname{sgn}(\varrho(x))$ at high frequency switching was conceived to obtain a low pass filter which can filter-out the high frequency component. Thus, the estimated value of equivalent control $\bar{v}_{eq}(t)$ satisfies

$$\tau \dot{\bar{v}}_{eq}(t) + \bar{v}_{eq}(t) = -(k(t) + \eta)\operatorname{sgn}(\varrho(x)) \quad (27)$$

where small time constant $\tau > 0$ and the output of the filter $\bar{v}_{eq}(t)$ is relatively close to the actual control i.e.,

$$\|v_{eq}(t) - \bar{v}_{eq}(t)\| \leq \epsilon_0$$

where ϵ_0 is any positive number and a condition $|\varrho(x) \leq \Delta|$ is obtained in [33] such that a constant δ $(0 < \delta \leq \epsilon_0)$ provided $0 < \tau \leq \delta$ and $\frac{\Delta}{\tau} \leq \delta$ to define the filter bandwidth $\frac{1}{\tau}$. It should be noted that error in the estimation of equivalent control $\epsilon \to 0$ as $\tau \to 0$ and $\frac{\Delta}{\tau} \to 0$.

Furthermore, the bandwidth of filter helps to decide an appropriate level of accuracy for the estimated value of equivalent $\bar{v}_{eq}(t)$. Here, the estimated value is used to design the adaptive scheme for $k(t)$. We introduce a safety margin ϵ to maintain the sliding motion and controller gain should be

$$k(t) > \frac{1}{\beta}|\bar{v}_{eq}| + \epsilon \quad (28)$$

where scalar constant $0 < \beta < 1$ and $\epsilon > 0$ are chosen to ensure the estimated control $\bar{v}_{eq}$ satisfies

$$\frac{1}{\beta}|\bar{v}_{eq}(t)| + \frac{\epsilon}{2} > |v_{eq}| \quad (29)$$

Therefore, the error variable $\delta(t)$ as

$$\delta(t) = k(t) - \frac{1}{\beta}|\bar{v}_{eq}| - \epsilon \quad (30)$$

When adaptive gain exactly matches the control design i.e., $\delta(t) = 0$, then $k(t) = \frac{1}{\beta}|\bar{v}_{eq}| + \epsilon > |v_{eq}(t)| = |d(x,t)|$. Thus, the controller gain $k(t)$ is sufficiently large for enforcing sliding motion and forcing the error variable $\delta(t)$ to converge toward zero ($\delta(t) \to 0$).

The error variable $\delta(t)$ in (29) for exact estimation of equivalent control is described as

$$\dot{\delta}(t) = \dot{k}(t) - \frac{1}{\beta}\phi(t) \quad (31)$$

where $\phi(t) = \frac{d}{dt}|\bar{v}_{eq}(t)|$ and considered as the rate of change of exogenous/model uncertainty for $|\phi(t)| \leq q|\dot{d}(x,t)|$, where $q > 1$ is the safety margin to ensure the inequality (30) even for the worst case analysis. Then the controller gain can be exploited in the adaptation scheme is explicitly described as

$$\dot{k}(t) = -\Upsilon(t)\operatorname{sgn}(\delta(t)) \quad (32)$$

where the time varying parameter $\Upsilon(t)$ allows the condition that $\Upsilon(t) > \frac{qA}{\beta}$ to ensure the convergence of error variable to sufficiently close to zero. This adaptive scheme helps the sectored nonlinear system to achieve asymptotic stability with relatively lesser amount of control action.

Theorem 1: For the nonlinear-affine system subjected to exogenous/model uncertainty (3) satisfying $|d(x,t)| < D$ and $|\dot{d}(x,t)| < A$ where D and A are finite quantities representing the maximum bound for uncertainty and its derivative respectively. Then the feedback law (23), under the objective to ensure the decrease in Lyapunov-like storage function to achieve an asymptotically stable results is given as

$$w(t) = u(t) + v(t) \quad (33)$$

where the *Hands-Off* control $u(t)$ allows a switching logic $\chi(\varrho(x), \xi(x))$ on $\varrho(x)$ and $\xi(x)$ as defined in (17). The [$\mathcal{K}$, $\mathcal{KL}$] sector $\wp$ (14) is designed in Lemma 2 with $\xi^2(x) = \rho\alpha_3(\|x\|)$ and $\hat{\alpha}_3(\|x\|) = (1-\rho)\alpha_3(\|x\|)$ for any Class $\mathcal{K}_\infty$ function $\alpha_1, \alpha_2, \alpha_3$ and Class $\mathcal{K}$ function $\hat{\alpha}_3$ along with some positive functional ρ $(0 \leq \rho < 1)$ to obtain a good region for disturbance-free system. While, the adaptive sliding mode control $v(t)$ in (26) forces $\delta(t) = 0$ in finite-time and consequently compensates an exogenous/model uncertainty with adaptation gain as small as possible to maintain the sliding motion. Then, the control design for inner and outer side of [$\mathcal{K}$, $\mathcal{KL}$] sector (14) are respectively defined as

$$u_i(t) = -(k(t) + \eta)sgn(\varrho(x)) \quad \forall x \in \wp \quad (34)$$

$$u_o(t) = -\left(\frac{\partial \mathcal{S}}{\partial x} g(x)\right)^{-1} \left(\frac{\partial \mathcal{S}}{\partial x} f(x) + K\varrho(x)\right) - (k(t) + \eta)sgn(\varrho(x)) \quad \forall x \in \mathbb{R}^n \backslash \wp \quad (35)$$

where K is sufficiently large positive constant and an adaptive scalar function $k(t)$ satisfy the following inequalities as

$$2K\rho\alpha_3(\|x\|) + \left(\frac{\partial \mathcal{S}}{\partial x} f(x)\right)^\top \varrho(x) + \varrho^\top \left(\frac{\partial \mathcal{S}}{\partial x} f(x)\right) > 0 \quad (36)$$

$$k(t) \geq \frac{1}{\beta}\left|\bar{v}_{eq}\right| + \epsilon > |d(t,x)| \quad (37)$$

Proof 3: Consider the Lyapunov-like storage function in case of adaptation law is as $V(x,\delta)$ which shows its dependency on system state $x(t)$ and error variable $\delta(t)$. The simplified form for the adaptive-scheme in Lyapunov function is written as

$$V(x,\delta) = V(x) + \frac{\delta^2}{2} \quad (38)$$

The [$\mathcal{K}$, $\mathcal{KL}$] sector for the nominal system (3) can be designed by the use of control-Lyapunov function as shown in (13). Thus, it the initial state is assumed to be lied outside the [$\mathcal{K}$, $\mathcal{KL}$] sector $\wp$, a control algorithm $u_o(t)$ is mentioned to move the system state to the interior of the sector. Therefore, the derivative for square of invariant function $\varrho^2(x)$ for inequality condition $\varrho^2(x) > \xi^2(x)$ and control law $u_0(t)$ (35) is represented as

$$\begin{aligned}\frac{d}{dt}\varrho^2(x) &= 2\varrho(x)\dot{\varrho}(x)\\ &= 2\varrho(x)\left(\frac{\partial \mathcal{S}}{\partial x}(f(x)+g(x)(u_o+d(x,t)))\right)\\ &= 2\varrho(x)\left(\frac{\partial \mathcal{S}}{\partial x}(f(x)+g(x)d(x,t))\right)\\ &\quad -2\varrho(x)\frac{\partial \mathcal{S}}{\partial x}g(x)\left(\frac{\partial \mathcal{S}}{\partial x}g(x)\right)^{-1}\left(\frac{\partial \mathcal{S}}{\partial x}f(x)\right.\\ &\quad \left.+K\varrho(x)\right)-2(k(t)+\eta)\left(\frac{\partial \mathcal{S}}{\partial x}g(x)\right)|\varrho(x)|\\ &= 2\varrho(x)\frac{\partial \mathcal{S}}{\partial x}g(x)d(x,t)-2K\varrho^2(x)\\ &\quad -2(k(t)+\eta)\left(\frac{\partial \mathcal{S}}{\partial x}g(x)\right)|\varrho(x)|\end{aligned}$$

By assuming the adaptive gain $k(t) > |d(x,t)|$, we get

$$\frac{d}{dt}\varrho^2(x) \le -2K\varrho^2(x), \tag{39}$$

for all $x \in \mathbb{R}^n \backslash \wp$ and $\chi(\varrho(x), \xi(x)) = 1$. This implies that the value of $\varrho^2(x)$ decreases and system state moves to the interior of $[\mathcal{K}, \mathcal{KL}]$ sector in finite-time depending on the selection of constant K to sufficiently large and control input $u_i(t)$ (34) inside the $[\mathcal{K}, \mathcal{KL}]$ sector is as

$$u_i(t) = -(k(t)+\eta)\mathrm{sgn}(\varrho(x)) \tag{40}$$

to compensate the effect of exogenous/model uncertainty, while convergence of system state toward equilibrium point.

With the movement of system state from outside to the interior of $[\mathcal{K}, \mathcal{KL}]$ sector with combined control algorithm (35), the Lyapunov-like storage function decreases according to Lemma 1 with $\xi^2(x) = \rho\alpha_3(\|x\|)$ and $\hat{\alpha}_3(\|x\|) = (1-\rho)$ $\alpha_3(\|x\|)$ for any Class $\mathcal{K}_\infty$ function $\alpha_1, \alpha_2, \alpha_3$ and Class $\mathcal{K}$ function $\hat{\alpha}_3$. Due to the adaptation nature of the designed control, the derivative of simplified Lyapunov function based on adaptive scheme (30) for the closed-loop system is written as

$$\begin{aligned}\dot{V}(x,\delta) &= \dot{V}(x)+\delta\dot{\delta}\\ &= \frac{\partial V}{\partial x}f(x)+\frac{\partial V}{\partial x}g(x)(w(t)+d(x,t))\\ &\quad +\delta\left(\dot{k}(t)-\frac{1}{\beta}\frac{d}{dt}|\bar{v}_{eq}|\right)\end{aligned} \tag{41}$$

One can show the stability analysis with constraints to be outside of $[\mathcal{K}, \mathcal{KL}]$ sector i.e. $|\varrho(x)| > \xi(x)$ and a switch $\chi(\varrho(x), \xi(x))$ to be 1, such that

$$\begin{aligned}\dot{V} &= \varrho^2(x)-\xi^2(x)+2\varrho(x)(u_0(t)+d(t,x))\\ &\quad -\hat{\alpha}_3(\|x\|)+\delta\left(\dot{k}(t)-\frac{1}{\beta}\phi(t)\right)\end{aligned} \tag{42}$$

where $\varrho(x) = \frac{1}{2}\frac{\partial V}{\partial x}g(x)$.

We supplant the derivative of square of invariant function $\varrho^2(x)$ as given in (39) and the value of $\dot{k}(t)$ in (32) in above equation as

$$\begin{aligned}\dot{V} &= \varrho^2(x)-\xi^2(x)-\hat{\alpha}_3(\|x\|)+\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}\frac{d}{dt}\varrho^2(x)\\ &\quad -2\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}\varrho(x)\left(\frac{\partial S}{\partial x}f(x)\right)-\Upsilon(t)|\delta|-\frac{1}{\beta}\delta\phi\\ &\le \varrho^2(x)-\xi^2(x)-\hat{\alpha}_3(\|x\|)-2\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}K\varrho^2(x)\\ &\quad -2\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}\varrho(x)\left(\frac{\partial S}{\partial x}f(x)\right)-\Upsilon(t)|\delta|-\frac{1}{\beta}|\delta||\phi|\\ &= -\left(2\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}K-1\right)\varrho^2(x)-\xi^2(x)-\hat{\alpha}_3(\|x\|)\\ &\quad -2\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}\varrho(x)\left(\frac{\partial S}{\partial x}f(x)\right)-\Upsilon(t)|\delta|-\frac{1}{\beta}|\delta||\phi|\end{aligned}$$

Now substituting $\varrho^2(x) > \xi^2(x)$ and $|\phi| \le qA$ in the above equation

$$\begin{aligned}\dot{V} &\le -2\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}K\varrho^2(x)-\hat{\alpha}_3(\|x\|)-\Upsilon(t)|\delta|+\frac{qd_1}{\beta}|\delta|\\ &\quad -2\left(\frac{\partial S}{\partial x}g(x)\right)^{-1}\varrho(x)\left(\frac{\partial S}{\partial x}f(x)\right)\end{aligned}$$

If K is sufficiently large to satisfy the inequality (36) and $\Upsilon(t) > \frac{qA}{\beta}$, then

$$\dot{V} \le -\hat{\alpha}_3(\|x\|), \quad \forall x \in \mathbb{R}^n \backslash \wp \text{ and } \chi(\varrho(x), \xi(x)) = 1 \tag{43}$$

After being entered to the interior of $[\mathcal{K}, \mathcal{KL}]$ sector, the *Hands-Off* control is gone to be offline and only adaptive sliding mode control is active to compensate the exogenous/model uncertainty. The constraints on the interior of $[\mathcal{K}, \mathcal{KL}]$ sector is modified as $|\varrho(x)| \le \xi(x)$ and a switch $\chi(\varrho(x), \xi(x))$ is set to be zero. The derivative of Lyapunov-like storage function for an adaptive scheme is given as,

$$\begin{aligned}\dot{V} &= \varrho^2(x)-\xi^2(x)+2\varrho(x)(u_i(t)+d(t,x))\\ &\quad -\hat{\alpha}_3(\|x\|)+\delta\left(\dot{k}(t)-\frac{1}{\beta}\phi(t)\right)\end{aligned} \tag{44}$$

The adaptive term occurs in the Lyapunov function is calculated to be negative in the above discussion (43), then

$$\begin{aligned}\dot{V} &\le \varrho^2(x)-\xi^2(x)-\hat{\alpha}_3(\|x\|)\\ &\quad +2\varrho(x)(-(k(t)+\eta)\mathrm{sgn}(\varrho(x))+d(t,x))\end{aligned} \tag{45}$$

With the inequality $|\varrho(x)| \le \xi(x)$ and $k(t) > |d(x,t)|$, we get

$$\dot{V} \le -\hat{\alpha}_3(\|x\|), \quad \forall x \in \wp \text{ and } \chi(\varrho(x), \xi(x)) = 0 \tag{46}$$

Therefore, as the system state remains inside the $[\mathcal{K}, \mathcal{KL}]$ sector, the control will set at off position and if its state reaches at the boundary of $[\mathcal{K}, \mathcal{KL}]$ sector, then the control becomes active and forced to move the system state to inside of $[\mathcal{K}, \mathcal{KL}]$ sector. In this process, the energy of the system keeps decreasing and the following inequality holds:

$$\dot{V}(x) \le -\hat{\alpha}_3(\|x\|), \quad \forall x \in \mathbb{R}^n \tag{47}$$

Thus, the nonlinear system (3) is asymptotically stable about an equilibrium point.

V. DESIGN EXAMPLE

Consider a first-order input-affine nonlinear differential equation as

$$\dot{x} = -0.5x + x^3 + w(t) + d(x, t) \tag{48}$$

where the control signal $w(t)$ is the combination of two mutually independent controllers. The first controller allows the convergence of the solution towards the origin for disturbance-free nonlinear system. However, the second controller act as a disturbance observer to remove the unknown uncertainty. For simplicity in the simulations, the unknown uncertainty is chosen as $d(x, t) = 0.1 + sin(3t)$. However the disturbance $|d(x, t)| \leq D = 1.1$ and the bound for rate of change of disturbance $d(x, t)$ is to be $|\dot{d}(x, t)| \leq A = 3$. The design parameters for adaptation law follows $\eta = 0.05$, $\beta = 0.99$, $\epsilon = 0.01$ and $q = 1.1$. The time constant of the low-pass filter is $\tau = 0.01$. The initial value of design example is 5.0. However, initially the adaptive gain and an estimation of equivalent control is suggested as 4 and 3 respectively. Furthermore, the fixed gain for *Hands-Off* control is considered to be $K = 0.8$ and the variable gain based on adaptation law is able to track the variation in the disturbance with inequality $k(t) > |d(x, t)|$ such that the error variable is transposed towards zero.

As there is no proper approach to select an appropriate Lyapunov function, so one can intuitively select a smooth Lyapunov function which consists of system state and an error variable is shown as $V = x^2 + \delta^2$. However, the derivative of Lyapunov-like storage function is directed to a stable/good region with combined control algorithm for a case of perturbed system and afterward, only adaptive sliding mode control remains active for further movement of system state towards the origin.

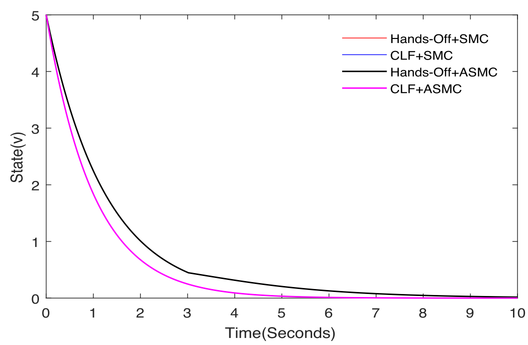

FIGURE 1. Evolution of system state.

For the design example, the convergence of the solution towards the origin is depicted in Fig. 1 for various controllers. In this, a *Hands-Off* and a control-Lyapunov function are allowed to define the convergence rate. While the sliding mode control and adaptive sliding mode are considered

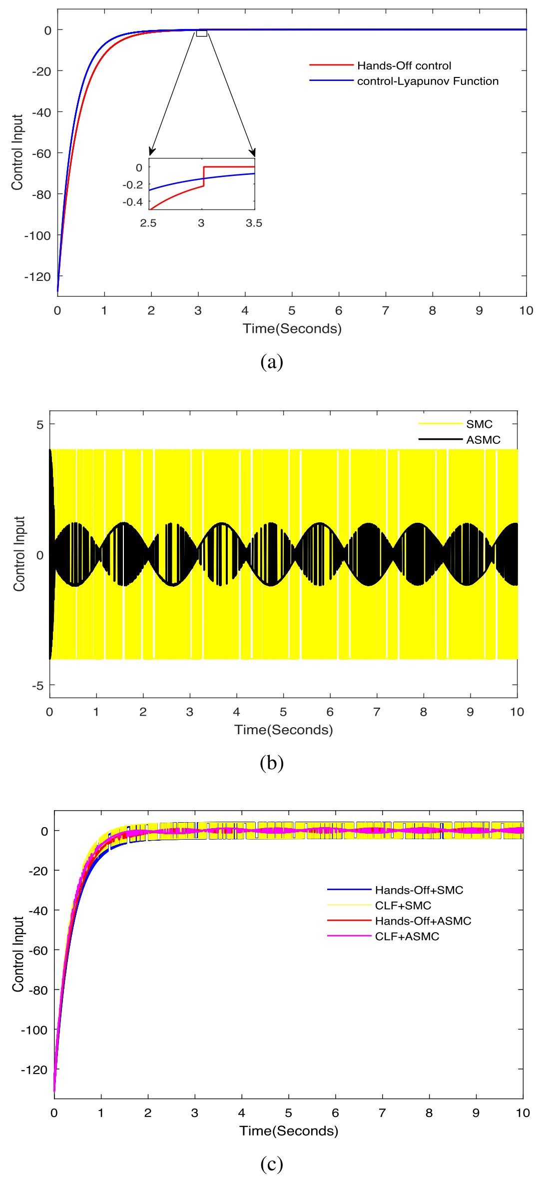

FIGURE 2. Simulation results. (a) Evolution of nominal control ($u(t)$). (b) Evolution of disturbance rejection controls ($v(t)$). (c) Evolution of combination of overall controls ($w(t)$).

as a disturbance observer to attenuate the unknown exogenous/model uncertainty. Thus, the state vector for combined controls are overlapped to one another. Here, the control action involved in shifting the system state to the interior of [$\mathcal{K}$, $\mathcal{KL}$] sector by an involvement of switching mechanism depends on the values of nonlinear functions $\varrho(x)$ and $\xi(x)$ such that the *Hands-Off* control show its presence only at the constraint condition $\varrho(x) > \xi(x)$. However, a control-Lyapunov function as discussed in [20], monotonically decreases toward the origin as time progresses to obtain asymptotic stability. Thus, both these controllers for disturbance-free nonlinear system is shown in Fig. 2(a). While the sliding mode control and the adaptive sliding mode control for attenuation of unknown disturbance are depicted

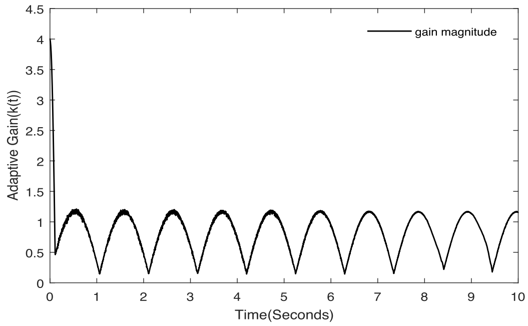

FIGURE 3. Evolution of time-varying adaptive magnitude $k(t)$.

in Fig 2(b) respectively for fixed gain and for adjustable gain to a minimum value to control the magnitude of numerical chattering. Finally, the response of all four combinations for above nominal controllers and disturbance observers are shown in Fig. 2(c). Moreover, the absolute tracking of unknown disturbance to extract a minimum admissible value of gain magnitude is depicted in Fig. 3.

VI. CONCLUDING REMARKS

This paper shows an elegant and succinct design of [$\mathcal{K}$, $\mathcal{KL}$] sector by decomposition of n^{th} dimensional space based on results of the derivative of Lyapunov function. The study of a nonlinear system is done in perspective of the comparison function. Here, the control algorithm is designed such that the minimum support is provided to the nonlinear system holding significant uncertainties to achieve asymptotic stability. Thus, the design of a *Hands-Off* control allows the system state to move to the interior of [$\mathcal{K}$, $\mathcal{KL}$] sector while the adaptive sliding mode obtains a low-pass filter to filter out the high frequency and cancel the effect of unknown disturbance throughout the system.

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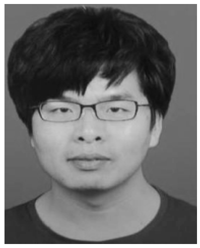

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