

PREFACE

This dissertation is focused on non-commutative aspects of the study of the generalizations of $C1$, $C2$, $C3$, and $D2$ modules. These generalizations include pure $C1$ modules, pure $C3$ modules, finite $C3$ modules, and copure $D2$ modules. Throughout the work, our aim is to discover new properties and characterizations of these modules over various classes of rings.

The dissertation comprises five chapters. In **Chapter 1**, we provide preliminary notions and results that we will use in the subsequent chapters.

In **Chapter 2**, we introduce the concept of pure $C1$ modules. An R module N is called a pure $C1$ module if every pure submodule of N is essential in a direct summand of N . These modules are a proper generalization of extending modules. We study several properties of pure extending modules and characterize several rings i.e. regular rings, semisimple rings, local rings, and PDS rings in terms of pure extending modules. We also introduce the pure continuous modules as the generalization of the continuous modules.

In **Chapter 3**, we introduce the concept of pure $C3$ modules, which is an extension of the concept of pure $C2$ modules. This class of modules generalizes the class of pure $C2$ modules as well as the class of $C3$ modules. Also, pure $C3$ modules are a strict generalization of $C3$ modules. We give new characterizations of many rings in respect of pure $C3$ modules, namely semisimple rings, pure-semisimple rings, von Neumann regular rings, Noetherian rings, pure-hereditary rings, pure- V -rings, etc. Moreover, we also discuss the pure $C3$ envelope and pure $C3$ cover of a module and introduce the pure quasi-continuous modules as the generalization of the quasi-continuous modules.

In **Chapter 4**, we introduce the concept of finite $C3$ modules, which is an extension of the concept of finite $C2$ modules. This class of modules generalizes the class of finite $C2$ modules as well as the class of $C3$ modules. Also, finite $C3$ modules are a strict generalization of finite $C2$ modules. We characterize hereditary rings, V -rings, regular rings, SI-rings and semisimple Artinian rings in terms of finite $C3$ rings. Moreover, we also discuss the finite $C3$ envelope and finite $C3$ cover of a module.

In **Chapter 5**, we introduce the notion of copure $D2$ modules, which is a generalization of the class of $D2$ modules and the dual notion of copure $C2$ modules. In this chapter, we study the direct sums and direct summands of copure $D2$ modules. Further, we characterize copure semi-simple rings, copure hereditary rings and PDS rings in terms of copure $D2$ modules.