CHAPTER - I

INTRODUCTION

INTRODUCTION

1.1 MOTIVATION FOR RESEARCH WORK

Fractional calculus is the generalization of classical calculus or integer calculus as the orders of integration and differentiation not necessarily to be an integer. Traditional calculus is based on integer order differentiation and integration. The concept of fractional calculus has tremendous potential to change the way we see, model, and control the nature around us. Denying fractional derivatives is like saying that zero, fractional, or irrational numbers do not exist. However, the concept of fractional calculus is not new topic it is having the history of more than 300 years. Control system is a branch of engineering and mathematics that deals with modeling of the dynamic systems and controlling of the systems. To obtain the desired behavior, a designed controller senses the operation of the system, compares the original output of the system to the desired output, takes the necessary actions based on user's specifications or reference models, and makes the system to deliver the desired output. Therefore, in order that the dynamics of a system or process might be properly modified, we need a model of the system, tools for its analysis, ways to specify the required behavior, methods to design the controller, and techniques to implement them. The concept of modeling the dynamic systems comes under the system identification area, the tools to model dynamic systems at a macroscopic level are integrals and derivatives. In the linear systems case, the algorithms that implement the controllers are mainly composed integrals and derivatives. Hence, it is not hard to understand that a way to extend the definitions of integrals and derivatives can provide a way to expand the frontiers for their applicability.

The applications of fractional order control grow vigorously in the last decades that makes researchers to develop new techniques for process identification and controller synthesis. Fractional order derivatives and integrals provide a powerful instrument for the description of memory and hereditary effects in various substances, as well as for modeling dynamical processes in fractal media. This is the most significant advantage of the fractional-order systems over the integer-order systems, in which, such effects or geometry are neglected. A control system is effective when the controller of the same order to that of a system which is controlled.

Many real dynamic systems are better characterized using a non-integer order dynamic model based on fractional calculus or, differentiation or integration of non-integer order. From a certain point of view, the applications of fractional calculus have experienced an evolution analogous to that of control, following two parallel paths depending on the starting point: the time domain or the frequency domain. In the control side, clearly, for closed-loop control systems, there are four situations. They are 1) IO (integer order) plant with IO controller; 2) IO plant with FO (fractional order) controller; 3) FO plant with IO controller and 4) FO plant with FO controller. This thesis dealt with integer order plant with fractional order controller.

In general, the basic control actions and their effects in the controlled system behavior in the time domain are Proportional, Integral and Derivative and their main effects over the controlled system behavior are

- To increase the speed of the response, and to decrease the steady-state error and relative stability.

- To increase the relative stability and the sensitivity to noise, for derivative action

- To eliminate the steady-state error and to decrease the relative stability, for integral action.

The positive effects of the derivative action (increased relative stability) can be observed in the frequency domain by the π/2 phase lead introduced, and the negative ones (increased sensitivity to high-frequency noise) by the increasing gain with slope of 20 dB/dec. For the integral action, the positive effects (elimination of steady-state errors) can be deduced by the infinite gain at zero frequency, and the negative ones (decreased relative stability) by the π/2 phase lag introduced. Considering this, it is quite natural to conclude that by introducing more general control actions of the form s^n , $1/s^n$, $n \in R^+$, we could achieve more satisfactory compromises between positive and negative effects, and combining the actions we could develop more powerful and flexible design methods to satisfy the controlled system specifications.

From control engineering point of view, doing something better is the major concern. Existing literature proves that the best fractional order controller outperforms the best integer order controller and can lead to more robust control performance. Thus, a renewed interest has been generated to fractional order systems in the area of automatic control, especially the fractional order $PI^{\lambda}D^{\mu}$ controller. The study of observability, controllability, stability, stabilizability of the fractional dynamical systems are the latest trending topics dealt with by researchers.

Fractional PID control is a useful control strategy since it provides five parameters to be tuned as opposed to the three available in ordinary PID control. It arises from the classical one after considering integrator and differentiator of arbitrary real order. Moreover, these two parameters still have a clear physical interpretation and the generalized fractional order operator are linear. Hence the same methods and rules applicable for linear control system design can also be applies for fractional order PID control system design in frequency domain.

In reality, most of the systems are of fractional order and therefore the fractional order controllers are needed to get an optimum result. Even for integer order systems, the fractional order controllers give better freedom to achieve desired performance. Several methods have been proposed for tuning fractional PID controller in both time domain and frequency domain. Zielger-Nichols type empirical rules and constrained optimization methods are well-known frequency domain approaches. Dominant pole placement tuning and optimal tuning based on minimization of integral time domain performances are widely used time domain approaches. Some methods involve both frequency domain and time domain approaches for tuning gains and fractional powers respectively.

Buck converters are employed in order to transform an unregulated DC voltage input (i.e. a voltage that possibly contains disturbances) in a regulated output voltage. For example, a DC-DC power converter can transform an unregulated (i.e. distorted) 9V input voltage in a regulated (i.e. "clean") voltage of 12V at the output. Some DC-DC power converters have a fixed output reference and ensure that such voltage is always delivered, no matter what the input is; some others can have a variable output reference, which can be therefore set depending on the current need of the device the power converter is used in. The buck converter discussed in this work belongs to this second category. In particular, the converter is able to deliver output voltages both higher as well as lower than (or even equal to) the input voltage; this is why it is referred to as a “buck-boost” (or “step-up/step-down”) power converter.

Such power converters are needed in a vast number of electrical devices, which on one side is a motivation for this project and on the other side, also explains why much research is still conducted on this topic. The thesis used the buck converter, which is a non-monotonic system.

1.2 CONTRIBUTION OF THE THESIS

- Designed and analyzed the fractional order CRONE controllers and Fractional order Lead-Lag Controller for the given system.
- Proposed a novel technique to design the FO-PI controller for Non-Monotonic Phase Systems. Compares the performance of the proposed controller with the conventional controller
- Proposed an Improved Analytical Design of FO-PD Controller for Non-Monotonic Phase Systems.
- Designed a fractional order phase shaper for delay systemrmances with Phase shaper and without phase shaper.
- Frequency Domain based a FOPID controller for process control systems and compares the performances proposed controller with the conventional PID.

1.3 ORGANIZATION OF THE THESIS

In this thesis Fractional order proportional derivative and Proportional Integral is designed for non-monotonic phase systems. The FOPID controller is for process control. The fractional order phase shaper is designed for delay system.

The work reported in this thesis is organized into seven chapters. In chapter 1, contains the importance of the fractional order control over the integer order control. Gives the brief introduction about the buck converter and in second part the complete literature review carried out on integer order modeling, fractional order modeling, tuning of integer order PID controllers and tuning of fractional order PI, fractional order PD and fractional order PID controllers. Also the applications of the fractional calculus have been discussed. Chapter 3 presents the design and analysis of the three generations of CRONE controllers and fractional order Lead-Lag Controllers. Simulation results prove that the designed controllers fulfill the pre defined specifications for the given system. In chapter 4, an improved analytical technique for the design of Fractional order proportional integral controller for Non-Monotonic system has been discussed. The design is carried out in terms of frequency domain specifications. In chapter 5, design A fractional-order Proportional derivative controller is proposed for non monotonic phase process**.** Emphasizing mainly on the open loop frequency response parameters of the control systems, a practical and systematic tuning procedure has been developed for the proposed FO-PD controller synthesis**.** In chapter 6, frequency domain based tuning of FOPID controller is carried out. The design process is carried out in terms of gain cross over frequency, phase margin, robustness constraint, sensitivity function and complementary sensitivity function. The proposed controller is compared with its conventional PID controller. In chapter 7, the design of fractional order phase shaper has been proposed. The design of phase shaper is carried out by using the Bodes integral formula. Also a classical PID controller is designed which is incorporated in series with the phase shaper. In chapter 8, the major contributions of this research work are summarized.

1.4 LITERATURE SURVEY

1.4.1 BUCK REGULATOR

The DC-DC buck converters are the simplest electronic circuits which converter one level of voltage into another level by switching action. The applications of these converters are numerous due to their use in electronic equipments like personal computer, office equipments, appliance control, automotive, telecommunication equipments, aircraft etc. the brief survey of these converters are presented in these section.

The analysis and implementation of the resonant converter is presented in Herman et al. (1992). Guichao hua and fred lee (1995) proposed some soft switching techniques aiming at combining the desirable features of conventional PWM converter and resonant converters. Zero voltage switching (ZVS) DC-DC converter proposed by poon and pong (1996). The analysis and behavior of ZVS active clamp fly back converter are presented Robert Watson, Fred Lee and Guichao Hua (1996). The nonlinear behavior of closed loop pulse width modulated DC-DC converter studied in Mariodi Bernardo et al (1998). A symmetrical zero voltage switched half bridge DC-DC converter is modified by Rais Miftakhutdinov et al (1999).

Marcelo Lobo Heldwein et al (2000) presented a simple clamping circuit for the ZVS PWM asymmetrical half bridge DC-DC converter. A control technique for DC-DC converters using an improved resonant switch model presented by Trevor Smith et al (2000). Peng Xu et al (2001) proposed a family of DC-DC converters which employs an innovative interleaving concept by using series primary windings and interleaved parallel secondary sides. Pit Leong Wong et al (2001) proposed integrated coupling inductors in multichannel interleaving voltage regulator modules. Jianhong Zeng et al (2002) designed and analyzed the optimization of a DC-DC converter which are used for applications. a single stage power factor correction converter with direct energy transfer feature presented in Khalid Rustom et al (2002). Mor Mordechai Peretz and Shmuel Ben Yaakov (2012) developed the time domain design method for the digital controller of pulse width modulation DC-DC converter. Lin and Hou (2012) proposed the analysis, design and implementation of a DC-DC converter with two series connected half bridge converters without an output inductor. Zhe Zhang et al (2012) proposed the converter that consists of push pull forward half bridge circuit and a high frequency transformer.

1.4.2 INTEGER ORDER PID CONTROLLER

In literature many authors contributed to the area of control systems and proposes their own algorithms to design controller in the area like adaptive control theory and/or robust control theory and implements for real systems. In spite of these complex and difficult algorithms exits, the simple and easy to implement proportional integral derivative (PID) controllers continue to be widely employed in process industries.

The main cause for the PID controller to gain the popularity in control engineering are summarizes as

- The PID controller is simple and easy to implement
- Each parameter plays an importance in order to stabilize the system
- The user's friendly controller and it is simple to tune.

Because of the reasons cited above, it is still popular among the other controllers. Due to their remarkable effectiveness, relatively explicable structure and simplicity of implementation PID controller are widely employed by control engineers. Some of the open loop and closed loop modeling methods and tuning of PID controllers are briefly described in the following paragraphs.

Ziegler Nichols (1942) is the first person to introduced a time domain approach to identify the Proportional gain, Proportional gain Integral gain and Proportional gain Integral gain Derivative gain of the PID controller based on parameters obtained from the process reaction curve (open loop response curve obtained for a step change in manipulated variable). He also proposed frequency domain approach in which the controller parameters are calculated from the system gain (which causes sustained oscillations in the closed loop step response) and ultimate period (which is the period of oscillation of sustained oscillations).

Chien et al. (1952) modified the Z-N technique to tune the PID parameters. He modified step response by using quickest response without overshoot or quickest response with 20% overshoot as a design criteria.

Cohen and Coon (1953) proposed empirical formulae for finding out the parameters of P, PI and PID controllers in terms of process parameters of FOPDT model. He found that the inclusion of the controller resulted in the oscillatory response and hence he removed the controller and opened the closed loop, and obtained the process reaction curve for a step change in the manipulated variable. He proved that open loop step responses of all the processes were sigmoidal curves and they can be approximated as FOPDT models. Then he used various techniques for calculating the parameters of P, PI and PID controllers.

Sundaresan and Krishnaswamy et al (1978) designed a PID controller based on the two time instants of the process reaction curve for estimation of dead time and time constant of FOPDT model.

Hougen et al (1979) derived a new strategy for tuning the parameters of PI controller by using synthesis of the process that is of third order. This technique is applicable for third order models with time delay (TOPTD). The tuning is based on steady state gain, open loop poles and delay time of the system.

Astrom and Hagglund et al (1984) developed a closed loop technique in which relay feedback controller was used to obtain the non parametric model of the process namely ultimate gain K_u and ultimate frequency ω_u, in analogy with the original idea of the ultimate sensitivity experiment of Ziegler Nichols (1942), where the control system led to the stability limit.

Polonyi et al (1989) proposed a technique for tuning of PID parameters based on performance index like Integral Square error (ISE), Integral Absolute Error (IAE). He calculated the parameters in closed loop environment.

Ho et al (1995) introduced frequency domain based strategy of tuning the PID parameters. The controller achieves the desired gain margin and desired phase margin. These type of tuning method are useful in the setting of adaptive control and auto-tuning, where the controller parameters have to be calculated on-line.

Skoczowski and Tarasiejski et al (1996) developed a analytical method of controller design for general process model with repeated poles. The method is based on gain margin and phase margin.

Hagglund et al (1996) designed a predictive PI controller. The tuning methods are suitable for process with long dead time. The proposed controller has advantage over the classically designed PID in terms of predicting the input signal even the process has long dead time.

Schaedel et al (1997) developed a tuning method to design the PID controller, which are applicable for third order plus time delay systems. The specialty of the tuning process is the plant can have the complex poles and zeros.

Wang and Shao et al (2000) designed PI controller by using optimization of load disturbance rejection with constraint that the Nyquist curve of the loop transfer function is tangent to a line parallel to the imaginary axes in the left-half of the complex plane.

Wang et al (2001) developed internal model control-based single-loop PID controller design method. Here he applied model reduction technique to find the best single loop controller approximation to the IMC controller.

Zhang et al. (2002) deigned a PID tuning method based on time-domain specifications, such as overshoot and rise time and frequency-domain specifications, such as resonance peak and stability margin.

Kaya et al (2003) presented model-based PI-PD controller design method. He considered PD feedback to change the poles of plant transfer function to more desirable locations for control by a PI controller.

Vrancic et al. (2004) changed the magnitude optimum criterion to optimize disturbance rejection performance, while tracking performance has been improved by an integral set-point filtering PI controller structure.

Leva et al (2005) presented an auto-tuned process controller that is aimed at rejecting load disturbances.

Hamamci and Tan et al (2006) has designed the PI controllers to achieve desired frequency domain specification like gain margin and phase margin and time domain specifications like rise time, settling time, over shoot simultaneously.

Gyongy and Clarke et al (2006) derived an alternative PID auto-tuning approach based on popular step response and Dey and Mudi et al (2009) proposed relay-based methods for auto-tuning of Z-N type PID controllers.

1.4.3 FRACTIONAL ORDER CONTROL

In literature several authors have been introduced their own techniques for modeling and control of fractional calculus. Notable researchers like Oldham and Spanier (1974), Ross (1975), Oustaloup (1981), Podlubny (1994, 1999) proposed their own techniques for modeling of fractional order systems and fractional order control. System identification has become a one of the standard tool for modeling the unknown systems. Since various techniques has been introduced in the literature for modeling the integer order system with experimental data. Here the complexity lies in modeling the fractional system from the experimental data, when fractional orders are present. However, the modeling of integer order systems has been done once the maximum order of the system to be modeled is chosen and the parameters of the model can be optimized directly. On contrary fractional-order systems, identification requires the choice of the number of fractional operators, the fractional power of the operators, and finally the coefficients of the operators. This make the identification of fractional order systems difficult (Bijoy 2009). Existing literature in this field has been limited (Sun 1984, Tsao 1989, Maia 1998). These authors' modeled electrode-electrolyte polarization and mechanical damping behavior by frequency domain techniques for specifically chosen transfer function forms.

(Podlubny 1999) shows Fractional order PID controllers have been increasingly used for process control over the last few years. Various technique has been introduced for tuning of fractional PI, fractional PD (Zhang and Pi, 2012), $PI^{\lambda}D^{\mu}$ controllers both in frequency domain and time domain (Biswas 2009) (Chen YQ 2006). Even some of the techniques take both frequency as well as time domain criteria for designing the controller. Vallerio and Sa Da Costa(2004) proposed a technique which uses frequency domain specifications based on Ziegler- Nichols type empirical rule. Tuning strategy based on the user specified desire phase margin (φ_m), gain cross-over frequency (ω_{gc}), iso-damping/robustness criteria (i.e. flat phase curve around ω_{gc}), high frequency noise rejection index(in dB) and sensitivity function(specified error in dB) using the NelderMead direct search simplex minimization method was introduced by Monje et al (2004, 2008). Bhaskaran et al (2007) proposed tuning of FOPI controller, Ho (1995), Chen YQ (2005) proposed tuning of FOPI/FOPD controllers for controlling integer order systems, Ying Luo et al (2010) proposed tuning of FOPI/FO[PI] controllers for controlling fractional order systems by using the afore mentioned first three constraints. The time domain approaches includes (Maiti Deepyaman 2008) dominant pole placement tuning strategy, (Cao Jun-Yi 2005) optimal tuning strategy , tuning strategy based on minimization of time domain integral performance indices (IAE, ISE, ITAE) was proposed by (Indranil Pan 2011), (Padula and Visioli, 2012). Bettou and Charef (2009) have proposed a combination of frequency domain and time domain approach for tuning FOPID controllers. Generalization is done in recently developed heuristic algorithms such as Particle Swarm Optimization (PSO), Bacterial Foraging Optimization (BFO), and hybrid optimization algorithms for fractional controller tuning. GA based tuning strategy used by (Saptarshi Dasa, 2012), PSO and BFO GA based tuning strategy used by (Sanjoy Debbarma and Lalit Chandra Saikia, 2012) for design of FOPID controllers are also presented in literature. (Chunna zhao 2008) shows the applications of fractional controllers to different types of processes.

Then there are short texts that contain but a short introduction to the field. Within this category, the paper by Kleinz et al.(2000) is most remarkable because of the pedagogical framework its material is set in.

The fractional order calculus is a generalization of the integer order calculus with order of derivatives and integrals are of non integer. Since fractional order systems are of irrational functions we can't directly apply it to the rational systems i.e., integer order transfer function. The difficult problem is to how to implement the irrational functions in to rational function. In literature some work has been done regarding the same, with hardware devices for fractional-order integrator, such as fractances. (e.g., RC transmission line circuit and Domino ladder network) and fractors, but there are some limitations, since it is difficult to design these materials. An alternative way to implement fractional-order operators and controllers is to use finite dimensional integer-order transfer functions. Theoretically, the integer order representation of the fractional operator s^α is infinite dimension. However it is to be noted that a band-limit implementation of fractional-order systems is important in practice, i.e., the finite-dimensional approximation of the fractional order systems should be done in a proper range of frequencies of practical interest.

Some different continuous time approximations or implementations of fractional-order operators and systems are explained below.

Carlson et al (1964) proposed a strategy based on Newton process used for iterative approximation of the α-th root. Predistortion of the algebraic expression is used for approximation of fractional operator. The resulting approximation in real variables (resistive networks) has the unique property of preserving upper and lower approximations to the α-th root of the real number

Oustaloup et al (1991) proposed a technique to approximate the fractional order differentiator which is well received by the research community. The approximation is valid in the pre defined frequency band limit.

Teukolsky et al (1992) proposed the Continued Fractional Expansion method evaluation of functions that frequently converges much more rapidly than power series expansions, and converges in a much larger domain in the complex plane.

Chareff et al (1992) proposed the approximation technique based on singular function method which Consists cascaded branches of pole-zero (negative real) pairs or simple RC section. The chareffs method is having some similarities of Oustaloup technique

Matsuda et al. (1993) proposed an approximation technique based on obtaining the rational transfer function from irrational transfer function by CFE and fitting the original function in a set of logarithmically spaced points.

Vinagre et al.(2000,2001) discussed the formulation of some possible models of fractional order systems, different approximation technique are discussed and analyzed. For continuous models, some methods for obtaining an approximated rational function using evaluation, interpolation and curve fitting techniques are studied. Discrete approximation technique are also discussed and evaluated with appropriate examples with the help of simulation.

Various discrete time approximations or implementations of fractional-order operators and systems are explained below.

Machado (1997) introduces the novel technique to implement the fractional order discrete controller. He confined to time domain which makes the implementation of digital controller easy. Here the approximation is based on MacLaurin series.

Machado (1999, 2001), Vinagre et al.(2000) mentions first order backwards finite difference, Tustin and Simpson approximations based upon MacLaurin series.

Tseng (2001) investigated the design of a fractional order FIR differentiator. He defined fractional derivative of power function. By solving linear equations of the Vandermonde form he obtained impulse response of the fractional order differentiator. He verified by taking an example and proved that fractional derivatives of digital signals are easily computed by using the proposed filtering technique.

Vinagre et al (2001) addressed the Identification of fractional transfer functions from frequency response data.

Chen et al (2003) proposed a novel technique for approximation of infinite impulse response type digital fractional order differentiator. He used stable inversion of the weighted sum of Simpson integration rule and the trapezoidal integration rule for obtaining integer first-order digital differentiators.

Hartley et al. (2003) proposed an identification technique of fractional order system and integer order system by continuous order-distribution concept. The identification is done in frequency domain. Least square technique has been employed for discritised order distribution.

Here a few techniques of modeling of fractional order system, tuning of fractional order controllers for integer order plants and tuning of fractional order controllers for fractional order systems are surveyed below.

Oustaloup et al (1991) Proposed three Crone generation controller i.e., 1st generation crone controller, 2nd generation crone controller, 3rd generation crone controller for the interval systems.. The salient features of the controller are they achieve the user specified gain crossover frequency and the phase margin for the interval systems. It also assures the iso damping property.

Petráš et al (1998) proposed a a novel technique to tuning of fractional order PID controllers for fractional order systems in the frequency domain for the determined stability and dumping level. They designed fractional PID and analyzed the stability of the control system in simulation. They show the drawback of non-adequate approximation of non-integer systems by integer order models and differences in their closed loop behavior. By help of simulation results they showed fractional order PID significantly improve static and dynamic control system properties and could be used as robust controllers because of their less sensitivity to controlled system parameters and controller parameters variations.

Petras (1999) proposed the tuning of fractional-order controllers. He outlines mathematical description of fractional controllers and methods of their synthesis and application. Synthesis method is a modified root locus method for fractional-order systems and fractional order controllers. He proved the inadequacy of approximation of fractional order system with integer order system for controller design and the robustness of the fractional order controllers to the process parameter variations and controller parameter variations.

Podlubny et al (1999) introduced the concept of fractional-order PID controller and derived explicit analytical expressions for the unit-step and unit-impulse response of a linear fractional order system with fractional order controller both for the open and closed loop. While designing the fractional order PID the plant dynamics are takes into consideration it is applicable for linear systems with constant coefficients. On the other hand, they considered a new class of dynamic systems (systems of an arbitrary real order) and new types of controllers (fractional order controllers).

Caponetto et al (2002) introduced a new tuning strategy to design the fractional order PID by using the frequency domain specifications they are gain crossover specification, phase margin specification, robustness specification, sensitivity specification and complementary specification.

Here both integral action and derivative action are of fractional order. Therefore, there are five tuning parameters. The five parameters of a non integer order $PI^{\lambda}D^{\mu}$ controller are validated by step by step extension of classical control theory.

Xue et al (2002) designed and compared the performance of four representative fractional-order controllers in the literature, namely, TID (Tilted Proportional and Integral) controller, CRONE controller (Controle Robuste d'Ordre Non Entier), $PI^{\lambda}D^{\mu}$ controller and fractional lead-lag compensator. They presented basic ideas and technical formulations of the four different types of fractional order controllers with some comparative comments.

Vinagre et al (2002) talked about historical introduction to fractional calculus, fractional calculus fundamentals, models or representations of fractional order systems and fractional order controllers in time domain, Laplace domain and Z domain to study their transient and steady state performances and to determine the conditions for stability, controllability and observability and to find the error static coefficients, frequency domain approach of analog and discrete approximations of fractional order operators. He also designed optimal fractional controller for a class of commensurate fractional order systems based on Wiener – Hopf design method.

Chen et al (2003) Introduces fractional order disturbance observer (FO-DOB) for vibration suppression applications such as hard disk drive servo control. The advantages of the controller are it achieves the major problem of trade off between the phase margin loss and strength of low frequency vibration suspension.

Monje et al (2004) proposed a Fractional Order Lead Compensator with respect to the traditional lead compensator, introduces a new parameter, α, the fractional order of the structure. Two methods are proposed for the design, one of them analytical and the other graphical. The salient features of the controller are by introduction of parameter α, fractional order of the structure, allows flexibility on the fulfillment of specifications of phase margin, gain crossover Frequency and static error constant

Chen et al (2004) Proposed a Fractional Order Phase Shaper which is designed on the concept of bode integral relationship. Here phase shaper is incorporated in series with the classical PID controller and to design phase shaper no prior plant model is considered. The advantage of the phase shaper is it achieved width of the phase flatness region adjustable. i.e, idea of “flat phase”,, the phase derivative w.r.t. the frequency is zero at a given frequency called the “tangent frequency” so that the closed loop system is robust to gain variations and the step responses exhibit an iso-damping property.

Zhao et al (2005) designed frequency domain based Fractional Order PD and Fractional Order PID controller for non integer order systems and for fair comparison he designed Integer Order PD and Integer Order PID controllers. He proved the proposed fractional order controllers outperform the integer order controllers with the help of simulation results. In their proposed design he used user specified gain crossover frequency and phase margin. For finding ω_p, ω_c , λ and μ he used optimization method based on some user specified constraints and derived the equations for K_p, K_i and K_d in terms of ω_p, ω_c , λ and μ.

Cao et al (2005) proposed the tuning of FOPID controller based on GA optimization technique, the objective of which is the weighted combination of ITAE and control input. He approximated the irrational function i.e., fractional order PID controller in to rational function i.e., integer order transfer function by using CFE.

Xue et al (2006) designed fractional order PID for a DC motor with elastic shaft, using minimization of ISE and ITAE by selecting the range of λ and μ between 0.5 and 1.5 randomly and proposed a modified approximated realization method for implementing fractional PID controller. he proved that the fractional order PID controller out performs the classical designed integer order PID controller in terms of gain margin , phase margin and robustness.

Cao et al (2006) designed enhanced PSO the objective of which is the weighted combination of ITAE and control input based FOPID controllers for different order processes. To design the parameters of FOPID controllers, the enhanced PSO algorithms was adopted, which guaranteed the particle position inside the defined search spaces with momentum factor. The numerical realization of FOPID controllers used the methods of Tustin operator and continued fraction expansion. Experimental results showed the effectiveness of the proposed design method in tuning the parameters of FOPID controllers and its performance was compared with GA based

FOPID controllers and the efficiency of the enhanced PSO based FOPID controllers over GA based FOPID controllers were proved.

Cheng et al (2006) designed fractional-order proportional derivative (PD) controllers for stabilizing unstable first-order plus time-delay (FOPTD) systems. They investigated how the fractional derivative order in the range (0, 2) affects the stabilizability of unstable FOTD processes. To estimate the stability domain in the space of process and controllers parameters they used D-partition technique. The necessity to characterize the stability boundary is to describe and compute the maximum stabilizable time delay as a function of derivative gain and/or proportional gain. They have proved that for the same derivative gain, a fractional-order PD controller with derivative order less than unity has greater ability to stabilize unstable FOTD processes than an integer-order PD controller. Thus the fractional-order PD controller can provide the use of higher derivative gain than an integer-order PD controller.

Natraj et al (2007) proposed the robust fractional-order controllers using the principles of quantitative feedback theory (QFT). He synthesizes proportional-integral-derivative (PID) and more general types of fractional-order QFT controllers for a fractional-order plant. The uniqueness of the controller is that, it can handle fractional-order system as well as takes into account the robustness issues like uncertainty and noise.

Bhambhani et al (2008) designed and analyzed the robust-jitter controller named it as optimum fractional proportional integral controller (OFOPI) and to show the superiority of the proposed controller he compared its performance with optimum proportional integral derivative (OPID) controller for systems with small value of τ. Depending on their previously proposed FOPI controller tuning rules using fractional Ms constrained integral gain optimization (F-MIGO), they tried to simultaneously maximize the jitter margin and ITAE performance (minimize ITAE performance index) for a set of hundred KLT systems having different time-constants and time delay values. They proved that the optimization results in enlarged jitter margin of all systems at expense of a slight decrease in ITAE performance of delay dominated systems. Further, the F-MIGO optimization based tuning rules were summarized by approximation of optimized gain parameters and fractional orders of the FOPI controller. By the help of Simulation results they proposed new tuning rules for best jitter margin and ITAE performance.

Wang et al (2009) proposed a novel technique to design FOPI and FO[PI] controllers by using the constrained optimization for the typical first-order velocity servo system in simulation. The user specification or desired specifications gain crossover frequency and the phase margin were achieved and the iso-damping property i.e., the open loop phase derivative with respect to the gain crossover frequency was zero at the gain crossover frequency, this provide closed loop system will be robust to gain variations and the step response exhibited an constant overshoot. The proposed FOPI and FO[PI] controllers designed by constrained optimization technique improves the performance and robustness of the first order velocity servo system. They also proved the better performance of FO[PI] controller over the FOPI controller among the two fractional control schemes.

Luo et al (2009) introduced a optimization based practical and systematic tuning procedure for the proposed FOPD and FO[PD] controller for a class of fractional order system and verified both in simulation and real time and were compared with IOPID controllers tuned based on the same constraints and proved that the proposed controllers are out performs the Integer order controllers in terms of settling time, steady state and rise time.

Chen et al (2009) talked about different types of fractional order controllers and simulation of Fractional Order Transfer Function (FOTF) in MATLAB environment and stability analysis of FOTF in both frequency and domain time domain.

Feliu-Batlle et al (2009) introduced a novel technique to design fractional integral controller combined applied in series with Smith predictors. These fractional order controllers are robust to high frequency model changes and applied in design of controllers for water distribution in a main irrigation canal pool. These results are compared with more complex control techniques as predictive control and robust H_1 controllers. Here special care is taken on time delay changes. The proposed fraction order controller out performs the Integer order PI and PID controller in terms of less sensitivity high frequency measurement noise and disturbances.

Al-Alaoui et al (2009) discussed about rational discrete approximation technique of continuous time parallel fractional PID controllers by direct and indirect discritization methods. To prove the superiority of the proposed technique he considered the step response of the unity feedback system with analog approximation and compared it with direct and indirect discretization approaches. The results shows that direct discretization methods yield shorter rise time than the analog approximation but indirect discretization using bilinear CFE can approximate at best.

Bettou et al (2009) proposed a new conception method of this fractional $PI^{\lambda}D^{\mu}$ controller is considered. The new tuning method is based on classical Ziegler–Nichols tuning method for calculating the K_p, K_i, K_d and parameters of the fractional $PI^{\lambda}D^{\mu}$ controller and for setting the integration action λ, differentiator action μ he use minimization of ISE index. which means setting the parameters of the classical PID controller, and on the minimum integral squared error criterion by using the Hall–Sartorius method for setting the fractional integration action order λ and the fractional differentiation action order μ. Illustrative examples were presented to show the effectiveness and the simplicity of the proposed method. From the simulation results it was proved that the fractional $PI^{\lambda}D^{\mu}$ controllers have significantly improved the performance characteristics of the feedback control systems compared to the classical PID controllers. In the proposed conception, method can use any other classical parameters tuning method. They also proved the robustness of the controller to model uncertainty. They concluded that their conception technique will be very suitable for already tuned PID controllers because in order to implement the fractional $PI^{\lambda}D^{\mu}$ controller the already existing classical PID controller can be used with given fractional order differentiator and fractional order integrator.

Bettou et al (2010) proposed a fractional order PI –PD controller for a first order plus integrator with time delay process to increase the closed loop control performances. They proved with the help of simulation results the proposed fractional order PI-PD controller is robust and well suited for models with noise compared with conventional integer order controllers.

Luo et al (2009, 2010) designed two fractional order proportional integral controllers, FOPI and FO[PI]. The design process is carried out in terms of user specified gain cross over frequency and user specified phase margin and robustness constraints i.e., open loop phase of the system is flat at gain cross over frequency. To prove the superiority of the designed controllers he compares with the integer order PID for the same plant.

Vale´rio et al (2010) discussed and explained the fundamentals of the theory of derivatives and integrals with arbitrary real or complex orders, fractional transfer functions and their approximations, identification of fractional transfer function models from experimental data.

Delavari et al (2010) introduced a fractional order controller for nonlinear system. fractional controller converts the system with integer derivatives into a system with desired fractional derivatives in order to increases the degree of freedom of the stability The proposed controller has increased the degree of freedom and rate of convergence. It can also reduce the reaching time and error in tracking control.

Narang et al (2010) introduced a novel design technique of fractional order PI controller for fractional order plants. The design is based on open-loop transfer function which is reference model is given by Bode's ideal transfer function. The parameters of the proposed controller are calculated by the non-linear constraints optimization problem. The performance of the proposed fractional order PI controller verified through three fractional order dynamic models. The important property of the closed loop system i.e., robustness is achieved that is proved by the iso-daming property which allows the closed loop over shoot to be constant. The proposed technique appears to have promise for the control of fractional order systems instead of designing a integer order counterpart.

Das et al (2011) designed the nonlinear process dynamics of an operating Pressurized Heavy Water Reactor (PHWR) as several linearized transfer function models from practical test-data with standard variants of Least Square Estimation, around various operating points and reduced as NIOPTD-I and NIOPTD-II (non integer order plus time delay) models.

Das et al (2011) designed the fractional order PID controller in both frequency domain and time domain for reduced fractional order models. He compared both the design techniques and concluded that frequency domain approach gives better performance in terms of Robustness (iso-damping property), better capability of high frequency noise rejection, lower value of control signal and good output disturbance rejection property. Therefore reduced size of the actuator where as time domain optimal tuning methodology is faster, has lesser robustness, high probability of building integral windup, a nice ability to suppress load disturbances and an inability to filter noise.

Luo et al (2011) presented a guideline for choosing feasible or achievable gain crossover frequency and phase margin specifications, and proposed a new FOPI/IOPID controller synthesis for all FOPTD systems. Using this synthesis scheme, the complete feasible region of the gain crossover frequency and phase margin were obtained and visualized in the plane. With this region as the prior knowledge, all combinations of the phase margin and gain crossover frequency were verified before the controller design. The areas of these two feasible regions for the IOPID controller and the FOPI controller were compared. This area comparison revealed, for the first time, the potential advantages of one controller over the other in terms of achievable performances. As a basic step, a scheme for finding the stabilizing region of the FOPI/IOPID controller was presented first, and then a new scheme for designing a stabilizing FOPI/IOPID controller satisfying the given gain crossover frequency, phase margin and flat phase constraint was proposed in details. Thereafter, the complete information about the feasible region of gain crossover frequency and phase margin was collected. This feasible region for the FOPI controller was compared with that for the traditional IOPID controller. This area comparison showed the advantage of the FOPI over the traditional IOPID clearly. Simulation illustration was presented to show the effectiveness and the performance of the designed FOPI controller comparing with the designed IOPID controller following the same synthesis.

Yeroglu et al (2011) proposed two different tuning methods. In the first method K_p and K_i values are tuned using Z-N method, K_d is tuned using Astrom–Hagglund method and λ, μ are tuned based on optimization technique for the required phase margin. In the second method, five non linear equations are formed based on five constraints to ensure the robustness and an optimization technique is used to tune the five parameters of the $PI^{\lambda}D^{\mu}$ for a first order and FOPDT systems with modeling uncertainties.

Vale´rio et al (2011) presented a study on the fundamentals of the theory of derivatives and integrals with arbitrary real or complex orders, fractional transfer functions and their approximations, identification of fractional transfer function models from experimental data, first- and second generation Crone controller, third-generation Crone control and fractional proportional-integral-derivative control.

Efe et al (2011) proposed a neural network based computationally simple $PI^{\lambda}D^{\mu}$ Control for a Quadrotor UAV. The neural network is used to find the coefficients of a Finite Impulse Response (FIR) type approximates, that approximates the response of a given analog $PI^{\lambda}D^{\mu}$ controller having time varying action coefficients and differ-integration orders. The results obtained showed that the neural network aided FIR type controller is very successful in driving the vehicle to prescribed trajectories accurately. He concluded that the response of the proposed scheme is highly similar to the response of the target PI D controller and the computational burden of the proposed scheme is very low.

Padula et al (2012) presented a set of optimal tuning rules for standard (integer-order) proportional-integral-derivative (PID) and fractional-order PID controllers for integral and unstable processes. Minimization of IAE is set as the objective function of optimization for both servo and regulatory performances.

Chenikher et al (2012) proposed a methodology based on optimization with constraints to minimize a cost function subject to H –norm for synthesis of a robust multi-variable fractional order PID controller. The FOPID controller was applied to MIMO plant with importantly multiple delays. The feedback control system with proposed controller guaranteed robustness and best performances. They addressed that these performance specifications are possible only with a good choice of the weighting functions. The obtained results showed the efficiency of the proposed method in time and frequency domains over a standard multi-variable PID controller.

Macias et al (2012) modeled the heating process as a fractional order system in frequency domain and validated in time domain. Fractional order PID controller is tuned with Z-N and minimization of ISE technique.

Debbarma et al (2012) designed a Bacterial Foraging Optimization based FOPID controller in automatic generation control (AGC) of an interconnected two-area reheat thermal System under deregulated environment.

Applications of fractional calculus are, as said above, most numerous. A few examples follow

Wang et al. (1999) proposed a novel technique that allows the diurnal variation of ground heat flux to be computed from the corresponding time series measurement of surface soil temperature. He derived the fractional operator for Soil temperature and soil heat flux over time at one location, when heat transfer in a soil matrix is described by a one dimensional diffusion equation with a constant diffusivity parameter. This theory relates to application of diffusion of soil by fractional calculus.

Fellah et al.(2004) made a attempt to interpret the fractional diffusion-wave equation (FDWE) via a new time-space fractional derivative wave equation which models frequency-dependent dissipations observed in such complex phenomena as acoustic wave propagating through human tissues, sediments, and rock layers. This theory relates to application of acoustic waves.

Lu et al. (2002) by applying Fractional Advection-Dispersion Equation (FADE) a 3-D analysis of transport of substance by water in soil is attempted. García-Fiñana et al.(2000) (this reference being an application of image processing to biomedicine, another field where several applications are found.

Mathieu et al. (2002) introduces an edge detector based on non-integer (fractional) differentiation can improve the criterion of thin detection, or detection selectivity in the case of parabolic luminance transitions. This strategy of immunity to noise, which can be interpreted in term of robustness to noise in general. This theory relates to application of image processing with reference to geo physics.

Tajahuerce et al.(2000). An optical implementation of the fractional Fourier transform (FRT) with broadband illumination is proposed by use of a single imaging element, namely, a blazed diffractive lens. . This fractional order can be tuned continuously by shifting of the input along the optical axis. This theory relates to application to optics.

Bagley et al. (1983) constructed stress-strain relationships for viscoelastic materials. These relationships are used in the finite element analysis of viscous elastically damped structures and closed form solution to the equations of motion is found.

MATLAB contains fractional system tool box and CRONE toolbox developed by Oustaloup (2000) which are very useful in the analysis of fractional derivatives and integrals and has resulted rapid growth in this field.

FOMCON toolbox developed by Aleksei Tepljakov It offers a set of tools for researchers in the field of fractional-order control. It includes the tuning rules of fractional controller and approximation techniques of fractional order systems in to integer order sytems.

NINTEGER MATLAB toolbox developed by Duarte Valerio and Jose Sa da Costa in 2004. The toolbox provided with modeling, designing and stability analysis of non integer order systems.