CHAPTER 5
A CONSENSUS MODEL TO MANAGE UNAVAILABILITY OF DECISION MAKERS IN GROUP DECISION MAKING

Conventional group decision-making assumes that the set of Decision Makers (DMs) participating in the decision process are fixed and static throughout the process. However, due to technical and non-technical reasons, the DMs cannot be continuously available. The possible reason for such a case is either the DM may not be able to provide the opinion (because of network failure), or the DM decides not to continue in the decision-making process (a malicious DM who intentionally want to delay the process). This chapter considers such a scenario wherein the DMs are sparsely present in the decision-making process. Obviously, the continuous presence of DMs will make him/her more informative and understanding than those who are sparsely available. Thus, based on the availability of the DMs and contributions, the DM's importance needs to be determined. The concept of bounded confidence is used in the proposed method to facilitate the process of decision-making by incorporating the opinions of unavailable DMs too. We propose to assign a higher weight to a DM based on their presence in the decision process. Consideration of the opinion of a DM in a particular iteration based on the opinion in the previous iterations in case of the absence of the concerned DM in an implementation shown here is shown to be useful.

5.1 Background

The continuous presence of a DM in the decision-making process means that the concerned DMs are interested in adequately contributing to the process. The technology also continuously enables their support, says the internet connectivity, and hence, the availability is high. The CRPs carried out in these contexts, some DMs or coalitions of them may not be available at times. Unavailability at times may be there because of personal, sociological, organizational, or technical reasons, and hence the various patterns of unavailability or absence would be observable.

Unavailability of DMs can be of two types:

a). Completely unavailable: This means once the DMs has left, they remained unavailable in the decision-making process. Either such DMs do not participate in the CRP at all or even if they join the process at some stages are considered to be the new participant leading to a dynamic scenario [76]. This type of unavailability has already been discussed in the literature with GDM models developed to deal with.

b). Partially unavailable: This means that the DMs, once unavailable, re-join CRP after some rounds: This needs to be dealt with properly as when previously present DMs re-join the discussion, then what should be the feedback given to him because if not given the feedback properly can retract the consensus. So, proper CRP needs to be developed to deal with those unavailable experts. To the best of our knowledge, this has not been discussed in the literature.

There are situations when the decision time for the negotiation process is limited, or the quorum group size is defined, and then it is hard to reach a consensus when the DMs are sparsely present. The unavailability of DMs can be expected to be feasible for decision-making process management because of the presence of mobile or internet-based new technologies. For example, the DMs may become unavailable on social media or web communities' platforms and re-join the discussion after some gaps. The absences marked could be examined to reflect either an intentional behavior or unintentional behavior of DMs which in any case has the potential to alter the problem's final solution.

Thus, to make a quality and mature decision incorporating the opinions of the complete set of DMs at a given time, a decision-making framework is required. Therefore, it is necessary to address the problem of the unavailability of DMs or partial availability in the active decision-making process.

This chapter discusses a novel consensus framework based on the self-management mechanism to manage the partial availability of DMs in the CRP. In this novel consensus framework, not only the preference information of available experts is considered but also the preference information of the unavailable DM at the time is also brought into the current consensus rounds. The unavailable DMs' preference is generated by the moderator itself based on their preference in the recent past and its bounded confidence value. An approach to obtain the weights of the set of DMs using the availability vector is proposed. We propose detailed simulation experiments and a comparison analysis to justify the validity of the proposed consensus framework in managing the unavailability of DMs.

5.2 Proposed Methods

In this section, a GDM model is presented that is suitable to work in a dynamic environment, and its CRP framework based on bounded confidence is presented. Different from general GDM models, our proposed model allows partial involvement of DMs. To deal with the partial availability, the possible solutions could be to either ignore the unavailable DMs or wait for them to be available back and provide them. In case of considering only the available set of DMs, the CRP would face the loss of information, and the latter case would introduce a sufficient delay in reaching consensus. Thus, both approaches would not suffice for a better-quality decision. Our CRP model overcomes this issue by considering the opinions of the unavailable DMs at times using their preference in the recent past. This is done by consistently updating the opinion of the unavailable DMs along with the available ones.

5.2.1 Dynamically Generating Weights Process

The importance of DM will depend on the contributions made. The continuous presence of DMs in the decision-making process will make him/her more informative and understanding than those who are sparsely present. This implies that the available one can contribute more in decision-making. Also, the likelihood of consistency in the consensus process (or the consensus value) leading to better acceptable decision-making will heavily depend on the patterns of the presence of DMs during the consensus-reaching process. Thus, it would be appropriate to deprecate the weight of such participants who were not available in the recent past. In the pursuit of obtaining as good a decision as when all are present, we weighted the DMs as per their availability, meaning thereby deprecation of the weightage based on the abstinence of DMs that is the highest weightage to the one who is always present. Thus, depending upon the availability of a DM in the decision-making process, their importance should be derived.

Let $d = \{d_1, d_2, d_3, \ldots, d_m\} (m \geq 2)$ be a set of DMs who provide their opinions on a set of alternatives $X = \{x_1, x_2, \ldots, x_n\} (n \geq 2)$ with the aim of obtaining a common solution. Without loss of generality, we here assume, in this chapter that the DM d_p provides his/her preference over X using fuzzy preference relation given by $V_p = \left(v_{ij}^p\right)_{n \times n}$, where $v_{ij}^p \in [0,1]$ denotes the preference degree of alternative x_i over x_j and $v_{ij}^p + v_{ji}^p = 1$.

The dynamic importance of DM will be determined based on the cumulative duration of presence in the decision-making process till then. High importance is given to those who is present for a greater number of rounds than those who keeps on leaving and joining the process. This even will apply for the new DMs joining the process. Therefore, based on the contributions made by a DM in the CRP, the importance will be a function of duration of presence. Specifically, the availability of the DM is determined based on whether he/she has provided their opinion at that particular round or not, is defined as follows:

$$Avail_r^p = \begin{cases} 1, & \text{when } d_p \text{ provides opinion at round } r \\ 0, & \text{when } d_p \text{ does not provides opinion at round } r \end{cases} \quad (5.1)$$

Let $\#Avail_r^p$ denotes the cumulative duration of availability of a DM d_p calculated up to round r, i.e., $\sum_{initial=1}^{r} Avail_{initial}^p = \#Avail_r^p$. Let $w_p^{ext} \in [0,1]$ be the external weight given to the DM d_p and $\sum_{p=1}^{m} w_p^{ext} = 1$, which remains constant throughout the process. Based on the availability of the DM, we calculate the presence factor of each DM termed as pf^p where $p = 1,2, \ldots, n$

$$pf_r^p = \frac{\#Avail_r^p * w_p^{ext}}{r} \quad (5.2)$$

The presence factor can be used to provide importance to the DMs as follows:

$$w_p^{r+1,int} = \frac{pf_r^p}{\sum_{i=1}^{n} pf_r^p} \quad (5.3)$$

The weight function is designed in such a way that when all the DMs are available for all the time then internal weight of a DM d_p at some round r i.e., $w_p^{r,int} = w_p^{ext}$, which is true also. The discussed case is equivalent to the traditional GDM where DMs are supposed to be present throughout the CRP. Also, with increase in the number of rounds if the availability vector $Avail_r^p$ of some DM d_p goes down then from Eq. (5.2) and (5.3), the internal weight $w_p^{r,int}$ of d_p decreases accordingly. This is conducive to the idea of penalizing the DMs who were not available after initially providing their opinions at any successive rounds of decision making.

5.2.2 Consensus Measure

To improve the level of consensus among the DMs the important two key steps are included in the CRP. The first one, consensus measure specifies measuring the degree of agreement or the degree of consensus among the DMs, while the second one, feedback process refers to providing the feedback process to the DMs to improve the consensus level when the group consensus is not acceptable.

Using Manhattan distance, the consensus level of the DM d_p is defined as follows:

$$cl(d_p) = 1 - \frac{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (v_{ij}^p - v_{ij}^c)}{n.(n-1)/2} \tag{5.4}$$

The consensus level of the group is given as:

$$cl = \sum_{p=1}^{m} wt_p cl(d_p) \tag{5.5}$$

The consensus level on a position (i, j) of a preference for decision maker d_p is defined as:

$$cp_{ij}^p = 1 - \left|\left(v_{ij}^p - v_{ij}^c\right)\right|, \text{ where } d_p \in \mathrm{D} \tag{5.6}$$

The consensus level on an alternative x_i for decision maker d_p is defined as

$$ca_i^p = \sum_{j=1, j\neq i}^{n} cp_{ij}^p/(n-1) \tag{5.7}$$

5.2.3 Feedback Mechanism based on Availability of DMs

In the general framework of CRP described in Section 2.1.1, the preference relation of the DMs present in the current consensus round is used to generate the collective preference relation, which is then used to produce the feedback suggestion. Different from general GDM models, our proposed model allows sparse involvement of DMs. Therefore, if a DM re-joins after being unavailable for one or more rounds, this might delay the CRP or negatively impact the existing DMs. To overcome this issue, in this section, we propose a feedback process that automatically updates the opinion of the unavailable DMs and the available ones using the interactive consensus rule. The CRP introduces a feedback mechanism where the DMs, when re-joins, receive suggestions based on the current consensus state. This kind of feedback can make the entire group develop consistency and reach a consensus faster.

When the obtained consensus level is less than the predefined consensus threshold, the feedback mechanism is adopted to improve the consensus level. To do that, we need to identify the alternatives and positions of the DMs preference that contributes less to a high consensus level. To find out the set of positions that the DMs should modify, the following strategy is provided:

Identification of Decision Makers: The set of decision makers DMS that should make modifications is identified as

$$DMS = \{d_p \middle| cl(d_p) < \lambda\} \tag{5.8}$$

Identification of Alternatives: For the DMs identified by (5.8), the set of alternatives ALT for them that should be modified are identified as

$$ALT = \{x_i \middle| ca_i^p < \sum_{i=1}^{n} ca_i^p / n \wedge d_p \in \text{DMS}\} \tag{5.9}$$

Inspired by (5.7), we use the average consensus degree $\sum_{i=1}^{n} ca_i^p / n$ of all alternatives for d_p as the threshold.

Identification of Positions: For the alternatives identified by (5.9), the set of positions of preferences that should be modified is identified as

$$POS = \{(i,j) | cp_{ij}^{p} < \sum_{i=1}^{n}\left(\sum_{j=1,i\neq j}^{n} cp_{ij}^{p}\right) \Big/ (n^2 - n) \wedge x_i \in ALT\} \tag{5.10}$$

where $\sum_{i=1}^{n}\left(\sum_{j=1,i\neq j}^{n} cp_{ij}^{p}\right)/(n^2 - n)$ is the average consensus value at position level. Since degree of consensus at the alternative level is calculated based on all positions associated with the alternatives, we have $\sum_{i=1}^{n}\left(\sum_{j=1,i\neq j}^{n} cp_{ij}^{p}\right)/(n^2 - n) = \sum_{i=1}^{n} ca_{i}^{p}/n$. Once the positions that should be modified are identified, the detailed suggestions will be generated by the feedback mechanism for the DMs.

The feedback mechanism based on bounded confidence consists of two rules: designed for two cases: 1) adjustment direction when DM is unavailable in a consensus round and 2) adjustment direction when DM returns back. Suppose that IR (Identification rule) identifies a DM d_p. Let σ_p be the bounded confidence of d_p, V_a be the feedback advice generated for the DM d_p and let D_{ap} be the distance between the preference relation V_p of DM d_p and the advice generated V_a. Similarly, let D_{pc} be the distance between the collective preference relation V_c and V_p. Regarding the availability of DM in the consensus round, given below are the two possible cases of adjustment suggestion based on the feedback mechanism discussed.

Case (1): Adjustment Direction for unavailable DM

DM d_p who was present in earlier states of the process but is now unavailable. Their modified preference relation in next consensus round after receiving the feedback advice which includes two kinds of advice, discussed as follows:

a) If $D_{pc} \leq \sigma_p$ then it means the collective preference relation V_c will lie within the bounded confidence region of the DM d_p and therefore it can be considered that d_p is willing to accept the generated advice as a personalized advice, i.e., $V_a = V_c$. Consequently, in this case the adjustment direction will be given as follows:

$$\bar{v}_{ij}^{p} = \alpha . v_{ij}^{p} + (1 - \alpha). v_{ij}^{c} \tag{5.11}$$

where $\alpha \in [0,1]$ is the control parameter used to control the degree of adjustment in DMs opinion. For $\alpha > 0.5$ means that the modified opinion will be more influenced by the individual opinion and for $\alpha < 0.5$ means that the modified opinion pays more attention to the collective opinion.

b) If $D_{pc} > \sigma_p$ then V_c can be considered to differ more than a given confidence level of the DM d_p, i.e., V_c does not verifies the bounded confidence of σ_p. Thus advice V_a provided to the DM d_p will be different from the V_c. Therefore, to satisfy the relation $D_{pa} \leq \sigma_p$ and advice being acceptable to the DM d_p, V_a can be produced as follows:

$$V_a = V_p + \frac{\sigma_p}{D_{pc}}\left(V_c - V_p\right) \tag{5.12}$$

The above equation will satisfy

$$D_{pa} = \frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|\frac{\sigma_p}{D_{pc}}\left(v_{ij}^{c} - v_{ij}^{p}\right)\right|}{n.(n-1)/2} = \sigma_p \tag{5.13}$$

Specifically, the Eq. (5.12) can be written as

$$v_{ij}^{a} = \frac{\sigma_p}{D_{pc}} v_{ij}^{c} + \frac{D_{pc} - \sigma_p}{D_{pc}} v_{ij}^{p} \tag{5.14}$$

This guarantees that $v_{ij}^{a} + v_{ji}^{a} = 1$ and $v_{ij}^{a} \in \left[min\left(v_{ij}^{p}, v_{ij}^{c}\right), max\left(v_{ij}^{p}, v_{ij}^{c}\right)\right] \subset [0,1]$. Thus, in order to improve the consensus level of the identified DM, the opinion of the unavailable DM can be determined as:

$$\bar{v}_{ij}^{p} = \gamma . v_{ij}^{p} + (1 - \gamma). v_{ij}^{a} \tag{5.15}$$

Where $\gamma \in [0,1]$ is the control parameter used to control the degree of adjustment in DMs opinion. For $\gamma > 0.5$ means that the modified opinion will be more influenced by the individual opinion and for $\gamma < 0.5$ means that the modified opinion pays more attention to the generated advice.

Case (2): Adjustment Direction when DM becomes available

Not being present for certain rounds of consensus and then returning, the aim is to provide feedback to the DM based on the current consensus state. Once the DM shows willingness to join the consensus process, the system confirms its consensus level. Obviously, the returned DM whose consensus level is below the consensus threshold, is given the feedback suggestion. Thus, the DM updates her opinion by themselves without being automatically updated. It is important to note that the DM may have been absent for one or more rounds of consensus, yet our model still allows them to interact and update the opinion on its current consensus state. The following procedures are being made use of for proving suggestions to the DMs:

The similarity among each pair of DMs d_p and d_q will be computed as follows:

$$Sim_{pq} = 1 - \frac{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|\left(v_{ij}^{p} - v_{ij}^{q}\right)\right|}{n.(n-1)/2} \tag{5.16}$$

These similarities will be used to provide information to each DM about those who share similar opinion about the alternatives. Thus, for each DM $d_p \in D$ the set of neighbors can be defined as:

$$N^p = \{d_{\beta_1}, d_{\beta_2}, \dots, d_{\beta_{NNh}}\} \,\middle|\, Sim_{p\beta_k} > Sim\_avg_p \tag{5.17}$$

where NNh is the number of neighbors to d_p, d_{β_i} is the i^{th} nearest similar DM to d_p and Sim_avg_p is the average similarity index for DM d_p, which can be computed as $Sim\_avg_p = \sum_{i=1, i\neq p}^{m} Sim_{pq}/(m-1)$. Note that instead of average similarity index value a similarity threshold value can also be used depending on which the number of similar neighbors presented to the DMs d_p may vary. Once the neighbors of the decision-maker d_p is identified, their cumulative opinion S_{N^p} is used to provide the adjustment direction to d_p.

Let S_{N^p} be the cumulative preference of N^p, which is calculated by

$$s_{ij}^{N^p} = \frac{1}{\#N^p} \sum_{d_{\beta_k} \in N^p} v_{ij}^k, \qquad i, j \in \{1, 2, \dots, n\} \tag{5.18}$$

Since, $S_{N^p} = \left(s_{ij}^{N^p}\right)_{n \times n}$ is the advice assigned to the DM d_p. The following way can be used to provide specific suggestions to the identified DM d_p which uses two forms of advice as follows:

Next, the following way can be used to provide specific suggestions to DM d_p.

i). If $v_{ij}^p < s_{ij}^{N^p}$, then d_p should increase the value with respect to the position (i, j).

ii). If $v_{ij}^p > s_{ij}^{N^p}$, then d_p should decrease the value with respect to the position (i, j).

Note 1: In this study we assume that the DMs accept the feedback suggestions and do not show non-cooperative behaviors.

5.3 Illustrative Example

In order to clarify how the proposed model works, in this section, an application example is shown. Imagine that a set of 8 DMs $d = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8\}$ are invited to evaluate the IaaS (Infrastructure as a Service) for migration purpose. The DMs preferences have been captured using a decision support approach which allows the DMs to be connected over the internet. They are going to discuss over the 4 different cloud service provider alternatives $X = \{x_1, x_2, x_3, x_4\}$. Initially, all DMs provide their preferences along with their bounded confidence. Preferences provided by the DMs are specified below:

$$V^1 = \begin{pmatrix} .50 & .85 & .56 & .93 \\ .15 & .50 & .70 & .58 \\ .44 & .30 & .50 & .82 \\ .07 & .42 & .18 & .50 \end{pmatrix} \quad V^2 = \begin{pmatrix} .50 & .88 & .99 & .01 \\ .12 & .50 & .87 & .61 \\ .01 & .13 & .50 & .99 \\ .99 & .39 & .01 & .50 \end{pmatrix}$$

$$V^3 = \begin{pmatrix} .50 & .53 & .48 & .80 \\ .47 & .50 & .23 & .50 \\ .52 & .77 & .50 & .90 \\ .20 & .50 & .10 & .50 \end{pmatrix} \quad V^4 = \begin{pmatrix} .50 & .57 & .85 & .74 \\ .43 & .50 & .59 & .25 \\ .15 & .41 & .50 & .67 \\ .26 & .75 & .33 & .50 \end{pmatrix}$$

$$V^5 = \begin{pmatrix} .50 & .08 & .63 & .66 \\ .92 & .50 & .73 & .89 \\ .37 & .27 & .50 & .98 \\ .34 & .11 & .02 & .50 \end{pmatrix} \quad V^6 = \begin{pmatrix} .50 & .77 & .58 & .93 \\ .23 & .50 & .58 & .02 \\ .42 & .42 & .50 & .12 \\ .07 & .98 & .88 & .50 \end{pmatrix}$$

$$V^7 = \begin{pmatrix} .50 & .86 & .48 & .84 \\ .14 & .50 & .21 & .55 \\ .52 & .79 & .50 & .63 \\ .16 & .45 & .37 & .50 \end{pmatrix} \quad V^8 = \begin{pmatrix} .50 & .03 & .61 & .36 \\ .97 & .50 & .05 & .49 \\ .39 & .95 & .50 & .19 \\ .64 & .51 & .81 & .50 \end{pmatrix}$$

Additionally, some related parameter values are set as follows: The acceptable consensus threshold in this process is set to 0.85 and the control parameter α is set to be 0.5. Initially, the availability vector values of all the eight DMs are 1, i.e., $Avail_r^p = 1$ for all $d_p \in D$. The external weight w_p^{ext} of all the DMs d_p is set to be equal.

a). First Round: To calculate the dynamic weight $w_p^{int,r}$ of DMs by Eq. (5.2) and Eq. (5.3). At round $r = 1$, since $Avail_r^p = 1$ then $w_p^{int,r} = w_p^{ext}$. By Eq. (2.1) the collective preference matrix.

$$V_c = \begin{pmatrix} .50 & .57 & .65 & .66 \\ .43 & .50 & .49 & .49 \\ .35 & .51 & .50 & .66 \\ .34 & .51 & .34 & .50 \end{pmatrix}$$

By Eq. (5.4) the DMs' consensus levels are: $cl(d_1) = 0.8184$, $cl(d_2) = 0.6446$, $cl(d_3) = 0.8548$, $cl(d_4) = 0.8972$, $cl(d_5) = 0.7543$, $cl(d_6) = 0.7283$, $cl(d_7) = 0.8296$, $cl(d_8) = 0.7024$. By Eq. (5.5), the consensus level of the group is $cl = 0.7787$. Because $cl < \lambda = 0.85$, the DMs need to adjust their preferences. According to (5.5), the DMs $\{d_1, d_2, d_5, d_6, d_7, d_8\}$ with lower consensus level will undergo the feedback mechanism. Let the availability vector is given as $Avail_p^2 = \{1,1,0,1,0,1,1,0\}$. Since d_3 is already in consensus therefore Case 1 of the feedback mechanism would only be applicable to d_5 and d_8 and Case 2 for the remaining available. Using Eq. (5.16) and Eq. (5.17), identify the neighbors of the inconsistent available DMs given as follows:

$N^1 = \{d_3, d_4, d_6, d_7\}$ $\quad$ $N^2 = \{d_1, d_3, d_4, d_5, d_6, d_7\}$

$N^6 = \{d_1, d_3, d_4, d_7\}$ $\quad$ $N^7 = \{d_1, d_3, d_4\}$

Based on the identified neighbors, their opinions are used to assign advice S_{N^p} (using Eq. (5.18)) to the DM d_1, d_2, d_6 and d_7 are as follows:

$$S_{N^1} = \begin{pmatrix} .50 & .68 & .60 & .83 \\ .32 & .50 & .40 & .33 \\ .40 & .60 & .50 & .58 \\ .17 & .67 & .42 & .50 \end{pmatrix} \quad S_{N^2} = \begin{pmatrix} .50 & .58 & .60 & .80 \\ .42 & .50 & .49 & .55 \\ .40 & .51 & .50 & .80 \\ .20 & .45 & .20 & .50 \end{pmatrix}$$

$$S_{N^6} = \begin{pmatrix} .50 & .70 & .59 & .83 \\ .30 & .50 & .43 & .47 \\ .41 & .57 & .50 & .75 \\ .17 & .53 & .25 & .50 \end{pmatrix} \quad S_{N^7} = \begin{pmatrix} .50 & .65 & .63 & .82 \\ .35 & .50 & .50 & .44 \\ .37 & .50 & .50 & .79 \\ .18 & .58 & .21 & .50 \end{pmatrix}$$

With reference to these suggestions the DMs will modify their opinions accordingly.

b). Second Round: We assume that the DMs accept the advice suggested to them and provide the updated advice as per the recommendation. The modified preference provided by the DMs are as follows:

$$V^1 = \begin{pmatrix} .50 & .76 & .56 & .84 \\ .24 & .50 & .56 & .58 \\ .42 & .45 & .50 & .82 \\ .16 & .42 & .18 & .50 \end{pmatrix} \quad V^2 = \begin{pmatrix} .50 & .89 & .99 & .68 \\ .12 & .50 & .87 & .61 \\ .01 & .13 & .50 & .99 \\ .32 & .39 & .01 & .50 \end{pmatrix}$$

$$V^6 = \begin{pmatrix} .50 & .77 & .58 & .93 \\ .23 & .50 & .58 & .07 \\ .42 & .42 & .50 & .58 \\ .07 & .93 & .42 & .50 \end{pmatrix} \quad V^7 = \begin{pmatrix} .50 & .70 & .48 & .84 \\ .30 & .50 & .40 & .55 \\ .52 & .60 & .50 & .63 \\ .16 & .45 & .37 & .50 \end{pmatrix}$$

Using Case (1), the modified preferences of decision maker d_5 and d_8 generated as follows:

$$V^5 = \begin{pmatrix} .50 & .13 & .63 & .66 \\ .87 & .50 & .73 & .85 \\ .37 & .27 & .50 & .98 \\ .34 & .15 & .02 & .50 \end{pmatrix} \quad V^8 = \begin{pmatrix} .50 & .04 & .61 & .36 \\ .96 & .50 & .06 & .49 \\ .39 & .94 & .50 & .20 \\ .64 & .51 & .80 & .50 \end{pmatrix}$$

At round $r = 2$, the availability vector is given as $Avail_p^2 = \{1,1,0,1,0,1,1,0\}$. Then by Eq. (5.2) and (5.3) the weights $w_p^{int,2}$of the DMs are (0.1538, 0.1538, 0.0769, 0.1538, 0.0769, 0.1538, 0.1538, 0.0769). By Eq. (2.1) the collective preference matrix.

$$V_c = \begin{pmatrix} .50 & .62 & .66 & .76 \\ .38 & .50 & .54 & .46 \\ .34 & .46 & .50 & .73 \\ .24 & .54 & .27 & .50 \end{pmatrix}$$

By Eq. (5.4) the consensus level of DMs are: $cl(d_1) = 0.9091$, $cl(d_2) = 0.7651$, $cl(d_3) = 0.8601$, $cl(d_4) = 0.9051$, $cl(d_5) = 0.7556$, $cl(d_6) = 0.8372$, $cl(d_7) = 0.8899$, $cl(d_8) = 0.6581$. By Eq. (5.5), the consensus level of the group is $cl = 0.8374$. Because $cl < \lambda = 0.85$, the inconsistent DMs $\{d_2\ d_5, d_6, d_8\}$ need to adjust their preferences. The availability vector for the DMs is given as $Avail_p^3 = \{1,1,1,0,1,1,0,1\}$. Using (5.16) - (5.18) identify the neighbors of the inconsistent available DMs using whose preferences the adjustment direction will be suggested. The identified neighbors are:

$$N^2 = \{d_1, d_3, d_4, d_5, d_7\} \qquad N^5 = \{d_1, d_2, d_3, d_4\}$$

$$N^6 = \{d_1, d_3, d_4, d_7\} \qquad N^8 = \{d_3, d_4, d_5, d_7\}$$

Based on the identified neighbors, their opinions are used to assign advice S_{N^p} (using Eq. (5.18)) to the DM d_2, d_5, d_6 and d_8 are as follows:

$$S_{N^2} = \begin{pmatrix} .50 & .54 & .60 & .78 \\ .46 & .50 & .50 & .55 \\ .40 & .50 & .50 & .80 \\ .22 & .45 & .20 & .50 \end{pmatrix} \qquad S_{N^5} = \begin{pmatrix} .50 & .68 & .72 & .76 \\ .32 & .50 & .56 & .49 \\ .28 & .44 & .50 & .84 \\ .24 & .52 & .16 & .50 \end{pmatrix}$$

$$S_{N^6} = \begin{pmatrix} .50 & .64 & .59 & .80 \\ .36 & .50 & .44 & .47 \\ .41 & .56 & .50 & .75 \\ .20 & .53 & .25 & .50 \end{pmatrix} \qquad S_{N^8} = \begin{pmatrix} .50 & .48 & .61 & .76 \\ .52 & .50 & .49 & .54 \\ .39 & .51 & .50 & .79 \\ .24 & .46 & .21 & .50 \end{pmatrix}$$

c). Third Round: The DMs after receiving the advice suggested to them provided the updated preference as per the recommendation. The updated preference of the DMs are as follows:

$$V^2 = \begin{pmatrix} .50 & .86 & .65 & .68 \\ .14 & .50 & .84 & .61 \\ .35 & .16 & .50 & .99 \\ .32 & .39 & .01 & .50 \end{pmatrix} \quad V^5 = \begin{pmatrix} .50 & .21 & .63 & .66 \\ .79 & .50 & .73 & .77 \\ .37 & .27 & .50 & .91 \\ .34 & .23 & .09 & .50 \end{pmatrix}$$

$$V^6 = \begin{pmatrix} .50 & .77 & .58 & .91 \\ .23 & .50 & .58 & .23 \\ .42 & .42 & .50 & .58 \\ .09 & .77 & .42 & .50 \end{pmatrix} \quad V^8 = \begin{pmatrix} .50 & .42 & .61 & .36 \\ .58 & .50 & .07 & .49 \\ .39 & .93 & .50 & .76 \\ .64 & .51 & .24 & .50 \end{pmatrix}$$

By Eq. (5.4) the DMs consensus levels are: $cl(d_1) = 0.9312$, $cl(d_2) = 0.8373$, $cl(d_3) = 0.8824$, $cl(d_4) = 0.8785$, $cl(d_5) = 0.8104$, $cl(d_6) = 0.8588$, $cl(d_7) = 0.8977$, $cl(d_8) = 0.8205$. By Eq. (5.5), the consensus level of the group is $cl = 0.8664$. which is greater than the predefined consensus threshold $\lambda = 0.85$. Hence the expected consensus level is reached. The final collective preference value is:

$$V_c = \begin{pmatrix} .50 & .63 & .60 & .74 \\ .32 & .50 & .42 & .57 \\ .25 & .58 & .50 & .49 \\ .51 & .43 & .52 & .50 \end{pmatrix}$$

According to [107]'s method, the ranking of the alternatives is $x_1 > x_3 > x_2 > x_4$ (0.6194, 0.4713, 0.5391, 0.3702)

5.4 Simulation and Comparison Analysis

In this section, a simulation analysis that explores the effectiveness of the proposed model on consensus reaching is presented, together with its comparison with a consensus model dealing with the dynamic set of DMs is presented to illustrate the characteristics of the proposed model.

5.4.1 Effect of number of unavailable and available DMs

In this section, we performed an experiment based on the number of available and number of unavailable DMs. The experiment is performed for different number of DMs at consensus threshold $\lambda = 0.85$ with control parameter α = 0.5 for about 1000 instances.

The following experiment is performed to understand the effect of the number of unavailable and the number of available DMs in the CRP. For the purpose of experiment, the availability vector at each round is generated randomly for each of the following two cases: (i). when the number of available DMs say nUA are greater than the number of unavailable DMs nA and (ii). when the number of available DMs are smaller than the number of unavailable DMs. From Fig. 5.1 the following observations can be drawn:

a) The average round to reach consensus increases as the number of DMs increases which means that the speed of convergence to consensus will slow down when number of DMs increases as it is obviously expected.

b) For any number of total DMs, the average consensus rounds will be greater when the number of available DMs are smaller than the number of unavailable DMs, i.e., the group consensus is achieved more quickly when majority of the DMs are available. This is because even though the opinions of unavailable DMs are considered, the proposed model gives high credence to the available ones in comparison to the unavailable DMs.

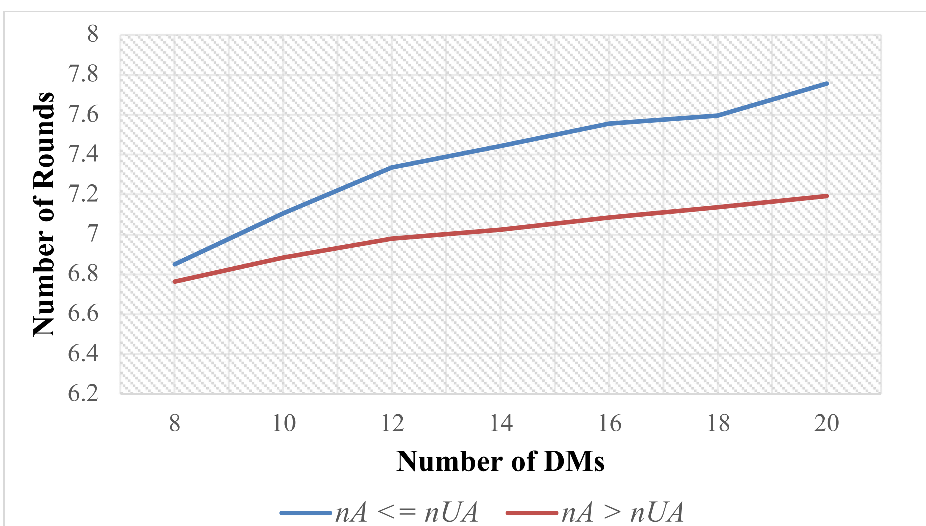

Fig. 5.1: Number of Round for Different Number of DMs under Different Values of nA and nUA

5.4.2 When the opinion of unavailable DMs is not considered

In order to compare the proposed method with the existing CRP, the rounds required to reach consensus is used as a criterion. The proposed model is compared with the modified model where only the opinions of the available DMs at each round are taken into consideration. That means only Case (2) of the feedback mechanism will be the only way to suggest the feedback suggestions to the DMs. Fig. 5.2 depicts the convergence rate of the proposed model and the modified model from which the following observations can be drawn:

1). The speed of convergence of the proposed model is faster than the modified CRP for a given consensus threshold. This is because the consensus process in the proposed model (Proposed CRP) grows gradually as the DMs are involved from the start and therefore understands the discussion process accurately. While in the modified model (Modified CRP) where the DMs are allowed to leave and new DMs are allowed to join the process mid-way can delay the ongoing decision-making process. Thus, on considering only the opinion preference of the available DMs the number of rounds to reach the consensus increases in comparison to the case when the opinion of the overall set of DMs are taken.

2). With increase in the number of DMs the average rounds to reach consensus increases which is obviously as expected. This is because with increase in the number of DMs the diversity of opinions adding complexity and hence increases the consensus rounds.

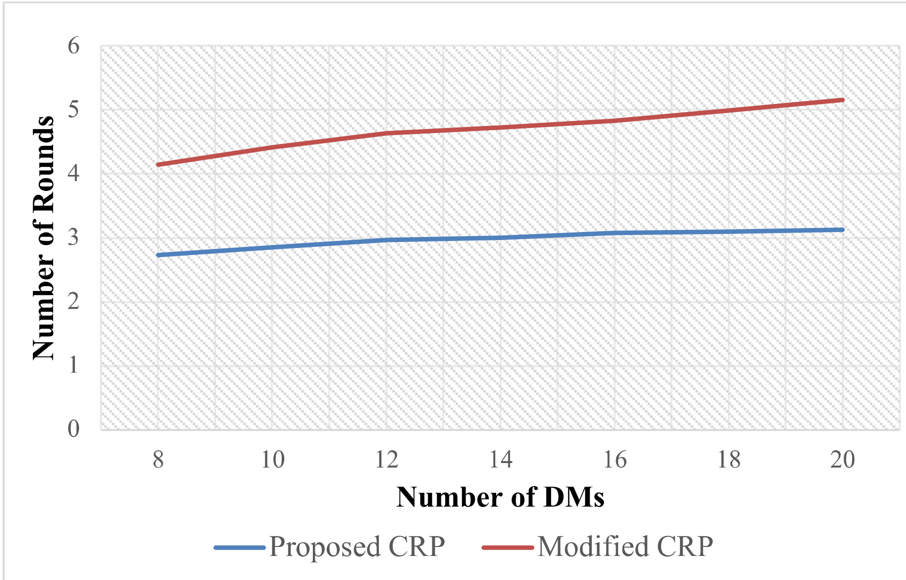

Fig. 5.2: Number of Round for Different Number of DMs in Proposed and Modified models

5.4.3 Discussion

In this chapter, a novel group decision making method that is applicable in dynamic DMs environment is presented. The consideration that a DM may be allowed to be unavailable at times necessitates the design of the proposed method. The proposed method is designed in such a way that it is applicable in situations where the set of DMs are fixed throughout the decision process. That is analogous to the traditional GDM discussed in the Chapter 2. While other methods present in the literature focuses on the way DMs participate throughout the decision-making process once they join, the proposed method in this paper has been designed to focus on the following aspects:

Incorporating opinion of unavailable DMs at times: Our method allows the incorporation of the opinions of the DMs in the current consensus state, who may not be available at each round of the decision-making process. The modified preference relation of the unavailable ones is automatically generated based on the bounded confidence value and, thus, staying within their confidence range. This way, instead of considering the resumed DMs as new participant are contemplated as the one who is unavailable for the time being and whose opinion has been incorporated continuously for achieving consensus. Using such an opinion, the system can consistently run the decision-making process and compute the required result satisfying all the participants.

Modelling real decision-making scenario: Most decision-making frameworks discussed in the literature consider the fixed number of DMs. However, in realistic situations, the set of DMs can be modified at any time during the process due to technological, organizational, or sociological reasons. Considering the same, authors in [76] discussed the dynamic DMs scenario where the set of DMs may leave and re-join the process. Consequently, it's better to incorporate their opinion in each consensus state rather than starting from the beginning or picking up where they left off.

The developed method design has the following weak points that can be addressed in the future:

1. ***The use of weight method based on the availability or Giving credence to the DM based on the availability:*** Although providing weightage based on the availability of the DMs is a comfortable way of prioritizing the preference information provided, it restricts the presence of DM in the GDM process. In contrast, this has the advantage of forcing the DM to understand the discussion, which helps in consensus building. Consequently, understanding and abiding by the process gives DM an edge over frequently joining and leaving the GDM process. Nevertheless, it would be desirable to design a dynamic weighting method considering the availability and the DM's other information, such as by using their consensus degree, reliability, or preference consistency. It should be noted that it would be necessary to provide the computational system with some information that allows it to understand the DM

clearly. Due to the complexity of predicting the DMs' behaviors, it is obviously not an easy job, leading it to be an interesting line of research in future.

2. ***Working with larger/smaller number of DMs:*** Although the problem statement applies to a smaller number of DMs, it can also be relevant when dealing with large-scale group decision-making, that is, in social networks with many DMs. This is because in large-scale scenario, especially in social networks, the DMs may become unavailable due to network failure etc.

It is very obvious that many internal and external factors affect the decision environment. Consequently, the consensus-reaching process (CRP) has been recently studied as a dynamic process. Adding to the same, some researchers have contributed to diverse, dynamic parameters incorporated in static consensus. The exact definition of the dynamic can be stated as when the exact parameter value cannot be predefined, and the parameters can change during the consensus process. However, there are situations when the parameter value, say group size, is predefined. Still, due to some technical faults or sociological, organizational, or physiological reasons, DMs may become unavailable at times. The same DM would resume when he/she wants to. Thus, a CRP framework is required to be developed that deals with the uncertainty regarding the availability of the DMs at a particular consensus round. To the best of our knowledge, such a scenario is untouched in the literature. That said, we developed a CRP model that includes the preferences of an entire group, regardless of who is present or absent.

Some of the early proposals in classical GDM problems that deals with the strategic preference manipulation were proposed by Yager in [108] where the preferences of DMs are penalized before moving onto the selection process. These weight penalizations are done by analyzing how drastic and biased their opinions are. Later on, another approach focused on consensus based GDM problems was proposed in [66] where the authors proposed a consensus model for dealing with non-cooperative behaviors in GDM. In these penalizing schemes the weights of the DMs are modified if they reflect a non-cooperative behavior thereby penalizing their current state value only. Nevertheless, there exist several GDM models including weight penalization in recent literature based on their behavior in whose framework the set of DMs showing non-cooperative behaviors is penalized only.

Regarding the previous cases, making use of the availability of the DMs in the decision-making process is as far as we know, a challenge not properly addressed in this research field yet. If tackled properly this aspect would allow a more accurate and appropriate management of such dynamic environment. Nevertheless, there exist some proposals for dynamic decision making in recent literature [76][77] in whose framework the set of alternatives, the set of DMs and the set of criteria varies over time and each such alternatives, criteria and DMs are assessed over time. In these frameworks the global (dynamic) assessment of each DM is computed who is currently present. Thus, the DM who joins after being absent for some one or more rounds of iteration is considered here as a fresh participant.

5.5 Summary

This chapter proposed a novel consensus-reaching model considering the opinion of the unavailable DMs at times that consider the DM's availability for providing credence to them. The proposed model is driven by the bounded confidence model in opinion dynamics. Due to technical or non-technical reasons, a DM may sometimes leave and join the group. The proposed method keeps using their bounded confidence for automatically generating the preferences. In these situations, the DM may lose some understanding while they are absent, and a dynamic weighting method is presented to address this issue. The detailed simulation analysis shown in this paper shows the effectiveness of the proposed method. This is mainly due to the consideration of the unavailable DM's opinion based on their bounded confidence that reduces the number of rounds to reach consensus even when the DMs leave and join at times.