Chapter 1

Introduction

"Nature isn't classical, and if you want to make a simulation of nature, you'd better make it quantum mechanical." Richard Feynman

This captivating insight propelled my fascination with the intricacies of quantum devices, recognizing the pivotal role of comprehending and harnessing such complexities in the near future. Quantum mechanics and suitable devices for observing and utilizing these quantum effects are two fundamental pillars of modern physics and engineering that, when intertwined, provide a powerful framework for understanding the macroscopic properties of the world around us through the lens of their microscopic constituents. Quantum mechanics lays the groundwork by dictating the behavior of matter at the atomic and subatomic level, where classical mechanics breaks down. It introduces concepts like wave-particle duality, quantization of energy, and superposition, which are crucial for comprehending the peculiar nature of the quantum world.

The application of quantum theory to suitable devices, on the other hand, focuses on the collective behavior of a large number of interacting quantum particles[Xiang et al., 2013; Kimble, 2008; Li et al., 2020]. It aims to bridge the gap between the microscopic quantum description provided by quantum mechanics and the macroscopic observable properties of quantum devices. By employing sophisticated mathematical techniques and leveraging the principles of quantum mechanics, its application for quantum material endeavors to explain a vast array of phenomena, ranging from photon-photon and light-matter interactions to quantum information technologies and quantum materials[Pozar, 2021; Moiseyev, 2011; Kimble, 2008].

The synergy between quantum mechanics and its application to the devices and exploration has led to groundbreaking discoveries and technological advancements[Xiang et al., 2013; Kimble, 2008; Li et al., 2020; Pozar, 2021; Moiseyev, 2011]. For instance, the theory of light-matter interactions, which describes the phenomenon of a photon interacting with the matter or with other photons, has revolutionized numerous fields, including quantum computing and medical imaging. In particular, the understanding of its application with other quasi-particles like magnons etc., facilitated by condensed matter theory, has paved the way for the development of new magnetic materials with applications in data storage and spintronics[Xiang et al., 2013; Kimble, 2008; Li et al., 2020; Pozar, 2021; Moiseyev, 2011].

Furthermore, the exploration of quantum phenomena at the nanoscale has opened doors to the realm of nanotechnology. By manipulating materials at the atomic level, scientists can engineer novel properties with potential applications in electronics, photonics, catalysis, healthcare etc[Xiang et al., 2013; Kimble, 2008; Li et al., 2020; Pozar, 2021; Moiseyev, 2011]. This interplay between quantum mechanics and harnessing it for suitable devices and further insight are at the forefront of modern materials research, holding the promise of revolutionary breakthroughs in various technological sectors.

The development of quantum devices and related theoretical tools has been a very active research avenue in recent times. We have identified two areas with substantial room for improvements. The first one is the development of simpler numerical techniques which are able to capture nonlinear effects beyond simplest cases, for example quantum blockade. The well established quasi-classical methods fail when we are dealing with systems at high input intensity in absence of internal dissipation. In the first part of the thesis we have addressed this problem and developed a modified quasi classical method that increases the range of applicability of such models beyond what was possible before [Tiwari et al., 2024]. A second aspect of theoretical research that has room for improvement is generalizing the theory of multi-mode interaction in hybrid systems including any kind of real or quasi modes. Such theory is required to explain the linear analogue of the well-known electromagnetically induced transparency (EIT) and electromagnetically induced absorption (EIA) [Ma et al., 2017; Lv et al., 2023] phenomena. There are several models and approaches that are available in the literature but a unified approach that is applicable to a wide range of systems with little modification is lacking. In particular, most of the approaches have not adequately modeled the microstripline/ cavity present in these experimental systems. In the second half of the thesis we have developed a unified model explicitly including the microstripline along with all the other components and explained a range of exotic phenomena observed in numerical simulations using CST studio suite. A key feature of our generalized approach [Shrivastava et al., 2024a] is that very different physical phenomena can be explained by just changing parameters and do not require qualitatively different models, which is what it should be to be consistent with simulations and experiments.

The physics of mesoscopic collections of quantum particles has been extensively explored in the past for investigating quantum effects such as size confinements, interference, interactions, and band-topology [Akkermans and Montambaux, 2007]. The quantization of conductances, persistent currents, and Coulomb blockade are well-known examples of quantum effects in lower dimensional systems. Though these phenomena were particularly popular for mesoscopic electrical devices, recent efforts in quantum photonics have also demonstrated many interesting mesoscopic quantum effects in various cavity [Walther et al., 2006], circuit [Girvin et al., 2009] and waveguide quantum electrodynamics (QED) [Roy et al., 2017; Gu et al., 2017] set-ups. Some such recent discoveries are the photon blockade and quantum nonlinearity for propagating single photons through an elongated ensemble of laser cooled atoms in an optical-dipole trap [Peyronel et al., 2012; Firstenberg et al., 2013], a dissipative phase transition in a one-dimensional circuit QED lattice [Fitzpatrick et al., 2017], and the cooperative Lamb shift in a mesoscopic array of ions suspended in a Paul trap [Meir et al., 2014].

In this thesis, we have explored some of such quantum phenomena, and have also designed and simulated a few suitable planar devices to observe and utilize these effects at macroscopic levels. Motivated by these fantastic developments in experiments, we study nonlinear quantum transport of light through one-dimensional (1D) QED lattices connected to radiation fields at the boundaries. Particularly, in chapters 2 and 3 we investigate 1D open QED lattices either (a) in the absence of optical confinement (cavity) along the light propagation direction or (b) when the coupling to and from the cavity(ies) is dominating the internal system losses in the so-called overcoupled regime [Roy et al., 2017; Aoki et al., 2009]. The above special features of the 1D open QED lattices separate our study from other recently explored cavity and circuit QED lattices [Underwood et al., 2012; Schmidt and Koch, 2013; Raftery et al., 2014; Naether et al., 2015b; Le Hur et al., 2016; Noh and Angelakis, 2016; Fitzpatrick et al., 2017; Kollár et al., 2019; Orell et al., 2019; Ma et al., 2019].

Photons being charge-neutral do not interact with each other and can have very long spatial and temporal coherence. An effective interaction between photons can be realized through their coupling to matter. Since the effective interaction is induced by the medium for photons, we also expect such interaction to affect/influence the photon transmission differently than interaction in electron transport. Our present study employs such light-matter coupling to investigate the effects of interaction in photons' quantum transport in a 1D mesoscopic array of qubits. These qubits can be considered to be made of alkali atoms

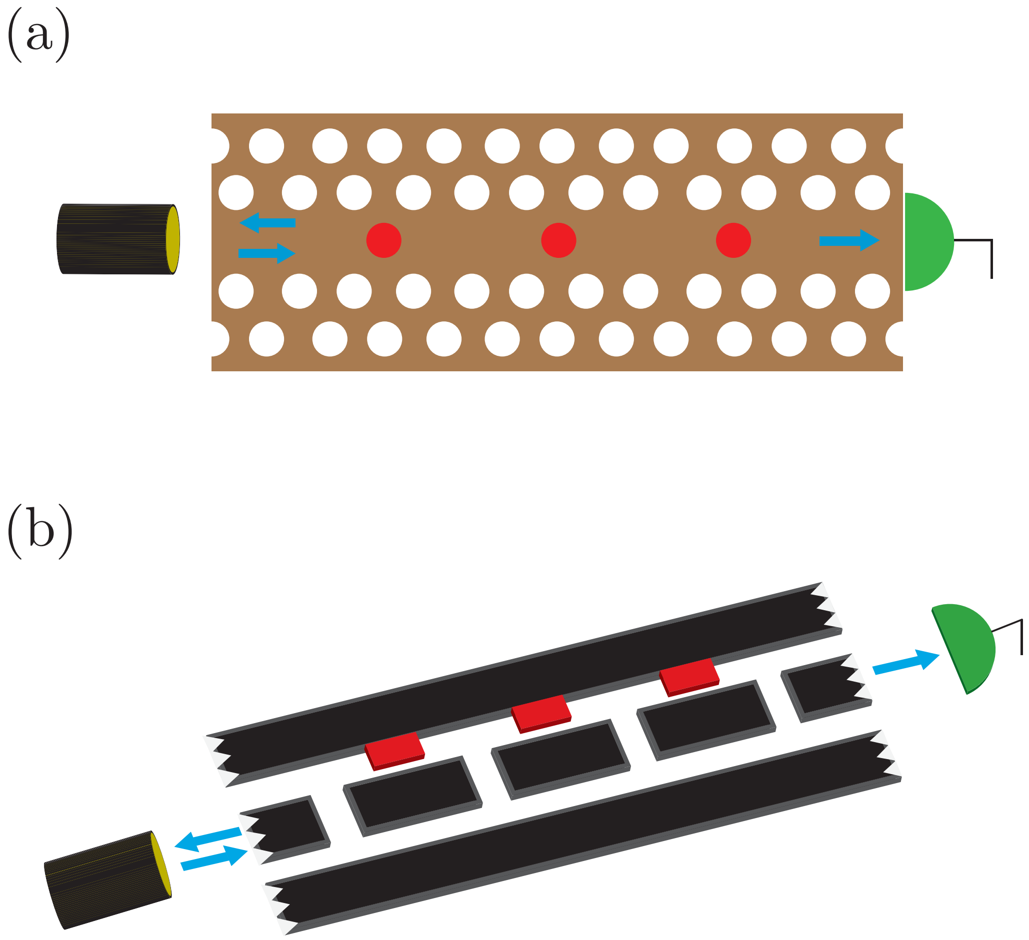

Fig. 1.1 Cartoons showing physical realizations of the direct-coupled (a) and side-coupled (b) qubit systems. The direct-coupled qubit system is an experimental arrangement in which the qubits are effectively coupled to each other directly along a chain. There is some medium between the qubits but the phenomena of interest is such that the intermediate medium can be ignored and replaced with a coupling among the qubits. It can be realized using quantum dot qubits (shown as red circles) embedded in a line-defect waveguide of a photonic crystal. On the other hand, the side-coupled qubit system is an array of resonators to each of which a qubit is coupled. In this setup there is no direct coupling among the qubits. It can be realized as an array of superconducting qubits (shown as red squares) coupled to a lattice of resonators made of transmission lines (black rectangles in the middle). We also show the sources of input photons and the detectors at the left and right of the systems respectively as well as the incident, reflected and transmitted (blue arrows) light for each system.

(e.g., rubidium atoms in [Peyronel et al., 2012; Firstenberg et al., 2013]) or superconducting qubits (e.g., transmon qubit in [Fitzpatrick et al., 2017]) or ions (e.g., Sr^+ ions in [Meir et al., 2014]). The features of the above qubits and their interactions with light can be very diverse. Because of the differences in the underlying physical systems and microscopic mechanism, these qubits have a very wide range of resonance frequencies, coupling strengths and other qubit parameters. Also the coupling to photons can be engineered to achieve various physical effects which are exploited to develop quantum gates for example.

In a modern quantum device, qubits and resonators can be coupled in many ways giving rise to a wide range of models, for example the 2D network of qubits and resonators of any real chip-based quantum processor. For simplicity, we mainly choose two different models for the 1D lattice of qubits; these are (a) a chain of qubits with direct coupling between the nearest neighbors, discussed in chapter 2 and (b) a chain of connected resonators to each of which a qubit is side-coupled, discussed in chapter 3. While we do not have any confinement along the light-propagation for the direct-coupled qubits, we work in the overcoupled regime [Roy et al., 2017; Aoki et al., 2009] for the chain of connected resonators with side-coupled qubits. In Fig. 1.1, we show representative physical implementations of the two 1D models considered here. The direct-coupled qubit system (Fig. 1.1(a)) can be realized by placing qubits such as nano-diamonds in a line-defect waveguide of a photonic crystal operating in the optical regime [Wolters et al., 2010]. The direct coupling between the qubits is generated by exchange of photons propagating through the waveguide between the qubits. Here, we integrate out these photons in the intermediate waveguide, and model the effective interaction by a constant exchange term (e.g., J_x) for the separation between the qubits being comparable to the wavelengths of the photons [van Loo et al., 2013]. The side-coupled qubit system is routinely realized in superconducting photonic circuits in the microwave regime [Fitzpatrick et al., 2017] (Fig. 1.1(b)). A lattice of resonators can be formed by a series of capacitive gaps in the middle conductor of the 1D transmission line waveguide.

There has also been considerable effort towards the applications of insights from quantum theory and light-matter interactions particularly for development of devices. In the last few decades, studies centered around the control and manipulation of electromagnetic (EM) waves have flourished and lead to many interesting physical phenomena with applications in diverse fields ranging from optoelectronics to quantum technology [Lvovsky et al., 2009; Wei et al., 2022; Zhang et al., 2016]. Emerging phenomena stemming from atomic coherence in light-matter interactions are deemed critical for advancing future quantum technology, while the pursuit of precise control over electromagnetic waves continues to inspire extensive research in this domain. One such example is EIT and its inverse effect (EIA) are a pair of exotic phenomena of light-matter interactions that provided a foundation for many new and exciting possibilities based on quantum interference effect [Lv et al., 2023; Lezama et al., 1999; Ma et al., 2017; Harris et al., 1990; Fleischhauer et al., 2005]. EIT relies on the coherent interaction between two optical fields, leading to reduced resonant absorption and rendering transparency in a medium or the coupling center, with a wide range of applications including slow light, optical signal processing, quantum switching, four-wave mixing and quantum computation. On the other hand, EIA results from dissipative interaction that enables the absorption of light at coupling center thereby potentiating many applications such as fast light, narrowband filtering, absorption switching, and optical modulators [Brazhnikov et al., 2005; Ning et al., 2019]. Recent discoveries have identified analogues of EIT and EIA in various systems such as photonic crystals, plasmas, and metasurfaces composed of materials like graphene, vanadium dioxide, and photosensitive silicon [Li et al., 2011; Lv et al., 2023; Shi et al., 2023; Cao et al., 2018; Sun et al., 2022]. Although these analogue effects have garnered significant attention from the scientific communities, there has been comparatively less emphasis on the integration and manipulation of both phenomena within a single device.

More recently, various coupled hybrid systems, such as photon-magnon systems, opto-mechanical systems, and optical microcavity systems, have been proposed to achieve both EIT-like CIT and EIA-like CIA effects [Bhoi et al., 2022; Rao et al., 2021b; Zheng et al., 2023; Bernier et al., 2018; Li et al., 2022; Maurya et al., 2024]. While the, EIT and EIA typically owe their existence to non-linear behaviour of the systems, the CIT and CIA effects were observed through the manifestation of mode splitting and merging at or near their coupling center, referred to as level repulsion (LR) and level attraction (LA), respectively [Bhoi et al., 2022; Rao et al., 2021b; Zheng et al., 2023; Bernier et al., 2018; Li et al., 2022]. Nevertheless, generating CIT or CIA phenomena experimentally often necessitates demanding conditions, including ultra-low temperatures and intricate three-dimensional designs with additional infrastructure, substantially restricting their practical applications [Zheng et al., 2023; Bernier et al., 2018; Li et al., 2022; Harder et al., 2021]. We briefly summarize these two phenomena and explain their differences.

Coupling induced transparency (CIT): Level repulsion(LR)/ normal anti-crossing

In coupling induced transparency, coherent coupling is characterised by its dispersion relation that shows level repulsion in the transmission profile of the hybridised modes around the coupling centre, Fig. 1.2(d). The real and imaginary parts of the complex eigenvalues show different behaviour. The real part shows level repulsion around the coupling centre, Fig. 1.2(e), i.e. eigenfrequencies have upper and lower branches even at the coupling centre. The imaginary part representing its linewidth profile shows crossing (or attraction), Fig. 1.2(f). This kind of coupling is very ubiquitous in nature and plays an important role in applications involving transduction, allowing efficient energy exchange between two modes.

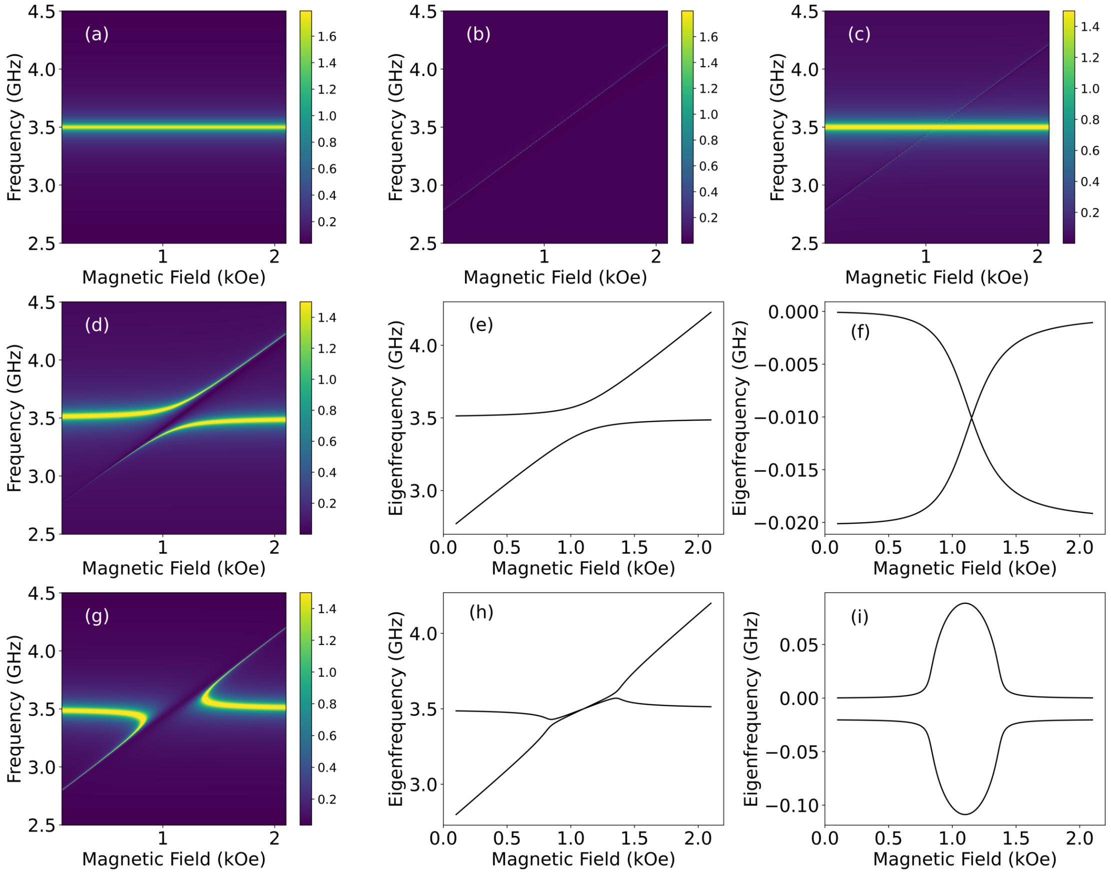

Fig. 1.2 (a) Transmission profile for photonic modes, (b) transmission profile for magnonic mode, (c) combined transmission profile for photonic and magnonic modes without any coupling. Coherent coupling (d, e, f), (d) transmission profile showing repulsive dispersion in nature, (e) real part of eigenvalue showing level repulsion and (f) imaginary part of eigenvalue showing attraction in linewidth. Dissipative coupling (g, h, i), (g) transmission profile showing attractive dispersion in nature, (h) real part of eigenvalue showing level attraction and (i) imaginary part of eigenvalue showing repulsion in linewidth.

Coupling induced absorption (CIA): Level attraction (LA)/ opposite anti-crossing

In coupling induced absorption, transmission profile of the phenomena near the coupling centre characterised by dissipative nature of the coupling and shows attraction in the dispersion profile Fig. 1.2(g). The real part shows level attraction around the coupling centre, Fig. 1.2(h), i.e. the upper and lower branches of the eigenfrequencies will get merged at/ near the coupling centre. The imaginary part representing its linewidth profile shows repulsion, Fig. 1.2(i). In 2018, a distinct magnon–photon dissipative coupling was discovered and it has been quickly verified in a variety of setups with different cavity configurations. A distinct feature of a dissipatively coupled system is the level attraction (LA) of the hybridized modes as shown in Fig. 1.2(g,h) and repulsion in their linewidth Fig. 1.2(i) , which is in strong contrast to the results induced by coherent coupling.

The study of CIT and CIA is presently an important area of research, addressing both fundamental principles and practical applications. Moreover, they are usually observed separately and appear to have different underlying mechanisms. In the quest to understand the simultaneous observation of CIT and CIA, different research groups have proposed various theoretical models. Some models suggest a need of an auxiliary mode [Bernier et al., 2018; Li et al., 2022; Hu et al., 2022], while others suggest use of a hypothetical negative energy mode [Bernier et al., 2018]. On the other hand, in chapter 4, we propose a model that model systematically takes into account the microstripline, the resonators and their coupling parameters, capturing all the details and factors of our hybrid system with a linear approach that effectively explains the observed phenomena [Rao et al., 2021b; Zheng et al., 2023; Bernier et al., 2018; Li et al., 2022; Hu et al., 2022; Tiwari et al., 2024; Rao et al., 2020; Harder et al., 2021; Maurya et al., 2024]. Also, there is a keen interest among researchers in the development of a miniaturized planar device with a robust theoretical model capable of hosting both types of CIT and CIA quantum phenomena concurrently that would allow a single device to encompass the functionalities of multiple systems. We have also designed a device by performing high frequency electromagnetic simulations to observe CIT and CIA and confirm our findings.

Another aspect of CIT and CIA that has caught some attention recently is the transition between them through coupling of other modes in multimode hybrid systems. In a previous study, Bhoi et al. reported a planar structure in which one magnonic mode (YIG) was coupled with three ISRR photonic modes [Bhoi et al., 2022]. The ISRR photonic modes were concentric square rings in the ground plane and YIG lied on the centre of these concentric rings but on the front plane touching the microstrip line. Individual square rings were of different dimensions having different resonance frequencies. When the individual rings were forming the hybrid system with YIG separately they showed level attraction at their respective resonance frequencies. This happened when the YIG resonance frequency was tuned to the vicinity of ISRR resonance frequencies by changing the magnetic field. But when the three concentric ISRRs were combined with YIG together the behaviour of their dispersion profile at the three crossings were different, one showing level repulsion, another showing level absorption and one showing level attraction. A classical theory was used to explain the phenomena, but we show that this phenomenon is possible even in the quantum domain by developing a full quantum theory for similar observations. Another study [Hu et al., 2022] described a cavity magnonic system of three modes in which the coupling between cavity and magnon modes were mediated by an SRR auxiliary mode. By controlling the damping of auxiliary mode the group has achieved both normal (CIT/LR) and opposite (CIA/LA) anticrossing. In yet another experiment [Rao et al., 2021a] with a hybrid system composed of a single YIG sphere and two orthogonally crossed coplanar waveguide they observed both normal (CIT/LR) and opposite (CIA/LA) anticrossing. They have also analysed how destructive interference between magnon-dipole and magnon-quadruple determines the interactions and have discussed perfect absorption at the opposite anticrossing in the S_{21} spectrum. Continuing the approach of the previous chapter, in chapter 5 of the thesis we have applied the quantum theory to the CIT and CIA for hybrid systems having multiple modes and explored multiple phenomena of CIA and CIT in a single device and also achieved transition between them.

All the physical realizations and devices considered in this thesis are examples of open quantum systems, and a powerful formalism to study them is the Heisenberg-Langevin equations. The phenomena discussed in chapters 2 and 3 involve strongly correlated transport of light highlighting the nonlinear properties, the theoretical study of which is a nontrivial problem. Many different theoretical approaches, including analytical [Shen and Fan, 2007; Yudson and Reineker, 2008; Yudson and Reineker, 2010; Roy, 2010; Zheng et al., 2010; Liao and Law, 2010; Fan et al., 2010; Koshino and Nakamura, 2012; Roy, 2013; Xu et al., 2013; Lalumière et al., 2013; Schmidt and Koch, 2013; Li et al., 2014; Fang and Baranger, 2015; Le Hur et al., 2016; Hartmann, 2016; Li et al., 2016; Noh and Angelakis, 2016; Roy, 2017] and numerical [Longo et al., 2010; Roy, 2011; Sanchez-Burillo et al., 2014; Caneva et al., 2015; Manasi and Roy, 2018] methods, have been explored in recent years. One major challenge in applying these theories is the scaling of the system sizes from single or multiple qubits to a mesoscopic collection of qubits.

We first study two nonlinear models in the chapters 2 and 3, using the truncated Heisenberg-Langevin equations (THLE), which have been applied in recent years for studying correlated light transmission in a chain of two-level systems [Manasi and Roy, 2018]. Next, we compare the THLE approach to a quasi-classical analysis (QCA) [Naether et al., 2015a] recently used in the theoretical analysis of experimental observation of dissipative phase transition in a 1D circuit QED lattice [Fitzpatrick et al., 2017]. The system [Fitzpatrick et al., 2017] has relatively large qubit relaxation as well as intrinsic photon loss from every site of the medium. We are here interested in a 1D open QED system with negligible or no loss from the middle sites of it as we wish to understand the effects of interaction. It is surprising that such a QCA badly fails, in the absence of internal losses in bulk, in capturing inelastic scattering of the photons and the related effective interaction between photons generated in our models. We specifically notice that contrary to the results from the THLE, there is always some frequency that has perfect transmission within the QCA. The main

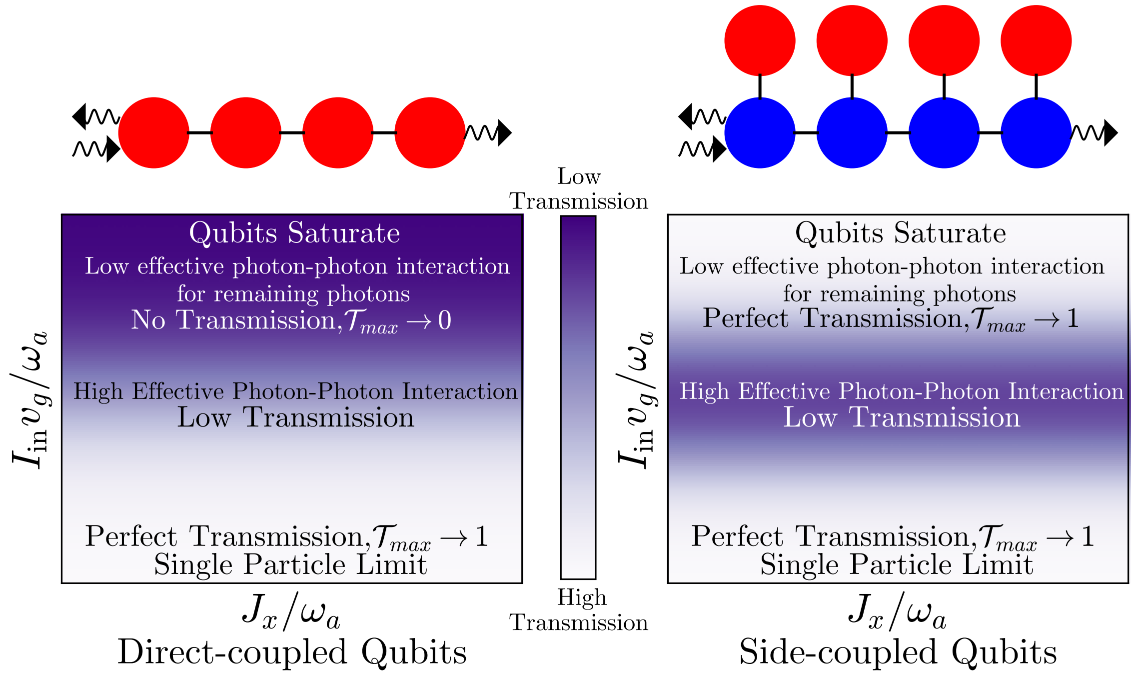

Fig. 1.3 Schematic diagrams showing the arrangement of the direct and side-coupled qubits mediums through which the photons are incoming and getting reflected on the left and being transmitted to the right. The red circles represent the qubits while the blue ones represent the resonators. For each medium we show, in the parameter space of input intensity (I_{in}) and inter-site coupling (J_x), the regions of high transmission with the color gradient. With increasing intensity we observe an initial increase in effective photon-photon interaction followed by the saturation of qubits at very high intensity. The saturation of qubits has opposite effect on transmission in the two mediums, causing a complete reduction of transmission in the direct-coupled system while a revival of full resonant transmission through the resonators for the side-coupled one.

contribution of chapters 2 and 3 is a systemic modification of the interaction parameter to the complex plane such that we revive the QCA even in the absence of bulk dissipation. We develop a simple scheme to find the complex interaction parameter for the modified QCA, which only uses single qubit analysis. Quite remarkably, the modified QCA works quite well when it is compared to the more accurate THLE even for larger systems. Unlike the QCA, the modified QCA is able to cause dissipation in the medium through the complex interaction parameter, which gives accurate results even at higher intensities at a much less computational cost than the THLE (Fig. 1.3).

At low intensities of the incident light, we find perfect resonant transmission in both the models, which can be investigated by any of THLE, QCA, and modified QCA. The interaction dominates light propagation for increasing intensities in an array of direct-coupled qubits. The transmission probability in this model falls with an increasing power due to the photon-photon blockade, which is generated by the nonlinearity (interaction) at the qubits. This regime can be adequately analyzed only by either THLE or modified QCA. For the side-coupled qubits, the effect of interaction on light propagation is non-monotonic with increasing intensity of input light. The light transmission probability at resonant condition first decays with growing power, and then it again enhances with further increase in intensity when photons saturate the qubits. Our work provides an insightful understanding of the role of excitation blockade on the transmission by proposing a simple calculation method to capture their effects.

In chapter 4, we establish a comprehensive theoretical framework for CIT and CIA, elucidating these phenomena within a microwave photon-photon coupled system. A notable difference from EIT/EIA or several earlier works on CIT/CIA is that our demonstration of the phenomena uses only linear couplings. We demonstrate how a coherent and dissipative coupling between the photon modes leads to the emergence of level repulsion (LR) and attraction (LA), which correspond to CIT and CIA, respectively. Further, we substantiate both LR and LA via numerical simulations using a planar hybrid system comprising two resonating elements that excite photon modes at microwave frequencies. Finally, we achieve substantial control not only in the coupling strength but also a controllable transition between CIT and CIA by manipulating the dissipation rates. This work presents potential avenues for controlling interactions between two modes, offering a feasible solution for demonstrating CIT and CIA in a single device at room temperature. Furthermore, the ability to reproduce the result using a quantum model with planar resonators suggests a possibility of observing and utilizing these phenomena in quantum devices which may have technological applications in future.

In chapter 5, we extend the findings of the previous chapter by developing a quantum theoretical framework for the hybrid quantum systems having multiple modes. We use two different types of modes, one having tunable properties (TM) while other modes are static (SM) in nature. We explicitly consider two cases, the first case having three modes, 1TM and 2SMs. When an individual SM is getting coupled with TM, LA is observed but when both SMs are getting coupled with TMs we observe transition from LA to LR. The second case has four modes, 1TM and 3SMs. Again when individual SM is getting coupled with TM, LA is observed but when all SMs are getting coupled with TM the transition from LA to LR occurs. When the transition from LA to LR is happening in the midway of the transition, passing from level absorption is also observed. Here for the two simple cases we achieved controllable transition from LA to LR in the multimode hybrid quantum system by tuning the coupling parameters. Different nature of transitions e.g. from LR to LA, LR to LR, LA to LA, LA or LR to level absorptions and vice versa may also be achieved with different sets of parameters. Many groups have observed some of these phenomenon [Bhoi et al., 2022; Hu et al., 2022; Rao et al., 2021a]. We have developed a general model explaining them and generalised our findings for multiple modes having different hybrid quantum systems of *N* modes. Our findings unveils the complex dynamics of the interaction between different modes of hybrid quantum systems; it may be expected to advance the application for future quantum technologies. In the last chapter we present a summary of all the work discussed in this thesis along with some of the later work by our group and few experimental results, discuss limitations and provide an outlook for future extensions.