PREFACE

Optimization is a crucial and versatile field that applies to many scientific disciplines, such as operations research, management science, economics and finance, and all engineering domains that involve some kind of extrema. Optimization process aims to achieve the best possible outcome among all feasible alternatives even under certain constraints. In essence, optimization is used to identify, describe and calculate the maximum or minimum of a function for a set of admissible points and specific conditions.

Optimization is a broad field that encompasses various methods and applications. It can be used to find the best solutions for different types of problems, such as maximizing efficiency, minimizing costs, or increasing profits. These problems may have different characteristics, such as convexity, linearity, nonlinearity, quadraticity, semidefiniteness, dynamism, integrality, or stochasticity. Optimization has a rich theoretical background and sophisticated algorithms. In general, optimization involves finding the optimal values of one or more decision variables that minimize or maximize a function or a set of functions subject to some constraints. Depending on the number of objectives, optimization problems can be classified into single or MOPs.

This thesis is devoted to the extensions of various classical methods for singleobjective optimization problems to MOPs. MOPs involve finding a set of solutions that are optimal with respect to two or more conflicting objectives. The main challenges of multiobjective optimization are to handle the trade-offs between the objectives, to deal with the possible existence of multiple Pareto optimal solutions, and to generate a representative and diverse approximation of the Pareto front.

The classical methods for multiobjective optimization can be broadly classified into three categories: scalarization methods, nonscalarization methods, and population-based methods. Scalarization methods transform the original problem into a singleobjective problem by using a scalar function that combines multiple objectives. Nonscalarization methods are techniques that do not transform a multiobjective problem into a singleobjective one by using a scalaring function. Instead, they try to find a set of solutions that represent the trade-offs between the conflicting objectives. Population-based methods maintain a set of solutions that are updated iteratively by using some selection and variation operators. All the above categories have their advantages and disadvantages, and the choice of the method depends on the characteristics of the problem and the preferences of the decision maker.

The aim of this thesis is to present some theoretical and practical aspects of classical methods for multiobjective optimization and to propose some novel extensions and improvements. The thesis consists of six chapters.

The first chapter introduces the basic concepts and definitions of multiobjective optimization and provides an overview of the existing methods and challenges. The second chapter introduces a nonmonotone Polak-Ribière-Polyak conjugate gradient method for MOPs and demonstrates its effectiveness on various test problems. The third chapter introduces projection-type hybrid conjugate gradient methods for MOPs with an application to an optimal control problem and demonstrates its effectiveness on various test problems. The fourth chapter introduces a nonmonotone quasi-Newton methods for unconstrained strongly convex MOPs. We explore two well-known types of nonmonotone line searches: one that considers the maximum of recent function values and the other that calculates their average. Moreover, we demonstrates its effectiveness on various test problems. The fifth chapter introduces a nonmonotone condition gradient method for MOPs and demonstrates its effectiveness on various test problems. The sixth chapter proposes a new scalarization-based method that incorporates the augmented Lagrangian technique for MOPs with an application to an optimal control problem. The seventh chapter summarizes the main contributions and findings of this thesis and suggests some directions for future research.