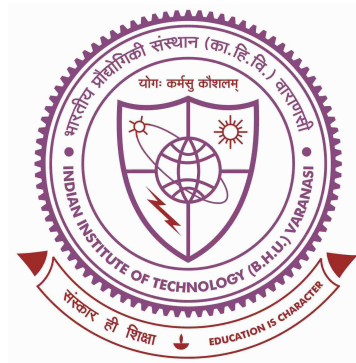


Accelerated Optimization and Control Using Predefined-Time Convergent Dynamical Systems



Thesis submitted in partial fulfillment
for the award of degree

Doctor of Philosophy

by

Sunidhi Pandey

DEPARTMENT OF ELECTRICAL ENGINEERING
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Varanasi

Roll No: 20181501

2025

To my mother, for her unconditional love and support.

CERTIFICATE

It is certified that the work contained in the thesis titled **Accelerated Optimization and Control Using Predefined-Time Convergent Dynamical Systems** by **Sunidhi Pandey** has been carried out under my supervision and this work has not been submitted elsewhere for a degree. It is further certified that the student has fulfilled all the requirements of the Comprehensive Examination, Candidacy, and State-of-the-Art Seminar for the award of Ph.D. Degree.

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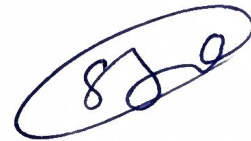


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I, **Sunidhi Pandey**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Prof. Devender Singh** and **Dr. Shyam Kamal** from December 2020 to September 2025, at the Department of Electrical Engineering, Indian Institute of Technology (BHU) Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports, dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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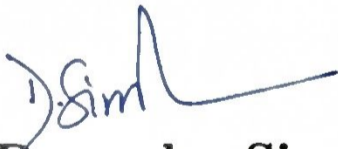
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Nomenclature

List of Greek and Roman Symbols

\mathbb{R}	Set of real numbers
\mathbb{R}_+	Set of positive real numbers
\mathbb{R}^n	n -dimensional vector space over the real number field \mathbb{R}
$\mathbb{R}_{\geq 0}$	Set of non-negative real numbers
$\mathbb{Z}_{\geq 0}$	Set of non-negative integers
$\mathbb{R}_{\geq k}$	Set of real numbers greater than or equal to k
$\mathbb{R} \rightarrow \mathbb{R}$	Element-wise mapping
\mathcal{K}	Class \mathcal{K} function
\mathcal{K}_∞	Class \mathcal{K}_∞ function
\mathcal{L}	Class \mathcal{L} function
\mathcal{KL}	Class \mathcal{KL} function
\mathcal{GK}	Generalized- \mathcal{K} function
\mathcal{GKL}	Generalized- \mathcal{KL} function

Abbreviations

TVO	Time-Varying Optimization
OFP	Output Feedback Passivity
IFP	Input Feedforward Passivity
RAP	Resource Allocation Problem
FTS	Finite-Time Stable
ISS	Input-to-State Stable
FTISS	Finite-Time Input-to-State Stable
PUBST	Predefined Upper Bound of Settling Time
GF	Gradient Flows
LLS	Linear Least Squares
PGF	Projected Gradient Flows
LTI	Linear Time Invariant
PT-GF	Prescribed-Time Stable Gradient Flows
DFTP	Discrete Finite Time Passive
ZSO	Zero State Observability
DOF-FTP	Discrete Output Feedback Finite Time Passive
KKT	Karush-Kuhn-Tucker
min	Minimum
max	Maximum