

Chapter 1

An introduction to vortex dynamics and atmospheric whirlwinds

1.1 Atmospheric whirlwinds

*Born of Sun and Earth's embrace,
The vortex stirs in nature's grace.
Dust-devils whirl on arid plains,
Cyclones roar with stormy reigns.
A god unseen, yet plainly shown,
Through mortal eyes is its power known.
With maths and science, we chart the skies,
Seeking the truth where tempest lies.
Boundless is its untamed art,
A riddle vast to tear apart.
Oh Nature's breath kind and wild,
A fearsome king, yet nature's child.*

The series of the twisting and dancing sand-whirls I witnessed on the terrains of the Nevada desert hallucinating glimpses of catwalks by gorgeous models on ramps overwhelmed me with thrills on my way from Canada to the US this February. The likes were the snow-whirls in Canadian winters I used to behold off and on from the terrace of my apartment lying with solitude in the loneliness of the scarcely populated country. Both the kinds of the whirlwinds rejuvenated my endeavour unearthing the mechanism of formation, stability and the gradual decay of such swirling motions in the atmosphere. Let us discuss these vortices in detail.

In meteorology, atmospheric vortices include tornadoes, cyclones, hurricanes, and dust devils—powerful whirlwinds that influence weather patterns and facilitate the transfer of heat, momentum, and energy between atmospheric layers. Their duration, magnitude, and intensity vary across the globe and are influenced by local environmental factors.

For instance, cyclones are characterised by winds spiralling around a low-pressure centre, with their direction determined by the Coriolis effect—counter-clockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere. Tropical cyclones—commonly referred to as hurricanes and typhoons, form over warm ocean water, with their naming conventions based on size, strength and geographic location.

1.1.1 Dust Devil

Dust devils are tiny whirlwinds, generally less intense and short-lived compared to other atmospheric vortices such as tornadoes, hurricanes and typhoons, etc. They occur globally and can be observed throughout the year, although they are most frequent in the arid regions. They are all driven by surface heating, which leads to superadiabatic lapse rates

near the ground. Unlike tornadoes, dust devils form suddenly, ascend into the atmosphere at once, and may dissipate within minutes. (Fig.(1.1)).



Figure 1.1: Dust devil (Source: Red Bluff Daily News)

Excessive solar heating of the Earth's surface causes rapid evaporation and making air near the ground hot. This hot air near the ground becomes less dense and lighter as well, starts moving upward, creating a depleted void of low pressure. In order to fill the void region, the colder air flows inward from all directions, and sometimes culminates in the formation of a rotating columnar mass of air mixed with dust, leaves, etc., thus a dust devil is formed.

Generally, dust devils are seen in unsaturated air. Sometimes, they cannot be visible, if there is no dust or debris present, even if the vortex is active. The visibility of a dust devil may vary as it moves across various terrains, flickering depending on the presence of dust particles on the ground. Until the vortex dissolves, this fleeting visibility persists. Dust devils are harmless, but occasionally they may cause little damage, like collapsing lightweight structures, breaking branches, or even lifting tiny things.

While fire whirls, steam devils, and dust devils are all atmospheric vortices, they vary in their behaviour and geographical origin. The primary source of these phenomena is heating of the Earth's surface, which creates super-adiabatic lapse rates near the ground. For example, fire whirls, which are associated with wildfires, are solely caused by the presence of intense heat sources. Conversely, steam devils occur when very cold air passes over a relatively warm water surface, causing a localised pressure gradient and the development of vortices.

1.1.2 Fire Whirls

Fire whirls, also known as fire tornadoes or fire devils, are powerful, revolving columns of fire and air. They originate during industrial fires, volcanic eruptions, or wildfires. They are solely dependent on intense heat sources, unlike to dust devils, which are driven by solar heating, or tropical cyclones, which are fuelled by oceanic processes. They can grow hundreds of feet tall, with wind speeds up to 50 m/s and diameters from a few to several tens of meters, making them impressive yet dangerous

The formation of fire whirls starts with intense heat—often exceeding 1000°C—generated by a fire, creating super-adiabatic lapse rates and powerful buoyancy. This drives rapid updrafts, with speeds of 10–30 m/s, that lift smoke, ash, and flames into a spiralling vortex. Ambient wind shear or terrain-induced convergence initiates the rotation, which is then amplified by baroclinic torque from sharp temperature gradients, as described by the vorticity equation. The fire sustains the whirl by heating the air and drawing in oxygen, thus intensifying the upward motion and vorticity.

Distinct from waterspouts or funnel clouds, fire whirls are surface-driven and short-lived, typically lasting only minutes. The extreme heat causes a low-pressure zone near the ground, pulling in cooler air that begins to rotate when influenced by local wind

patterns or terrain. This process, which is driven by the fire, entrains burning debris into a large spiral. When the fire diminishes or air conditions stabilise, the vortex dissipates.

Despite their brevity, fire whirls pose significant dangers, capable of lofting burning debris, exacerbating fire spread, and challenging firefighting efforts, as evidenced in events like the 1923 Tokyo firestorm. Their compact scale and thermal dependence make them ideal for mathematical modelling, using tools like the Boussinesq-adjusted Navier-Stokes equations. Such models provide valuable insights into heat-driven rotation and vortex dynamics within the broader study of atmospheric whirlwinds.

1.1.3 Steam Devils

Small revolving columns of air known as steam devils are created over warm bodies of water, such as lakes, rivers, or hot springs, when a cold wind blows over the warmer surface (Fig.(1.2)). This temperature difference creates a localised low-pressure zone by accelerating the evaporation and upward movement of warm, humid air.



Figure 1.2: Steam devils on Lake Champlain, USA (Source: <https://www.mynbc5.com>).

Due to wind shear or mild environmental vorticity, rising air may start to spin when colder air fills it, creating a visible vortex of condensed water vapour. Steam devils, which are typically only a few meters in diameter and their wind speeds hardly ever get over $10 - 15\text{m/s}$, persist for minutes. They are weaker and less powerful than tornadoes or fire whirls. They are most common in autumn or winter, when the air-water temperature gradient is huge, and often appear as delicate, wispy spirals rising from the surface. Steam devils are harmless still offer a fascinating glimpse into small-scale atmospheric dynamics driven by thermal and moisture gradients.

1.1.4 Tornado

Among the most powerful and destructive atmospheric phenomena, tornadoes are distinguished by rapidly spinning aerial columns that reach from clouds to the surface of the earth. Intense wind speeds they can produce—often surpassing 300km/hr in the strongest cases and their funnel-like appearance are well-known. They are not long-lived, but their localized impacts can be severe, ranging from significant property damage to loss of life (Fig.(1.3)).

Within the storm, an updraft tilts the rotating air into a vertical orientation, forming a mesocyclone. A funnel cloud forms when the rotation tightens and extends downward; if it reaches the ground, it becomes a tornado. The tornado reaches its peak intensity when the storm's updraft weakens or the supply of warm air is cut off, after which it begins to dissipate. Tornadoes form under specific atmospheric conditions, typically arising from severe thunderstorms, particularly supercells, which feature a well-defined rotating updraft known as a mesocyclone. Key factors contributing to their formation include wind shear—where changes in wind speed and direction with height create atmospheric rotation—

and strong updrafts, where rising warm, moist air intensifies this rotation, helping the development of a mesocyclone.



Figure 1.3: A tornado approaching Elie, Manitoba, Canada in June 2007 (Source: photo supplied by Justin Hobson Twitter @JustinHobson85).

Additionally, downdrafts of cooler air descending from the thunderstorm help concentrate the rotation, eventually leading to the formation of a tornado. The life cycle of a tornado includes the initial funnel cloud stage, touchdown on the ground, and eventual dissipation as the atmospheric conditions become less favourable.

Tornadoes are associated with severe weather systems such as cold fronts, warm fronts, and squall lines. These weather features create the instability and lift needed for tornado formation. The presence of warm, moist air near the surface and colder, drier air aloft enhances the likelihood of tornado development by increasing atmospheric instability.

Although they exist across the world, tornadoes are most common in areas with flat terrain and favourable weather. The United States, particularly in Tornado Alley, experiences the highest frequency of tornadoes globally due to the interaction of moist Gulf air, dry air from the Rockies, and polar air masses. Other hotspots include the Pampas in Argentina, parts of Bangladesh and India, and southern Canada. Tornadoes are less common but not unheard of in Europe, Australia, and parts of Africa.

Advances in meteorology, particularly the use of Doppler radar, have significantly improved tornado detection and prediction. Early warning systems play a critical role in reducing casualties by providing timely alerts to affected areas. However, challenges remain in accurately forecasting tornado intensity and exact paths, highlighting the need for continued research and technological improvements.

Tornadoes have significantly impacted history through several notable events. The Tri-State Tornado of 1925 remains the deadliest tornado in U.S. history, claiming 695 lives. In 2011, the Joplin Tornado, classified as an EF5, caused widespread destruction in Missouri and resulted in 158 fatalities. Globally, the Daulatpur–Saturia Tornado of 1989 is considered the deadliest, with an estimated 1,300 deaths in Bangladesh. These devastating events highlight the immense destructive power of tornadoes and emphasize the critical need for preparedness and effective mitigation strategies.

Tornadoes are part of a broader class of atmospheric vortices, sharing similarities with hurricanes, dust devils, and waterspouts. These phenomena involve rotating air masses but differ in scale, intensity, and formation mechanisms. Tornadoes offer a unique perspective on energy transfer and atmospheric instability, providing valuable insights for meteorologists and climate scientists.

1.1.5 Mesocyclones

Mesocyclones are cyclonically rotating vortices, typically 2–10 km in diameter, embedded within convective storms, most commonly supercells, and characterized by vorticity of the order of 10^{-2} s^{-1} or higher, making them a significant intermediate-scale atmospheric whirlwind. They form when strong vertical wind shear (e.g., 20–30 m/s over 0–6 km) generates horizontal vorticity, which is tilted into the vertical by a robust updraft (often 20–50 m/s), as described by the vorticity equation where stretching and baroclinic torque amplify rotation. Frequently associated with updrafts in supercells, mesocyclones can persist for hours, detected by Doppler radar as velocity couplets, and occasionally intensify near the surface to spawn tornadoes, particularly when the rear-flank downdraft (RFD) enhances low-level rotation. Unlike dust devils, which rely on surface heating, or tropical cyclones, driven by oceanic moisture, mesocyclones depend on deep convective instability and shear, bridging small-scale and large-scale vortex dynamics.

1.1.6 Funnel Clouds

Funnel clouds are rotating columns of air extending downward from a cumuliform cloud—typically a cumulonimbus—but not reaching the ground, distinguishing them as unique atmospheric whirlwinds. Their diameters range from tens to hundreds of meters, with wind speeds potentially exceeding 20, m/s, though they are often less intense than fully developed tornadoes. Formed in environments with strong vertical wind shear and moist, unstable air, they appear as condensation funnels when humidity and low pressure cause water vapour to condense into a visible, tapered structure. According to meteorological definitions, they occur beneath but connected to a cumuliform buoyant convective cloud. Their rotation, driven by vorticity within the parent cloud, may not always connect to surface circulation. In the absence of dust or debris, funnel clouds may remain invisible

unless condensation occurs; when wrapped in rain, they can completely vanish from view. Unlike dust devils, which are surface-driven, or tropical cyclones, which operate on much larger spatial scales, funnel clouds are aloft phenomena that can transition into tornadoes if their rotation extends to the surface—a process described by vortex stretching as the updraft intensifies. Frequently observed in convective storms, including supercells, funnel clouds signal potential severe weather and present compelling cases for mathematical modelling to investigate their dynamics and improve tornado prediction, thereby deepening our understanding of atmospheric vortices.

1.1.7 Tropical Cyclone

Large, rotating columns of wind associated with a low-pressure centre, strong winds, and copious amounts of rainfall are called cyclones (Fig.(1.4)). They develop over warm ocean waters when moist air rises and forms a low-pressure area that gathers surrounding air, which spirals due to the Coriolis effect of the Earth's rotation. They are identified as the most destructive weather phenomena and have a huge impact on human lives and the ecosystem. Tropical cyclones, extra-tropical cyclones, and polar cyclones are among many different kinds that are categorised based on their size, strength, and area of origin.

Cyclones occur throughout the globe and are classified depending on their geographic location. Those forming in the Atlantic and northeastern Pacific Oceans are called hurricanes (Fig.(1.5)), while those forming in the northwestern Pacific Ocean are referred to as typhoons. In the Indian Ocean and South Pacific Ocean, they are simply called cyclones. Although these are named differently depending on the region, they all have similar features, such as powerful winds, plenty of rain, and a risk for life-threatening storm surges.

A cyclone is characterized by winds rotating around a central region of low pressure. Its rotation is influenced by the **Coriolis effect**, a force caused by the Earth's rotation that deflects anything that is moving to the right in the Northern Hemisphere and to the left in the Southern Hemisphere, resulting in counter-clockwise movement in the Northern Hemisphere and clockwise in the Southern Hemisphere.



Figure 1.4: Hurricane Catarina, a rare South Atlantic tropical cyclone viewed from the International Space Station on March 26, 2004).

Cyclones are formed under specific atmospheric and oceanic conditions, such as warm ocean water, with a surface temperature of at least 26°C , which provides the heat and moisture needed to sustain the cyclone. It also requires high humidity in the lower to mid-troposphere and low wind shear to build and maintain its structure. A sufficient Coriolis effect provides the necessary spin for cyclonic motion, which is why these cannot be developed near the equator, where the Coriolis effect is minimal. Lastly, a pre-existing low-pressure disturbance, such as a tropical wave, is required to start the phenomenon.

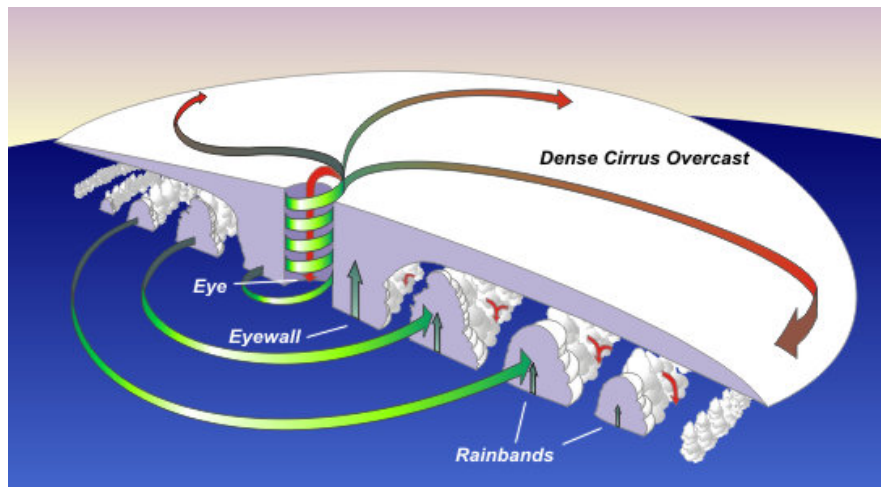


Figure 1.5: Cross-section of a typical hurricane (Source: NOAA National Oceanic and Atmospheric Administration).

1.1.8 Other Types of Cyclones

Extratropical cyclones are formed in the mid-latitudes, typically between 30° and 60° latitude. They are driven by the temperature gradient and are associated with weather systems like cold fronts, warm fronts, and occluded fronts. They are larger than tropical cyclones, often extending to 1000–2000 km. The interaction of these air masses creates a low-pressure system, with warm air rising and cold air descending, leading to the development of fronts. Unlike tropical cyclones, extratropical cyclones derive their energy from horizontal temperature gradients rather than heat from the ocean. They occur in North America, Europe, Asia, and the southern parts of South America and Australia.

Polar cyclones are smaller cyclones that form in polar regions, often over cold ocean waters or near the Arctic and Antarctic. They are short-lived and are smaller compared to tropical and extratropical cyclones, around 100–500 km in diameter. Because there is less heat and moisture available in polar regions, these storms are often weaker than tropical and extratropical cyclones. Strong winds and localized weather disruptions, especially in the form of snow or blizzards, are still possible with them, though. They

tend to form in areas with sharp temperature gradients, such as between open water and ice-covered regions, and can play a role in the redistribution of cold air masses to lower latitudes. They occur in Polar lows in the Arctic Ocean or near Greenland and Iceland.

Subtropical Cyclones are weather phenomena that have characteristics of both tropical and extratropical cyclones. They are formed over subtropical waters, often in transition zones between tropical and mid-latitude regions with a mixture of tropical (warm-core) and extratropical (cold-core) features. Compared to tropical cyclones, they feature a broader wind field, with the highest winds far from the centre. They frequently exhibit certain frontal features, such as extratropical cyclones, and typically lack an entirely warm core.

The Saffir-Simpson Hurricane Wind Scale is used to categorise tropical storms based on wind speed. Winds in a tropical depression can reach 61 km/h, although they have little effect. With speeds in range 62–118 km/h, a tropical storm can cause considerable damage and a lot of rain. Category 1 hurricanes 119–153 km/h, minor damage; Category 2 hurricanes 154–177 km/h, moderate damage; Category 3 hurricanes 178–208 km/h, major damage; Category 4 hurricanes 209–251 km/h, severe damage; and Category 5 hurricanes above 252 km/h, catastrophic damage are all classified as hurricanes.

Huge cyclones have had a tremendous effect on people. More over 300,000 people died in East Pakistan (now Bangladesh) during the 1970 Bhola Cyclone, showing the dangers to coastal communities. Disaster response concerns were brought to light by Hurricane Katrina, which destroyed the U.S. Gulf Coast in 2005. In 2008, Cyclone Nargis killed nearly 138,000 people in Myanmar, highlighting the need for better preparedness and infrastructure. The significance of predicting and mitigation measures is highlighted by these incidents.

1.1.9 Waterspouts

Waterspouts are rotating columns of air occurring over water surfaces, typically ranging from tens to hundreds of meters in diameter with wind speeds of $20 - 50\text{m/s}$, and are classified into two types-tornadic waterspouts, which resemble tornadoes extending from cumuliform clouds, and fair-weather waterspouts, which form from the surface upward due to localized heating and convergence, sharing dynamics with dust devils.



Figure 1.6: Image of Waterspout (Source: Wikipedia)

Tornadic waterspouts, linked to severe thunderstorms or supercells, descend from clouds with strong updrafts and vorticity, often transitioning to tornadoes if they move ashore, while fair-weather waterspouts arise in calmer conditions over warm water (e.g. $>26^{\circ}\text{C}$), driven by thermal instability and low-level wind shear, which sometimes responsible for land erosion. Their structure features a visible funnel of condensed water vapour or spray, sustained by moisture from the surface, and their dynamics, modelled via vortex stretching and the Navier-Stokes equations under moist conditions, mirror those of tornadoes but differ from tropical cyclones' oceanic scale and dust devils' dry convection. Waterspouts, observed globally over oceans, lakes, and rivers, can erode coastlines or capsize small vessels, making them a compelling subject for mathematical modelling

to explore their formation, intensity, and transition potential within the spectrum of atmospheric whirlwinds (Fig.(1.6)).

1.1.10 Supercells

Supercells are highly organized, severe convective storms distinguished by a single, quasi-steady rotating updraft, known as a mesocyclone, which persists for hours—far longer than the 10–20 minutes typical of ordinary thunderstorms. Forming in environments with high convective available potential energy (CAPE, often $> 2000 \text{ J/kg}$), strong vertical wind shear (20–30 m/s over 0–6 km), and atmospheric instability, supercells feature a mesocyclone 2–10 km wide with cyclonic vorticity around 10^{-2} s^{-1} . This rotation arises when horizontal vorticity from shear is tilted into the vertical by a powerful updraft (up to 50 m/s), sustained by a separated rear-flank downdraft (RFD) that tightens the vortex, often spawning tornadoes. Classified as classic, low-precipitation (LP), or high-precipitation (HP), supercells produce extreme weather—large hail ($> 2 \text{ cm}$), winds exceeding 40 m/s, and long-lived tornadoes (up to EF5 strength, $> 90 \text{ m/s}$)—detectable via Doppler radar’s hook echoes and bounded weak echo regions (BWERS).



Figure 1.7: A supercell thunderstorm over the Great Plains.(Source: Shutterstock).

Propagating unidirectionally at 10–20 m/s, supercells dominate regions like the U.S. "Tornado Alley", offering a rich case for mathematical modelling relevant to understanding atmospheric whirlwinds.

1.2 Fundamentals of Vortex Dynamics

Mathematical modelling of whirlwinds hinges on vortex dynamics, the study of rotating fluid flows. Below, we outline key concepts and equations, focusing on their relevance to dust devils and tropical cyclones.

1.2.1 Velocity and Vorticity

The velocity field $\mathbf{q} = [u, v, w]$ describes fluid motion in Cartesian coordinates $[x, y, z]$.

Vorticity is

$$\boldsymbol{\zeta} = \nabla \times \mathbf{q} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right).$$

In dust devils, $\boldsymbol{\zeta}$ peaks near the surface due to shear; in tropical cyclones, it concentrates in the eyewall, reflecting intense rotation. Vorticity is twice the angular velocity, making it a fundamental quantity for vortex analysis.

1.2.2 Circulation

Circulation $\Gamma = \oint_C \mathbf{q} \cdot d\mathbf{s} = \int_S \boldsymbol{\zeta} \cdot d\mathbf{S}$, quantifies rotation around a closed path C where Stokes' theorem relates it to vorticity flux over a surface S . For dust devils, Γ measures vortex strength; for tropical cyclones, it scales with the storm's size and intensity, aiding model validation.

1.2.3 Governing Equations

Atmospheric whirlwinds follow the Navier-Stokes equations, adapted for atmospheric flows via the Boussinesq approximation

$$\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \mathbf{q} + \mathbf{g} + \frac{\rho'}{\rho_0} \mathbf{g},$$

$$\nabla \cdot \mathbf{q} = 0,$$

where ρ_0 is reference density, ρ' is perturbation density, p is pressure, ν is viscosity, and $\mathbf{g} = [0, 0, -g]$ is gravity. The buoyancy term $\frac{\rho'}{\rho_0} \mathbf{g}$ drives dust devils, while latent heat effects dominate tropical cyclones.

1.2.4 Vorticity transport equation

The vorticity vector, defined as

$$\boldsymbol{\zeta} = \nabla \times \mathbf{q},$$

measures the local rotation of fluid elements based on the velocity field \mathbf{q} , with its magnitude indicating rotation strength and direction aligning with the rotation axis. The vorticity transport equation, for an incompressible, Newtonian fluid, is

$$\frac{D\boldsymbol{\zeta}}{Dt} = (\boldsymbol{\zeta} \cdot \nabla) \mathbf{v} + \nu \nabla^2 \boldsymbol{\zeta},$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{q} \cdot \nabla)$ is the material derivative and $\nu = \frac{\mu}{\rho}$ is kinematic viscosity, capturing vorticity evolution through advection, stretching/tilting, and viscous diffusion, essential for studying complex flows like turbulence.

1.2.5 Cyclostrophic balance

Cyclostrophic balance occurs when the pressure gradient force balances the centrifugal force in curved flow, neglecting Coriolis effects, given by

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{v^2}{r},$$

where ρ is density, $\frac{\partial p}{\partial r}$ is the radial pressure gradient, v is tangential velocity, and r is radius of curvature. It dominates in small-scale, fast-rotating systems like tornadoes, where the Rossby number is high, driving strong circular motion with a central pressure drop, key to analysing intense vortices.

1.2.6 Geostrophic balance

Geostrophic balance arises when the Coriolis force balances the pressure gradient force in large-scale atmospheric flows, typically away from the equator. It is expressed as

$$fv = \frac{1}{\rho} \frac{\partial p}{\partial r},$$

where f is the Coriolis parameter, v is the tangential velocity, ρ is the air density, and $\frac{\partial p}{\partial r}$ is the radial pressure gradient. This balance characterizes mid-latitude cyclones and anticyclones, where rotation due to Earth's spin is dominant and the Rossby number is small.

1.2.7 Gradient wind balance

Gradient wind balance incorporates both the Coriolis and centrifugal forces in balance with the pressure gradient force, applicable to curved atmospheric flows such as hurricanes.

The governing relation is

$$\frac{v^2}{r} + fv = \frac{1}{\rho} \frac{\partial p}{\partial r},$$

where v is the tangential velocity, r is radius of curvature, f is the Coriolis parameter, and ρ is air density. This balance applies to mesoscale vortices where both planetary rotation and centrifugal effects are important.

1.2.8 Hydrostatic balance

Hydrostatic balance governs the vertical equilibrium in the atmosphere, where the downward gravitational force is balanced by the upward vertical pressure gradient. It is given by

$$\frac{\partial p}{\partial z} = -\rho g,$$

where $\frac{\partial p}{\partial z}$ is the vertical pressure gradient, ρ is density, and g is the gravitational acceleration. This approximation is valid for large-scale, slowly evolving flows and is fundamental to atmospheric stratification.
