PREFACE

Gas dynamics is a particular branch of fluid dynamics which expanded significantly in the 20th century to analyze the fundamentals of high speed compressible flows. It has grown with the development of high-speed aerodynamics, supersonic flight, and space exploration, and has become an area of research for physicists, chemists, applied mathematicians, and astrophysicists to develop theories and experimental techniques. While the term, "gas dynamics" itself implies that the field is entirely related to the state of gaseous matter. Historically, gas dynamics played an important role in the theoretical treatment of detonation in gases, combustion process, aerodynamics. However, modern gas dynamics takes a broader perspective, nowadays it concerns with the causes and effects arising from high-speed flows, steady and unsteady phenomena, viscous and non-viscous behaviors, conducting and non-conducting mediums for which compressibility is important.

A wave is recognizable signal that travels from one part of a medium or space, to another with a recognizable velocity of propagation. The signal can be any disturbance, it may distort, change its magnitude and change its velocity but it is still recognizable. Waves can take various forms, such as sound waves, light waves, water waves, and seismic waves. These waves are broadly categorized into two main classes by Whitham [1] as, dispersive waves and wave motions formulated mathematically as hyperbolic system of partial differential equations. In dispersive waves the signal propagate at once with all the particles of the medium whereas in case of waves represented by hyperbolic PDEs, the signal is not felt at once by every point of the space. The hyperbolic equations depend on the form of equations rather than obtained explicit solution while dispersive wave is based on type of solution rather than type of equation.

The motion of fluid is said to be one-dimensional when all its properties depend on only geometric coordinate and on the time. Such motions in which distribution of flow variables remain similar to themselves with time and vary only as a result of change in scale is called self similar. Self similar are of great importance in gas dynamics. In this, flow variables depend upon particular combinations of time and spatial coordinates. This method is used to find approximate solution of certain problem of one-dimenstional motion of compressible fluid. In 1968, WD Hayes [2] published an article titled "Self similar strong shocks in an exponential medium", using Eulerian approach of Sedov to study self similar one dimensional propagation of strong shock wave.

The present thesis manifests the results of research carried out by me at the Department of Mathematical sciences, IIT (BHU), Varanasi, from July 2019 to June 2025 under the guidances of Prof. L. P. Singh. This thesis is devoted to the study of the propagation of non-linear waves in various gaseous medium governed by the quasi-linear system of partial differential equations. The main purpose of this study is to understand the propagation of wave and its evolutionary behavior under certain constraints using analytical methods such as method of characteristic, method of wavefront analysis etc. Also, we have used similarity method to study the imploding shock wave in non-ideal gas with gravitational effect and compared the results with CCW approximation methods. The entire thesis is divided into six chapters as follows:

Chapter - 1 serves as an introduction, providing a comprehensive overview of the research conducted in the area of non-linear wave propagation, along with its applications and methodologies. It also offers a concise discussion of the underlying mathematical theories and fundamental properties associated with this field. The essential characteristics of non-linear waves and their propagation are outlined. Additionally, a brief review of the physical properties of ideal gases, non-ideal gases, dusty gases, magnetogasdynamics, and concept of self similarity is presented. This chapter also includes several key results that will be referenced in the following chapters.

In **Chapter - 2**, we have studied one-dimensional non-ideal relaxing gas to study solution of shock wave for planar and non-planar flows, more specifically cylindrically symmetric, and spherically symmetric flows. The analytic solution of the problem is determined in the characteristic plane. It signifies that linear solution in this plane reveals non-linear behavior in the physical plane. Also, the condition of shock formation has been determined by analyzing the growth and decay of shock waves using transport equations. It is observed that nature of the solution completely depends upon relaxing gas parameters and non-idealness. All computations are done using computational package MATHEMATICA.

Chapter - 3 presents the study of the problem of propagation of planar, cylindrically symmetric and spherically symmetric shock waves of the one-dimensional motion of an inviscid, self-gravitating, non-ideal interstellar gas cloud. The analytic solution of the problem is resolved, which specifies non-linear behavior in the physical plane. The transport equation, which describes the evolution of weak discontinuity in non-ideal gas is derived. It is observed that the nature of the solution completely depends on the cooling-heating function and self-gravitating parameter. Using the computational package MATHEMATICA, all computations are performed.

Chapter - 4, demonstrates the study of propagation of shock waves in 2-D steady supersonic magnetogasdynamics flow of non-ideal dusty gas using wavefront analysis method. We derived the transport equation which determines the condition for the shock formation. Our aim is to analyze the effect of interaction of dust particles with magnetic field in non-ideal gas on the evolution of shock formation and to examine how the flow patterns of the disturbance vary with respect to the variations in the physical parameters of the medium. It is found that the presence of magnetic field plays an essential role in the wave propagation phenomena. The nature of the solution with respect to the Mach number is analyzed, and it is examined how the shock formation distance changes with an increase or decrease in the value of Mach number. Also, the combined effect of non-idealness, magnetic field and dust particles on the shock formation distance is elucidated and examined how the formation of shocks is affected by the increase in the value of corresponding physical parameters.

In **Chapter - 5**, we have investigated self-similar solutions of second kind for imploding cylindically symmetric shock waves in unsteady one-dimensional flow behind strong shock wave in non-ideal gas with gravitational field. It is assumed that the equation of state of the medium to be of the Mie-Gruneisen type. Similarity exponent has been computed numerically. A numerical description of the flow field was presented within non-ideal gravitational field. We compared the results obtained with the numerical solution derived using the CCW approximation method and it yields a highly accurate approximation for similarity exponents in non-ideal media affected by a gravitational field in both cases. Comprehensive analyses were conducted for both physically significant non-ideal media in the presence of gravitational field. Additionally, the effect of gravitational field on the profiles of density, pressure and velocity is presented graphically for two distinct and physically relevant media i.e. for dusty gas and condensed matter behind the wave front.

Lastly, in **Chapter - 6**, we have provided a comprehensive summary of the analysis conducted in this thesis. The key contributions of the thesis are briefly discussed, followed by a discussion on the future scope.