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Practical prescribed/prescribed time stabilization via event-triggered impulsive control with actuation delays

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ABSTRACT

The present work investigates the problem of practical prescribed / prescribed-time stabilization of nonlinear systems by virtue of event-triggered impulsive control with actuation delays. Two event-triggering mechanisms (ETM) based on continuous event detection are put forward in this context. In both cases, impulse control inputs are updated after a specific time, known as actuation delays, to the event-triggering instants. Lyapunov-like sufficient conditions are proposed to achieve practical prescribed/prescribed-time stability of impulse-controlled systems with actuation delays corresponding to the designed first and second ETM, respectively. Moreover, both of the proposed ETM is free from Zeno behavior. Furthermore, two examples are considered and corresponding numerical simulations are performed to convey and validate the efficacy of the proposed theoretical results.

1. Introduction

Among several control methodologies, the impulsive control strategy has grown leaps and bounds recently due to its discrete-time implementations on the dynamical systems to control it. Also, it transmits only a limited amount of information to the considered system and is able to control it accordingly. In the book [1], the significant contributions on impulsive control theory is presented. In majority of the fascinating works on impulsive control, impulsive instances used to be chosen arbitrarily according to the authors' choices. Though it is simple to implement, it suffers from the drawbacks of unnecessary use of control costs and information transmissions. Moreover, this methodology of controlling the dynamical systems with the help of arbitrarily chosen impulse instances is known as time-triggered impulsive control ($TTIC$).

To overcome the disadvantages of $TTIC$, mentioned above, the concept of event-triggered impulsive control ($ETIC$), based on Lyapunov method, has recently been introduced and studied in the article [2]. Some Lyapunov-based sufficient conditions, to achieve uniform and asymptotic stability, are provided there. In $ETIC$ strategy, the impulse controller is activated at specific instances, known as event/triggering instances, obtained through the violation of the proposed triggering condition by the systems' dynamics. Before delving into the concept of $ETIC$, it is worth noting that the pioneering work on the concept of event-triggered control (ETC) was conducted meticulously in the papers [3,4]. In summary, the author of the work [3] was the first to explore event-triggered execution of control tasks explicitly and demonstrated its major advantages over time-triggered execution of control tasks. Whereas [4] aimed to provide an overview of event-triggered and self-triggered control, as well as their implementation differences. They have also demonstrated their application in wireless communication. Now it is important to note that, $ETIC$ is different from the notion of event-triggered control (ETC) in the aspect: in ETC, the controllers used to be updated at each triggering instants and hold constant between two consecutive triggering instants. Whereas, in $ETIC$, the controller is activated at each event instances only and there is no control mechanism between two consecutive event instances. In brief, $ETIC$ take the advantages of both ETC and impulsive control. Hence, $ETIC$ mechanism has drawn significant attention of researchers of the control community in very short period of time. Some of the many fascinating works in this field can be found in the articles [5–7] and references therein. Specifically, [5] investigated the problem of input-to-state stability of nonlinear dynamical systems via $ETIC$ strategy. And, [6] provides some sufficient conditions to achieve input-to-state stability of impulsive and switched stochastic delayed systems under the framework of $ETIC$. Whereas, [7] studied the problem of exponential stability of nonlinear systems based on partial unknown states. The authors there have constructed event-triggering mechanism by considering only the partially known states. However, it can be observed from the articles [2,7] that, a forced impulse time sequence has been used there to obtain the asymptotic or exponential stability

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of the system, which somehow weaken the purpose of event-triggered mechanism and make its implementation difficult.

Moreover, it is worth mentioning that time delays are a normal part of any real-world system. Therefore, taking into account and examining time delays in models makes them more realistic and enhances their accuracy and performance. A rigorous study on time-delay systems and its stability analysis can be found in the books [8,9]. Since the $ETIC$ for nonlinear dynamical systems (without time-delay) was implemented successfully over several models, it thus became popular among the researchers to study $ETIC$ for time-delayed systems and obtain several stability results and provide significant advancement to the field of control theory. For instance, [10] studied the problem of exponential stability of time-delayed nonlinear systems by means of $ETIC$ mechanism. The authors there have constructed Lyapunov-based event-triggered mechanism to achieve exponential stability in the context of Lyapunov–Razumikhin method. Whereas, some sufficient conditions are presented in [11] to obtain input to state stability of nonlinear time-delay systems, consisting of external disturbances, by virtue of $ETIC$ strategies. And, the authors of the article [12] considered the stability problem of stochastic delayed system under $ETIC$ mechanism. Even though all of the previously stated articles produced remarkable outcomes and advanced the field of control theory, they did not take controller delays into account.

In general, there are two types of delays in the controller design. One is the communication delay that occurs between the sensor and the controller pair, often known as the *sensing delay*. The other is the communication delay between the controller and the actuator pair, known as *actuation delay*. Some of the notable articles on $ETIC$ by considering sensing delays into account can be found in [13,14] and the references therein. It should be noted that, in the case of sensing delays, the impulsive controller is acting specifically at the triggering instants. However, there used to be actuation delays in every practical scenario of control systems. Here, the impulse controller acts on the system after a particular time of the triggering instants. Hence, analyzing different stability properties by considering actuation delays makes it more practical and challenging. As per as the authors' knowledge, [15] is the first article which provided some Lyapunov-based sufficient conditions to achieve asymptotic stability of nonlinear systems by virtue of $ETIC$ with actuation delays. Following this initial endeavor, some of the works in this regard can be found in [16,17] and the corresponding references. Elaborately, based on Lyapunov theory, exponential consensus criteria for multi-agent systems is studied in [16]. Whereas, the authors in [17] studied hyper-exponential stability of neural networks. At this point it is essential not to forget that all the studies on $ETIC$ of nonlinear systems with actuation delays are basically based on attending the stability of the proposed system as the time advances towards infinity, i.e, either asymptotic or exponential. As a result, they cannot stabilize the associated nonlinear systems within a finite duration of time period. However, as noted in [18], the notion of stabilities like asymptotic or exponential may not fulfill the stability requirements in practical sense.

To dealt with this, the concepts of stability in the likes of finite-time and prescribed-time stability, which provide a fast convergence rate, have attracted a lot of researchers in the recent past. However, the concept of prescribed-time stability (PTS) has the upper hand over finite-time stability (FTS) in the sense that the upper bound of the settling time, in the case of PTS, is independent of the initial conditions and can be defined by the user in advance. Such flexibility in choosing the upper bound of settling time is absent in the case of FTS. Also, the settling-time depends on the states' initial conditions, in the case of FTS, which may only sometimes be available in practical scenarios. Hence, the notion of PTS has become an emerging topic in the field of control theory. Recently, [19] has provided some sufficient Lyapunov conditions to stabilized switched systems within some prescribed-time by virtue of $ETIC$. It is also noteworthy to mention that, many practical systems satisfies the unstable Lyapunov form like, $D^{+}V \leq b\,V$, where $b > 0$ and V is the Lyapunov function or non-vanishing external disturbances used to be there in the system (see [20,21]). In these situations, PTS cannot be achieved under the impulsive control. However, one can reach to the desired neighborhood of the equilibrium point, origin, within some pre-specified time and stayed there afterwards. This property is termed as practical prescribed-time stability ($PPTS$). Very recently, the authors of the article [22] studied the problem of $PPTS$ of a particular class of nonlinear systems via ETC. With regard to the discussion above, it is reasonable to wonder if we could provide some Lyapunov-like sufficient conditions to attain $PPTS$ and PTS of general nonlinear systems using $ETIC$ while taking *actuation delays* into consideration.

Inspired by the discussions above, this article proposes certain Lyapunov-like conditions via $ETIC$ for both $PPTS$ and PTS of nonlinear systems while taking the system's *actuation delays* into account. Further, due to the presence of actuation delays, the event instances are different from the time instants of updating of the impulsive control signals. Two different event-triggering mechanisms (ETM) are designed to obtain the event instances. The first ETM is designed to derive the event instances based on Lyapunov-like functions, specifically κ^1 functions. And some Lyapunov-like sufficient conditions are proposed, in this regard, to achieve $PPTS$. Whereas, the second ETM corresponds to the detection of event instances based on κ^1 functions. Also some Lyapunov-like sufficient conditions are proposed, in this regard, to achieve PTS of the general nonlinear systems. It should be noted that, both the ETM's functions are considered differently and serve different purposes. The first ETM does not require memory of the previous impulsive instants to obtain the current event instant; whereas, the derivation of the next event instant, in case of second ETM, requires the memory of the present impulsive instant.

The article's remaining content is arranged as follows: Definitions, system descriptions, and some useful notation are introduced in Section 2. The article's main findings, which include $PPTS$ and PTS theorems for general nonlinear systems, are given in Section 3. Section 4 shows the numerical validity of the proposed results. The article's conclusion in Section 5 emphasizes the important contributions and room for further research.

2. Preliminaries

This part, which acts as a preamble to the remainder of the article, contains a number of notations, an explanation of the system, and a few definitions.

2.1. Notations

$\mathbb{R}$ denotes the set of real numbers and $\mathbb{R}_+ = \{y \in \mathbb{R} : y > 0\}$; $\mathbb{R}_{\geq 0} = \mathbb{R}_+ \cup \{0\}$. $\cup$ denotes the union of two sets. The n-dimensional real space with the Euclidean norm $|\cdot|$ is denoted by $\mathbb{R}^n$. $\mathbb{Z}_+$ is the set of positive integers. The signum function is denoted by $sgn(y)$, where $y \in \mathbb{R}$. Λ be the set of locally Lipschitz functions from $\mathbb{R}^n$ to $\mathbb{R}_{\geq 0}$.

2.2. System's synopsis

Let us investigate the following impulsive control system:

$$\begin{cases} \dot{y}(t) = f(y(t)), t \geq 0, t \neq t_k, \\ y(t_k) = y(t_k^-) + h_k(y(s_k)), k \in \mathbb{Z}_+, \\ y(0) = y_0, \end{cases} \tag{1}$$

where $y \in \mathbb{R}^n$ is the state vector; $y_0 \in \mathbb{R}^n$ is the initial state; f, h_k are the continuous functions from $\mathbb{R}^n$ to $\mathbb{R}^n$ satisfying $f(0) = 0$ and $h_k(0) = 0$ for $k \in \mathbb{Z}_+$ and $\forall t \geq 0$. Further, $\{s_k, k \in \mathbb{Z}_+\}$ (in short $\{s_k\}$) is the set of time instances at which certain event has occurred and is going to be defined later on. Whereas, $\{t_k = s_k + \tau_k, k \in \mathbb{Z}_+\}$ (in short $\{t_k\}$) is the impulsive instances at which the impulsive control is acted on the system. And, $\tau_k \in \mathbb{R}_{\geq 0}$ is the time delay, at the kth instance, between the

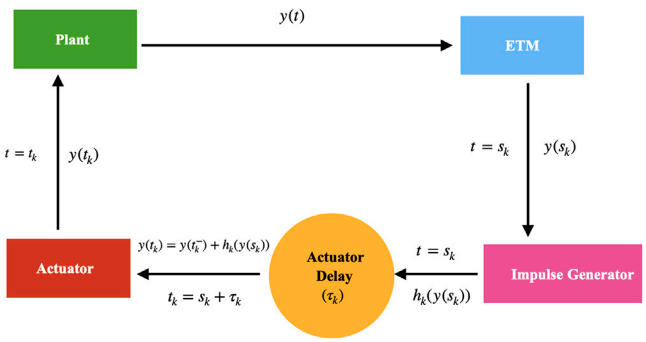


Fig. 1. Block diagram of event-triggered impulsive control loop with actuation delay.

controller and the actuator. The basic idea of what is happening behind here is that, the impulsive controller is activated at the event triggered instance but due to the presence of delay τ_k it is acted, at the time instance t_k, to the system. Further, please refer to Fig. 1 for a pictorial description of Eq. (1). Additionally, it is also assumed that, f and h_k satisfy every requirement that guarantees the unique existence of the solution $y(t) = y(t, 0, y_0)$ of the system (1) in forward time (see [1]). Furthermore, it is assumed that the solution $y(t)$ is right continuous and always has left limits at all times, meaning that $y(t_k^+) = y(t_k)$ holds for all $k \in \mathbb{Z}_+$. Throughout the manuscript, s_k will be called as event instance and t_k as impulse instance.

The purpose of the article is to propose some Lyapunov-like sufficient conditions to establish practical prescribed-time stability ($PPTS$) and prescribed-time stability (PTS) of the system (1) via Event-Triggered impulsive control with actuation delay. To ensure these, appropriate event-triggering conditions are designed to detect event instances and obtain several relations between the impulsive control input h_k, the actuation delay τ_k, the prescribed region and the prescribed-time. Prior to delving into the thorough analysis of the primary findings, let us first concentrate on a few definitions, which are as follows.

2.3. A few definitions and assumptions

In the framework of definitions provided in [15,20,23], let us provide the definitions of $PPTS$ (Definition 1) and PTS (Definition 2), respectively, corresponding to the system (1). This section of the manuscript also recalls an example (Example 1) and two definitions (Definitions 3 and 4) . Apart from these, two assumptions (Assumptions 1 and 2) are also proposed and will be used frequently throughout the article's research.

Definition 1. For any given $\varsigma > 0$ and prescribed constant $T > 0$, the trivial solution of system (1) is said to be practical prescribed-time stable ($PPTS$) with respect to T and ς, if for any $y_0 \in \mathbb{R}^n$, $| y(t, y_0) | \leq \varsigma$ $\forall t \geq T$. Further, the set $\{y \in \mathbb{R}^n : | y(t) | \leq \varsigma\}$ is known as the residual set.

Remark 1. Unlike the definitions of $PPTS$ defined in the articles like [20,21], where the required residual region is obtained through proper adjustment of the system parameters, here, we do not need tunable parameters to achieve $PPTS$. Instead, we will use properly designed impulsive control, obtained through an event-triggered mechanism law, to obtain $PPTS$ of the system (1). Further, one may not have the freedom of tunable parameters in every practical system, and hence to achieve $PPTS$ in such scenarios is one of the motivations of the present study.

Definition 2. For any prescribed constant $T > 0$, the trivial solution of system (1) is declared to be prescribed-time stable (PTS) with respect to time T, if

- for any $\sigma > 0$, there exists a positive real number $\mathcal{L}(\sigma)$ (here $\mathcal{L}(\sigma)$ denotes that, $\mathcal{L}$ is dependent on σ) such that whenever $| y_0 | < \mathcal{L}(\sigma)$ holds, it implies $| y(t) | < \sigma$ $\forall t \geq 0$.
- for any $y_0 \in \mathbb{R}^n$, $| y(t, y_0) | \equiv 0$ $\forall t \geq T$.

Definition 3 (*[24] κ_∞ Functions*)**.** The class of κ_∞ Functions is defined as the collection of strictly increasing continuous function $\varphi : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that $\varphi(0) = 0$ and $\varphi(\lambda) \to \infty$ as $\lambda \to \infty$ holds, and is denoted by $\varphi \in \kappa_\infty$.

Definition 4 (*[24] κ^1 Functions*)**.** Let $\Phi : \mathbb{R}_{\geq 0} \to [0, 1)$ be a strictly increasing continuous function. Φ is said to be a κ^1 function if, $\Phi(0) = 0$ and $\Phi(\lambda) \to 1$ as $\lambda \to \infty$. It is denoted by $\Phi \in \kappa^1$.

Example 1 (*[24]*)**.** Some examples of the κ^1 Functions are:

- $\Phi(r_0) = \frac{r_0}{r_0+\alpha}$, $\alpha > 0$.
- $\Phi(r_0) = \frac{2}{\pi} \arctan(r_0)$.
- $\Phi(r_0) = 1 - \exp(-r_0)$.

Assumption 1. There exist functions $V \in \Lambda$, $\Phi \in \kappa^1$, $\varphi_1, \varphi_2 \in \kappa_\infty$ and positive constants b, c such that for any $y \in \mathbb{R}^n$

(i) $\varphi_1(| y |) \leq V(y) \leq \varphi_2(| y |)$;
(ii) $D^+\Phi(y) \leq b\,\Phi(y)$, and
(iii) $\Phi(z + h_k(y)) \leq c\,\Phi(y)$,

holds, where $\Phi(y) \equiv \Phi(V(y))$ and $z = w(\tau_k)$. And, $w(t)$ is the solution of the initial value problem

$$\begin{cases} \dot{w}(t) = f(w(t)), \\ w(0) = y. \end{cases} \tag{2}$$

Remark 2. The Assumption 1, as mentioned above, is based on the Assumption 2.2 of the article [15]. Now, we will formulate some results for $PPTS$ corresponding to the system (1), and we do not need several Assumptions as mentioned in the article [15] (Assumption 2.1 and first part of the condition (iii) of Assumption 2.2). Further, the notion of κ^1 functions have been used here instead of Lyapunov functions. The advantages of this modification will be analyzed thoroughly later in the manuscript. Moreover, comparing Eq. (1), *condition (ii)* of the Assumption 1 and Eq. (2) one can obtain that, $w(0)$ plays the role of $y(s_k)$, $z = w(\tau_k)$ plays the role of $y(t_k^-)$. So, *condition (ii)* of the Assumption 1 provides some relation between the value of the κ^1 function Φ at the time instant $t = t_k$, the impulsive strength c and the value of the function Φ at the time instant $t = s_k$.

Assumption 2. There exist functions $V \in \Lambda$, $\Phi \in \kappa^1$, $\varphi_1, \varphi_2 \in \kappa_\infty$ and positive constants a_0, b_0, c_k, p such that $0 < p < 1$ and for any $y \in \mathbb{R}^n$

(i) $\varphi_1(| y |) \leq V(y) \leq \varphi_2(| y |)$;
(ii) $D^+\Phi(y) \leq -a_0\,\Phi^p\,(1 - \Phi)^{2-p} + b_0\,\Phi\,(1 - \Phi)$, and
(iii) $\Phi(z + h_k(y)) \leq c_k\,\Phi(y)$,

holds, where $\Phi \equiv \Phi(y) \equiv \Phi(V(y))$ and $z = w(\tau_k)$ and, c_k depends on k. And, $w(t)$ is the solution of the initial value problem

$$\begin{cases} \dot{w}(t) = f(w(t)), \\ w(0) = y. \end{cases} \tag{3}$$

3. Main results

There are primarily two subsections in this section of the manuscript. The first section provides some Lyapunov-like sufficient conditions to achieve practical prescribed-time stability of system (1) under Event-Triggered impulsive control. Whereas, the second section consists of establishment of some Lyapunov-like sufficient conditions for prescribed-time stability of system (1) under Event-Triggered impulsive control approach. Moreover, all the proposed results are free from Zeno behavior.

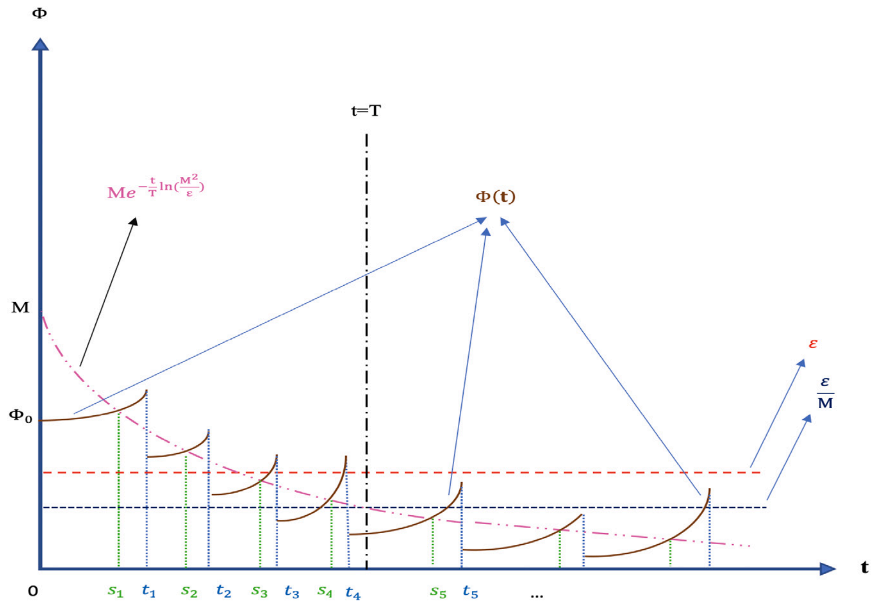


Fig. 2. Graphical representation of the ETM (4).

3.1. Event based results on practical prescribed-time stability

This subsection of the article is dedicated to the establishment of some Lyapunov-like sufficient conditions for $PPTS$ of the system (1) by virtue of event-triggered impulsive control with actuation delay. First, let us consider the following event-triggered mechanism (ETM):

$$s_{k+1} = \inf \left\{ t \geq t_k : \Phi(t) \geq M e^{-\frac{t}{T}\ln\left(\frac{M^2}{\varepsilon}\right)} \right\}, \tag{4}$$

where Φ is a κ^1 function, $\Phi(t) \equiv \Phi(V(y(t)))$, M is a positive constant such that $M > 1$, $T > 0$ is the prescribed-time within which the system (1) reaches the prescribed region $\Delta = \{y \in \mathbb{R}^n : \Phi(t) \leq \varepsilon\}$, where $0 < \varepsilon < 1$, and stayed there afterwards. Further, $V \equiv V(y(t))$ is the Lyapunov function. One can observe from Eq. (4) that, the threshold function $Me^{-\frac{t}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}$ is dependent on the prescribed-time T and the value ε. Here we have considered $t_0 = 0$ as the initial time instance. Then, the first event has occurred at the time instant s_1. But, due to the presence of actuation delay τ_1, the impulsive control acted on the system at some time $t_1 = s_1 + \tau_1$. So, t_1 is the first impulse instant. Now, due to the presence of actuation delay τ_1, the impulsive control needs to bring the value of Φ below the threshold function at the impulsive instant t_1. Similar process continues for all $t \geq 0$. An graphical illustration of the ETM (4) is illustrated in Fig. 2. Now in view of these, let us put forward a theorem, providing a Lyapunov-like sufficient condition for $PPTS$ of the system (1) by virtue of the ETM (4), along with some insights into this.

Theorem 1. *Let the following conditions, corresponding to the system (1) under the event instances determined by the ETM (4), hold:*

(i) Assumption 1 holds for all $y \in \mathbb{R}^n$ with $c \in (0,1)$;

(ii) $\tau_k < \min\left\{\frac{\ln\left(\frac{1}{c}\right)}{A}, T - s_k\right\}$ *if* $s_k < T$; *and*

(iii) $\tau_k < \min\left\{\frac{\ln\left(\frac{1}{c}\right)}{A}, \frac{1}{b}\ln(M)\right\}$ *if* $s_k \geq T$,

where A is given by, $A = \frac{\ln\left(\frac{M^2}{\varepsilon}\right)}{T}$. Then, the inter-event time $\{s_{k+1} - s_k\}$ is lower bounded by

$$\frac{b\tau_k + \ln\left(\frac{1}{c}\right)}{A + b}. \tag{5}$$

Further, the trivial solution of the system (1) is $PPTS$.

Proof. At the starting point, i.e, $t = 0$ the value of the κ^1 function Φ is denoted by $\Phi(V(y_0)) \equiv \Phi_0$. By Definition 4, one has $\Phi_0 < 1 < M = Me^{-\frac{0}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}$. Hence initially we are below the threshold function $Me^{-\frac{t}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}$. Now, depending on the event instances s_k two cases arises either $s_k < T$ or $s_k \geq T$. Whatever be the case, from *conditions (ii)* and *(iii)*,one have that $\tau_k < \frac{\ln\left(\frac{1}{c}\right)}{A}$.

Part 1: To make the ETM (4) working, we need to make sure that at each impulsive instance t_k, $\Phi(t_k)$ comes below the threshold function $Me^{-\frac{t_k}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}$. This is the first part of the proof which is given below.

By virtue of Assumption 1, one has

$$\begin{aligned}\Phi(t_k) &\equiv \Phi(V(y(t_k))) = \Phi(V(y(t_k^-) + h_k(y(s_k))))\\ &\leq c\,\Phi(V(y(s_k))) \equiv c\,\Phi(s_k).\end{aligned} \tag{6}$$

Combining Eqs. (4) and (6), it can be obtained that

$$\begin{aligned}\Phi(t_k) &\leq c\,M\,e^{-As_k}\\ &= M\,e^{-At_k}\,c\,e^{A\tau_k}.\end{aligned} \tag{7}$$

Using the inequality $\tau_k < \frac{\ln\left(\frac{1}{c}\right)}{A}$ in Eq. (7) one can deduce

$$\Phi(t_k) < M\,e^{-At_k}. \tag{8}$$

This completes the first part of the proof of the Theorem.

Part 2: Now, let us come to the second part of the proof of Theorem 1. That is to prove the validity of Eq. (5). For this, using *condition (ii)* of Assumption 1 one can obtain

$$\Phi(t) \leq \Phi(t_k)\,e^{b(t-t_k)}, \quad \forall t \in [t_k, s_{k+1}].$$

And thus, at $t = s_{k+1}$,

$$\Phi(s_{k+1}) \leq \Phi(t_k)\,e^{b(s_{k+1}-t_k)}. \tag{9}$$

Also, from the ETM (4) and using $A = \frac{\ln\left(\frac{M^2}{\varepsilon}\right)}{T}$ one has

$$\Phi(s_{k+1}) = Me^{-As_{k+1}}. \tag{10}$$

Eqs. (6), (9) and (10) collectively gives

$$\begin{aligned} Me^{-As_{k+1}} &\leq \Phi\left(t_k\right) e^{b\left(s_{k+1}-t_k\right)} \\ &= c\, M\, e^{-As_k} e^{b\left(s_{k+1}-t_k\right)} \\ &= c\, M\, e^{-As_k} e^{b\left(s_{k+1}-s_k\right)} e^{-b\,\tau_k}, \end{aligned}$$

which further reduces to

$$\frac{1}{c} \leq e^{A\left(s_{k+1}-s_k\right)} e^{b\left(s_{k+1}-s_k\right)} e^{-b\,\tau_k}. \tag{11}$$

From Eq. (11) one can easily obtain that

$$s_{k+1} - s_k \geq \frac{b\tau_k + \ln\left(\frac{1}{c}\right)}{A+b}.$$

Hence, the present ETM (4) is free from Zeno behavior. This completes the proof of the second part of the Theorem.

Part 3: Next our aim is to make sure that one can reach the prescribed region $\Delta = \{y \in \mathbb{R}^n : \Phi(t) \leq \varepsilon\}$ inside the prescribed-time T and remained there afterwards. For this, let us consider two cases depending on either $t < T$ or $t \geq T$.

Case 1: $t < T$.

Let s_{k_0} be the last event instance before the prescribed-time T. Then, from *condition (ii),* one has $\tau_{k_0} < T - s_{k_0}$ i.e, $t_{k_0} < T$. So, if there is any event instance s_k before the prescribed-time T then the impulse instance t_k will also act before the time T. Thus t_{k_0} is the last impulse instance before the time T. By virtue of Eq. (8) one can get that $\Phi\left(t_{k_0}\right) < M\, e^{-At_{k_0}}$. Since, s_{k_0} and t_{k_0} are the last event instance and impulse instance respectively we have

$$\Phi(t) < M\, e^{-At} = Me^{-\frac{t}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}, \quad \forall\, t \in \left[t_{k_0}, T\right]. \tag{12}$$

And thus at $t = T$,

$$\begin{aligned} \Phi(T) &\leq Me^{-\frac{T}{T}\ln\left(\frac{M^2}{\varepsilon}\right)} \\ &= \frac{\varepsilon}{M} < \varepsilon. \end{aligned} \tag{13}$$

Hence we have obtained that at the time instance $t = T$ we are within the prescribed region Δ. Now we have to make sure that we remain within the prescribed region Δ for all $t \geq T$. So, let us consider the following case:

Case 2: $t \geq T$.

At this stage let us consider the interval $t \in \left[T, t_{k_0+1}\right]$ which can be further splitted into $t \in \left[T, s_{k_0+1}\right] \cup \left[s_{k_0+1}, t_{k_0+1}\right) \cup \left\{t_{k_0+1}\right\}$. Now, combining Eq. (13) and the ETM (4) one can obtain that

$$\Phi(t) \leq \tfrac{\varepsilon}{M} < \varepsilon, \quad \forall\, t \in \left[T, s_{k_0+1}\right]. \tag{14}$$

However, for $t \in \left[s_{k_0+1}, t_{k_0+1}\right)$ one can deduce from *condition (ii)* of Assumption 1 that

$$\begin{aligned} \Phi(t) &\leq \Phi\left(s_{k_0+1}\right) e^{b\left(t-s_{k_0+1}\right)} \\ &< \frac{\varepsilon}{M} e^{b\left(t_{k_0+1}-s_{k_0+1}\right)} \\ &= \frac{\varepsilon}{M} e^{b\,\tau_{k_0+1}}. \end{aligned} \tag{15}$$

Combining *condition (iii)* of Theorem 1 and Eq. (15) it can be deduced that

$$\Phi(t) < \varepsilon, \quad \forall\, t \in \left[s_{k_0+1}, t_{k_0+1}\right). \tag{16}$$

Further, at $t = t_{k_0+1}$, we have from Eq. (6) that

$$\begin{aligned} \Phi\left(t_{k_0+1}\right) &\leq c\,\Phi\left(s_{k_0+1}\right) \\ &\leq c\frac{\varepsilon}{M} \leq \frac{\varepsilon}{M} \\ &< \varepsilon. \end{aligned} \tag{17}$$

Combining Eqs. (14),(16) and (17) one can get that

$$\Phi(t) < \varepsilon, \quad \forall\, t \in \left[T, t_{k_0+1}\right]. \tag{18}$$

Moreover, as the threshold function $Me^{-\frac{t}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}$ used in the ETM (4) decreases as time increases and time delay τ_{k_0+1} has been upper bounded by considering the worst possible case ($s_{k_0+1} = T$), $\Phi(t)$ remains upper bounded by ε $\forall\, t \geq t_{k_0+1}$. Thus, Eq. (18) in combination with the aforementioned statement concludes that

$$\Phi(t) < \varepsilon, \forall\, t \geq T. \tag{19}$$

This completes the last part of the proof of the Theorem.

Ultimately, Part 1, 2 & 3 combiningly completes the proof of Theorem 1. □

Remark 3. It can be observed that, by considering $\Phi(V) = \frac{V}{V+\alpha}$, *condition (ii)* of Assumption 1 reduces to $D^+V \leq b\,V + \frac{b}{\alpha}V^2$. And, this can be considered as a generalized version of the Lyapunov-form used in the articles [2,10,15,17], where the authors have considered the Lyapunov form $D^+V \leq b\,V$ and obtained several useful results diversifying the study of event-triggered impulsive control in various domains. In other words, one can assert that the ETM (4) proposed in the present work is capable of withstanding the effects of nonlinear terms arriving in the Lyapunov inequality. Apart form these, several other forms of Lyapunov-inequality, corresponding to different choices of the κ^1 functions Φ can also be obtained and thereafter surmounted by the proposed ETM (4) to achieve desired results. Hence, the proposed methodology further enhances the topic of event-triggered impulsive control and also extends the domain of practical implementations.

Remark 4. It is noteworthy to mention that all the existing articles in literature on the event-triggered impulsive control of non-linear systems considering actuation delay have considered either asymptotic [15], exponential [16] or hyper exponential [17] stability. But, it is more realistic to get to some desired domain around the equilibrium point within some pre-specified and staying there afterwards. This phenomenon has been achieved through Theorem 1 of the present manuscript and is named as $PPTS$. At this point it is to be noted that, as the origin of the system (1) can be unstable one, (can be assured from *condition (ii)* of Assumption 1), we may not have any stable region around the origin. And thus, only reaching the desired region around the origin is not enough but we had to make sure it stays there thereafter. All these issues have been taken care of by the above proposed theorem, Theorem 1.

Remark 5. Authors of the article [15,17] needed the initial values of the state trajectories so that depending on the initial value of the Lyapunov function, the parameters of the ETM, proposed there, can be chosen for functionality of the corresponding ETM. But, the initial values of the state trajectories used to be unavailable or poorly calculated in most of the physical systems. However, such kind of dependency among the initial values of the states and the parameters is not there in the ETM (4) proposed in the present work. Therefore, this increases its practical implementation. Further, the memory of the previous impulsive instant was required to be calculated from the upcoming event instant, corresponding to the ETM considered in the articles [16,25]. However, the ETM (4) proposed in the present work is based on the continuous sampling which reduces the complexity in the implementation.

3.2. Event based results on prescribed-time stability

In this subsection, some Lyapunov-like sufficient conditions, to obtain PTS of system (1), will be discussed via event-triggered impulsive control with actuation delay. In this regard, let us construct the

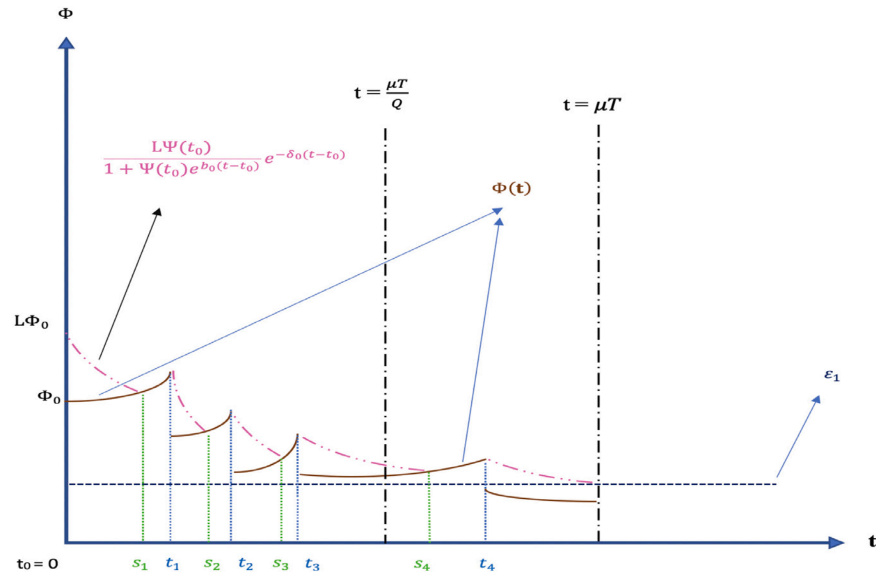


Fig. 3. Graphical illustration of the ETM (20).

following ETM:

$$s_{k+1} = \inf\left\{ t \geq t_k, t \leq \mu T : \Phi(t) \geq \frac{L\Psi\left(t_k\right)}{1+\Psi\left(t_k\right)e^{b_0\left(t-t_k\right)}} e^{-\delta_k\left(t-t_k\right)} \right\}, \quad (20)$$

where Φ is a κ^1 function, $\Phi(t) \equiv \Phi(V(y(t)))$, $\Psi(t) = \frac{\Phi(t)}{1-\Phi(t)}$, L is a positive constant such that $L > 1$, $\mu \in (0,1)$, δ_k is some positive constant, which will be revealed later, and $T > 0$ is the prescribed-time within which the system (1) reaches the origin and stayed there afterwards. Further, $V \equiv V(y(t))$ is the Lyapunov function. Consider $t_0 = 0$ as the initial time instance. First event has occurred at the time instant s_1. But, due to the presence of actuation delay τ_1 the impulsive control acted on the system at some time $t_1 = s_1 + \tau_1$. So, t_1 is the first impulse instant. An graphical insight to the ETM (20) is given in Fig. 3. Now, let us derive and analyze some results by providing a Lyapunov-like sufficient condition for PTS of the system (1) via proposed ETM (20).

Theorem 2. *Let $Q > 1$, $0 < q < 1$ be some constants. Further, denote $B = \frac{a_0(1-q)}{\left\{1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}\right\}^{2-p}}$, $\varepsilon_0 = \frac{\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}{1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}$ and choose $\mu_1 \in (0,1)$ such that $\varepsilon_1 = \left\{B(1-p)\mu_1 T\right\}^{\frac{1}{1-p}} \leq \varepsilon_0$. And, suppose the following conditions, for the system (1) under the event instances determined by the ETM (20), hold true*

(i) Assumption 2 holds for all $y \in \mathbb{R}^n$ with $c_k \in (0,1)$;

(ii) $\tau_{k+1} < \frac{\mu T}{Q} - s_{k+1}$, if $s_{k+1} < \frac{\mu T}{Q}$, $\forall k$;

(iii) $\tau_{k+1} < \mu T - s_{k+1}$ and $c_{k+1} \leq \frac{\varepsilon_1}{Le^{\delta_k(\tau_{k+1}+t_k)}}$, if there exist some k such that, $\frac{\mu T}{Q} \leq s_{k+1} < \mu T$;

where $\delta_k = \frac{1}{\mu T - t_k}\ln\left(\frac{L\Psi(t_k)}{\varepsilon_1\left(1+\Psi(t_k)e^{b_0(\mu T-t_k)}\right)}\right)$ and $\mu = 1-\mu_1$. Then, two consecutive event instances satisfy

$$s_{k+1} - s_k \geq \frac{\ln(L)}{b_0+\delta_k} + \tau_k \geq \frac{\ln(L)}{b_0 + \frac{Q}{\mu T(Q-1)}\ln\left(\frac{L}{\varepsilon_1}\right)} + \tau_k, \forall k \in \mathbb{Z}_+. \quad (21)$$

Moreover, the trivial solution of the system (1) is PTS.

Proof. Before dive deep into the proof of the Theorem, let us illustrate some concepts required to derive the proof. Which is as follows:

It can be observed from the threshold function $\frac{L\Psi(t_k)}{1+\Psi(t_k)e^{b_0(t-t_k)}} e^{-\delta_k(t-t_k)}$ of the ETM (20) that it reaches the region $\{y \in \mathbb{R}^n : \Phi(t) = \varepsilon_1\}$ exactly at the time $t = \mu T$ and is greater than ε_1 for all $t < \mu T$. So, if there exist some $k_{00} \in \mathbb{Z}_+$ such that $\frac{\mu T}{Q} \leq s_{k_{00}+1} < \mu T$ holds then, as mentioned in *Sub-case 2* of Theorem 2, there can only be one such instant. Also, combining *Sub-case 1* and *Sub-case 2* it can be obtained that $t_{k_{00}} < \frac{\mu T}{Q}$. So, the term $\mu T - t_{k_{00}}$, appeared in the denominator of $\delta_{k_{00}}$, is greater than $\mu T - \frac{\mu T}{Q}$. Hence, $t_{k_{00}}$ cannot be too close to μT to make the denominator $\mu T - t_{k_{00}}$ tends to 0 and thus making the speed of convergence rate $\delta_{k_{00}}$ tends to ∞. So, infinite speed of convergence is also not there in the ETM (20).

With this introduction let us focus on the proof of the Theorem explicitly.

Part 1: The first part of the proof is dedicated to establishing the validity of Eq. (21). From *condition (ii)* of Assumption 2 and considering the interval $t \in \left[t_k, s_{k+1}\right]$, one can have

$$D^+\Phi \leq b_0\Phi(1-\Phi),$$

which gives

$$\Phi(t) \leq \frac{\Psi\left(t_k\right)e^{b_0\left(t-t_k\right)}}{1+\Psi\left(t_k\right)e^{b_0\left(t-t_k\right)}}. \quad (22)$$

And, at $t = s_{k+1}$ one can obtain from Eq. (22) that

$$\Phi\left(s_{k+1}\right) \leq \frac{\Psi\left(t_k\right)e^{b_0\left(s_{k+1}-t_k\right)}}{1+\Psi\left(t_k\right)e^{b_0\left(s_{k+1}-t_k\right)}}. \quad (23)$$

Also from the ETM (20) it can be inferred that

$$\Phi\left(s_{k+1}\right) = \frac{L\Psi\left(t_k\right)}{1+\Psi\left(t_k\right)e^{b_0\left(s_{k+1}-t_k\right)}} e^{-\delta_k\left(s_{k+1}-t_k\right)}. \quad (24)$$

Combining Eqs. (23) & (24) it can be concluded that

$$\frac{L\Psi\left(t_k\right)}{1+\Psi\left(t_k\right)e^{b_0\left(s_{k+1}-t_k\right)}} e^{-\delta_k\left(s_{k+1}-t_k\right)} \leq \frac{\Psi\left(t_k\right)e^{b_0\left(s_{k+1}-t_k\right)}}{1+\Psi\left(t_k\right)e^{b_0\left(s_{k+1}-t_k\right)}},$$

which further deduces that

$$L \leq e^{-\left(b_0+\delta_k\right)\tau_{k+1}} e^{\left(b_0+\delta_k\right)\left(t_{k+1}-t_k\right)},$$

i.e,

$$t_{k+1} - t_k \geq \frac{\ln(L)}{b_0+\delta_k} + \tau_{k+1}.$$

It further generalizes to

$$s_{k+1} + \tau_{k+1} - s_k - \tau_k \geq \frac{\ln(L)}{b_0+\delta_k} + \tau_{k+1}.$$

Which then establish

$$s_{k+1}-s_k \geq \frac{\ln(L)}{b_0+\delta_k}+\tau_k. \tag{25}$$

Now, combining the fact $\mu T - t_k \geq \mu T - \frac{\mu T}{Q}$ along with the expression of $\delta_k == \frac{1}{\mu T - t_k}\ln\left(\frac{L\Psi(t_k)}{\varepsilon_1\left(1+\Psi(t_k)e^{b_0(\mu T-t_k)}\right)}\right)$ one can obtain

$$\delta_k \leq \frac{Q}{\mu T(Q-1)}\ln\left(\frac{L}{\varepsilon_1}\right). \tag{26}$$

Finally, from Eqs. (25) and (26) it can be concluded that

$$s_{k+1}-s_k \geq \frac{\ln(L)}{b_0+\frac{Q}{\mu T(Q-1)}\ln\left(\frac{L}{\varepsilon_1}\right)}+\tau_k, \forall k \in \mathbb{Z}_+.$$

Hence, the present ETM (20) is free from Zeno behavior. This completes the proof of the first part of the Theorem.

Part 2: Now let us come to the second part of the proof of Theorem 2. That is to prove the PTS of the system (1). For this, let us first divide the proof into two parts depending on $\Phi\left(V\left(y_0\right)\right) \equiv \Phi_0 \leq \varepsilon_0$ or $\Phi_0 > \varepsilon_0$. Further, let us denote $\Phi(V(y(t))) \equiv \Phi(y(t)) \equiv \Phi(t)$.

Case 1: $\Phi_0 \leq \varepsilon_0$. We further assert that, $\Phi(t) \leq \varepsilon_0$ for all $t \geq 0$. If this is not the case, then there exist some $t > 0$ for which $\Phi(t) > \varepsilon_0$ holds. Let us represent, $t^{\ominus} = \inf\left\{t \geq 0 : \Phi(t) > \varepsilon_0\right\}$. As $c_k \in (0,1)$ for every $k \in \mathbb{Z}_+$, $t^{\ominus}$ cannot be an impulsive instant. Thus,

$\Phi\left(t^{\ominus}\right) = \varepsilon_0$, $\Phi(t) \leq \varepsilon_0$ for all $t \in \left[0, t^{\ominus}\right]$ and $D^+\Phi(t)\mid_{t=t^{\ominus}} \geq 0$. Now from *condition (ii)* of Assumption 2, it can be obtained that

$$\begin{aligned}
D^+\Phi(t)\mid_{t=t^{\ominus}} &\leq -a_0\Phi^p\left(t^{\ominus}\right)\left\{1-\Phi\left(t^{\ominus}\right)\right\}^{2-p}+b_0\Phi\left(t^{\ominus}\right)\left\{1-\Phi\left(t^{\ominus}\right)\right\}\\
&= a_0\Phi^p\left(t^{\ominus}\right)\left(1-\Phi\left(t^{\ominus}\right)\right)\left\{-\left(1-\Phi\left(t^{\ominus}\right)\right)^{1-p}+\frac{b_0}{a_0}\Phi^{1-p}\left(t^{\ominus}\right)\right\}\\
&= a_0\Phi^p\left(t^{\ominus}\right)\left(1-\varepsilon_0\right)\left\{-\left(1-\varepsilon_0\right)^{1-p}+\frac{b_0}{a_0}\varepsilon_0^{1-p}\right\}\\
&= a_0\Phi^p\left(t^{\ominus}\right)\frac{1}{1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}\left\{-\left(\frac{1}{1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}\right)^{1-p}\right.\\
&\qquad\left.+\frac{b_0}{a_0}\left(\frac{\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}{1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}\right)^{1-p}\right\}\\
&= a_0\Phi^p\left(t^{\ominus}\right)\frac{1}{\left(1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}\right)^{2-p}}\{-1+q\}\\
&= -B\Phi^p\left(t^{\ominus}\right)\\
&< 0,
\end{aligned}$$

which is a contradiction. So, $\Phi(t) \leq \varepsilon_0$ for all $t \geq 0$. Thus, the region defined as $\Omega = \left\{y \in \mathbb{R}^n : \Phi(t) \leq \varepsilon_0\right\}$ is a stable region. In fact, the trajectories of the system (1) will be strictly decreasing as long as they are in the region $\Omega\backslash\{0\}$.

Further, for all $y \in \Omega$, we have from *condition (ii)* of Assumption 2 that

$$\begin{aligned}
D^+\Phi(t) &\leq -a_0\Phi^p(1-\Phi)^{2-p}+b_0\Phi(1-\Phi)\\
&= a_0\Phi^p(1-\Phi)\left\{-(1-\Phi)^{1-p}+\frac{b_0}{a_0}\Phi^{1-p}\right\}\\
&\leq a_0\Phi^p(1-\Phi)\left\{-\left(\frac{1}{1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}\right)^{1-p}+\frac{b_0}{a_0}\left(\frac{\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}{1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}}\right)^{1-p}\right\}\\
&\leq \frac{a_0}{\left(1+\left(\frac{qa_0}{b_0}\right)^{\frac{1}{1-p}}\right)^{1-p}}\Phi^p(1-\Phi)\{-1+q\}\\
&\leq -B\Phi^p.
\end{aligned} \tag{27}$$

Case 2: $\Phi_0 > \varepsilon_0$. Then, from *condition (ii)* of Assumption 2 it can be obtained that

$$D^+\Phi(t) \leq b_0\Phi(1-\Phi). \tag{28}$$

At this stage, our aim is to reach the region $\Omega_1 = \left\{y \in \mathbb{R}^n : \Phi(t) \leq \varepsilon_1\right\}$ within the time μT. Now, as B, p, T and ε_0 are known we can always choose $\mu_1 \in (0,1)$ such that $\varepsilon_1 = \left\{B(1-p)\mu_1 T\right\}^{\frac{1}{1-p}} \leq \varepsilon_0$ holds. Hence, $\Omega_1 \subseteq \Omega$. Also, we have

$t_0 = 0 < \frac{\mu T}{Q} < \mu T$. With these informations, let us consider and analyze the following sub cases.

Sub-case 1: $s_{k+1} < \frac{\mu T}{Q}$, $\forall k$. Then via *condition (ii)* one have $\tau_{k+1} < \frac{\mu T}{Q} - s_{k+1}$, i.e, $t_{k+1} < \frac{\mu T}{Q}$. Let k_0 be such that s_{k_0+1} is the last event instance and thus t_{k_0+1} be the last impulse instance. Then, for all $t \in \left[t_{k_0+1}, \mu T\right]$ one can use ETM (20) to find that

$$\Phi(t) < \frac{L\Psi\left(t_{k_0+1}\right)}{1+\Psi\left(t_{k_0+1}\right)e^{b_0\left(t-t_{k_0+1}\right)}}e^{-\delta_{k_0+1}\left(t-t_{k_0+1}\right)}. \tag{29}$$

Further, using the value of δ_{k_0+1} it can be obtained at $t = \mu T$

$$\begin{aligned}
&\Phi(\mu T)\\
&< \frac{L\Psi\left(t_{k_0+1}\right)}{1+\Psi\left(t_{k_0+1}\right)e^{b_0\left(\mu T-t_{k_0+1}\right)}}e^{-\frac{1}{\mu T-t_{k_0+1}}\ln\left(\frac{L\Psi\left(t_{k_0+1}\right)}{\varepsilon_1\left(1+\Psi\left(t_{k_0+1}\right)e^{b_0\left(\mu T-t_{k_0+1}\right)}\right)}\right)\left(\mu T-t_{k_0+1}\right)}\\
&\leq \varepsilon_1.
\end{aligned} \tag{30}$$

Sub-case 2: $\frac{\mu T}{Q} \leq s_{k_{00}+1} < \mu T$, for some $k_{00} \in \mathbb{Z}_+$. Then by *condition (ii)* we have $\tau_{k_{00}+1} < \mu T - s_{k_{00}+1}$, i.e, $t_{k_{00}+1} < \mu T$. Then, using Assumption 2 one have that, at $t = t_{k_{00}+1}$

$$\begin{aligned}
\Phi\left(t_{k_{00}+1}\right) &= \Phi\left(t^-_{k_{00}+1}+h_k\left(s_{k_{00}+1}\right)\right)\\
&\leq c_{k_{00}+1}\Phi\left(s_{k_{00}+1}\right)\\
&= c_{k_{00}+1}\frac{L\Psi\left(t_{k_{00}}\right)}{1+\Psi\left(t_{k_{00}}\right)e^{b_0\left(s_{k_{00}+1}-t_{k_{00}}\right)}}e^{-\delta_{k_{00}}\left(s_{k_{00}+1}-t_{k_{00}}\right)}\\
&\leq c_{k_{00}+1}Le^{-\delta_{k_{00}}\left(s_{k_{00}+1}-t_{k_{00}}\right)}\\
&= c_{k_{00}+1}Le^{\delta_{k_{00}}\left(\tau_{k_{00}+1}+t_{k_{00}}\right)}e^{-\delta_{k_{00}}t_{k_{00}+1}}\\
&\leq c_{k_{00}+1}Le^{\delta_{k_{00}}\left(\tau_{k_{00}+1}+t_{k_{00}}\right)}
\end{aligned}$$

Using the value of $c_{k_{00}+1}$, above equation becomes

$$\begin{aligned}
\Phi\left(t_{k_{00}+1}\right) &\leq \frac{\varepsilon_1}{Le^{\delta_{k_{00}}\left(\tau_{k_{00}+1}+t_{k_{00}}\right)}}Le^{\delta_{k_{00}}\left(\tau_{k_{00}+1}+t_{k_{00}}\right)}\\
&= \varepsilon_1.
\end{aligned} \tag{31}$$

Since $t_{k_{00}+1} < \mu T$ and the trajectories of the system (1) is strictly decreasing in the region $\Omega_1\backslash\{0\}$ one can deduce, from Eq. (31), that $\Phi(\mu T) \leq \varepsilon_1$. Hence from both the *Sub-cases 1 & 2* one can deduce that $\Phi(\mu T) \leq \varepsilon_1$.

Further, we have $\varepsilon_1 \leq \varepsilon_0$, and thus using Eq. (27) it can be obtained, for all $t \geq \mu T$, that

$$\begin{aligned}
\Phi^{1-p}(t) &\leq \Phi^{1-p}(\mu T) - B(1-p)\{t-\mu T\}\\
&\leq \varepsilon_1^{1-p} - B(1-p)\{t-\mu T\}.
\end{aligned} \tag{32}$$

And, at $t = T$, it is easily possible to acquire, from Eq. (32), that

$$\begin{aligned}\Phi^{1-p}(T) &\le \varepsilon_1^{1-p} - B(1-p)T\{1-\mu\}\\ &= \varepsilon_1^{1-p} - B(1-p)\mu_1 T.\end{aligned} \tag{33}$$

Further, using $\varepsilon_1 = \left\{B(1-p)\mu_1 T\right\}^{\frac{1}{1-p}}$ above equation reduces to

$$\Phi(T) \le 0, \tag{34}$$

which, along with the *condition (i)* of Assumption 1, makes sure $y(T) = 0$. But $y \equiv 0$ being an equilibrium point of the system (1) assures that $y(t) = 0$ for all $t \ge T$.

The proof of Theorem 2 is thus accomplished by virtue of Part 1 & 2. □

Remark 6. If we consider, $\Phi(V) = \frac{V}{1+V}$ then *condition (ii)* of Assumption 2 reduces to $D^+V \le -a_0 V^p + b_0 V$. This exact Lyapunov form has been used in the article [19] to achieve PTS of switched system under the event-triggered impulsive control approach. By virtue of this, it can be concluded that the Lyapunov-like form used in the present manuscript is a general one corresponding to [19]. Several such Lyapunov forms can also be deduced by considering different forms of κ^1 function Φ. Moreover, a more practical scenario, i.e, considering actuation delay into account, improves the efficacy of the proposed ETM (20). Also, none of the articles in the existing literature has accounted for actuation delay to obtain PTS corresponding to event-triggered impulsive control. Which further highlights the importance of the proposed study.

Remark 7. The ETM (4) is different from the ETM (20) as: ETM (4) does not have the memory of the previous impulse instant to obtain the upcoming event instant, it runs for all $t \ge 0$ and provides a sufficient condition to obtain $PPTS$ of system (1) corresponding to the Assumption 1. Further, the ETM (4) reaches a region $\Delta_1 = \left\{y \in \mathbb{R}^n : \Phi(t) = \frac{\varepsilon}{M}\right\}$ at the prescribed-time $t = T$. At this point, it is essential to concentrate on the fact that reaching the prescribed region $\{y \in \mathbb{R}^n : \Phi(t) = \varepsilon\}$ at the prescribed-time $t = T$, by the ETM (4), is not sufficient to achieve $PPTS$ of the system (1) as the presence of actuation delay may lead the trajectories to go beyond the desired prescribed-region $\Delta = \{y \in \mathbb{R}^n : \Phi(t) \le \varepsilon\}$. To get around this situation, a smaller region Δ_1 has been considered, and some relations of the delay with it have been derived in Theorem 1 to achieve $PPTS$ of the system (1) (please refer to Fig. 2 for a clear visualization of it). Whereas, ETM (20) uses the memory of the previous impulsive instant to obtain the next event instant, it runs only up to the time $t \le \mu T$ and provides a sufficient condition to obtain PTS of system (1) corresponding to the Assumption 2. Also, there exists a stable region $\left\{y \in \mathbb{R}^n : \Phi(t) \le \varepsilon_0\right\}$ in this case (please refer to Theorem 2), and the Lyapunov-like inequality is negative definite here. So, once the trajectories reach this region, they will reach the origin within some finite time. But, we need to do it (reaching of the trajectories to the origin) within some pre-specified time. Hence, a smaller region $\left\{y \in \mathbb{R}^n : \Phi(t) \le \varepsilon_1\right\}$ is considered here. And the ETM (20) is thus constructed in such a way that it reaches the region $\left\{y \in \mathbb{R}^n : \Phi(t) = \varepsilon_1\right\}$ with the pre-specified time μT. Once it reaches the region $\left\{y \in \mathbb{R}^n : \Phi(t) = \varepsilon_1\right\}$, the continuous part of the dynamics will take over and make the trajectories to reach to the origin within the prescribed-time T. Please refer to Fig. 3 for a pictorial representation of ETM (20). So, both the ETM serves different purposes and thus constructed differently as per the requirements.

4. Numerical simulations

The purpose of this section of the article is to give a numerical understanding and demonstrate the validity of the theoretical results suggested in Section 3.

Example 2. Consider the following one-dimensional impulsive control system:

$$\begin{cases} \dot{y}(t) = b\left\{y(t) + y^2(t)\,sgn(y(t))\right\}, & t \ne t_k\\ y(t_k) = y(t_k^-) + c\frac{|y(s_k)|}{1+(1-c)|y(s_k)|} - e^{b\tau_k}\frac{|y(s_k)|}{1+\left(1-e^{b\tau_k}\right)|y(s_k)|} & ;\\ \qquad \times sgn(y(s_k)), & k \in \mathbb{Z}_+ \end{cases} \tag{35}$$

where b, c are positive real numbers with $0 < c < 1$ and $\{s_k\}_{k\in\mathbb{Z}_+}$ is the event-instances determined from the ETM (4). Further, $\{t_k\}_{k\in\mathbb{Z}_+}$ denotes the impulse instances with $t_k = s_k + \tau_k$ and $\tau_k (\ge 0)$ denotes the actuation delay at the kth instant. Moreover, to make sure the denominator, $1 + \left(1 - e^{b\tau_k}\right) \mid y(s_k) \mid$, is always greater than zero, we consider $\tau_k < \frac{1}{b}\ln\left(\frac{1}{\Phi(s_k)}\right) = \frac{1}{b}\ln\left(\frac{1}{Me^{-\frac{s_k}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}}\right) \le \frac{1}{b}\ln\left(\frac{1}{Me^{-\frac{s_1}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}}\right)$. Now, let us consider the Lyapunov function as $V(y) =\mid y \mid$ and the κ^1 Function $\Phi = \frac{V}{1+V} \equiv \frac{|y|}{1+|y|}$. Then the system (35) satisfies condition *(i)* of Theorem 1 with $\varphi_1(y) = \varphi_2(y) =\mid y \mid$ and $h_k(y(s_k)) = c\frac{|y(s_k)|}{1+(1-c)|y(s_k)|} - e^{b\tau_k}\frac{|y(s_k)|}{1+\left(1-e^{b\tau_k}\right)|y(s_k)|}sgn(y(s_k))$. Further, for numerical simulation, let us consider $b = 1$. Then, one can obtain from Fig. 4 that the trajectories of the system (35) without impulsive control, corresponding to different non-zero initial conditions, are unstable. Now, let us examine the system (35) and perform numerical simulations with $M = 2$, $T = 1$, $c = 0.5$, $\varepsilon = 0.1$ and $y_0 = 2$. Then, one can obtain, $s_1 = 0.2353$ and thus $\tau_k < 0.24$ (from the above inequality on τ_k). Further, $\frac{\ln\left(\frac{1}{c}\right)}{A} \approx 0.187$ and $\frac{1}{b}\ln(M) \approx 0.69$. So, $\tau_1 = 0.02$ satisfies $\tau_k < 0.24$ and the condition *(ii)* of Theorem 1. Proceeding similarly one can easily find out that, $\tau = \tau_1 = \tau_2 = \tau_3 = \cdots = 0.02$ satisfies the conditions *(ii) & (iii)* of Theorem 1 along with the restriction $\tau_k < 0.24$. Thus, system (35) satisfies all the conditions of Theorem 1. One can now find out, from Fig. 5 that, the trajectory of the system (35) reaches the prescribed region $\Delta = \{y \in \mathbb{R}^n : \Phi(t) \le \varepsilon\}$ inside the prescribed time $T = 1$ and remains there afterwards. Also, it can also be observed that, $s_{k+1} - s_k \ge \frac{b\tau_k + \ln\left(\frac{1}{c}\right)}{A+b} \simeq 0.152$ holds. Hence, Fig. 5 validates the claims of Theorem 1 numerically.

Remark 8. In this remark we will show the necessity of the threshold values of the delay proposed in Theorem 1 corresponding to the system (35). Let us consider $y_0 = 1$ and keep all the parameters same as mentioned in Example 2. Then, one can easily have, $s_1 = 0.2956$ and thus $\tau_k < 0.39$ (from the inequality $\tau_k < \frac{1}{b}\ln\left(\frac{1}{\Phi(s_k)}\right) = \frac{1}{b}\ln\left(\frac{1}{Me^{-\frac{s_k}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}}\right) \le \frac{1}{b}\ln\left(\frac{1}{Me^{-\frac{s_1}{T}\ln\left(\frac{M^2}{\varepsilon}\right)}}\right)$). Further, it can be easily observed that, $\tau = \tau_1 = \tau_2 = \tau_3 = \cdots = 0.02$ satisfies $\tau_k < 0.39$ and the conditions *(ii) & (iii)* of Theorem 1. Now, one can easily observe, from Fig. 6(a) that, the trajectory of the system (35) reaches the prescribed region $\Delta = \{y \in \mathbb{R}^n : \Phi(t) \le \varepsilon = 0.1\}$ within the prescribed time $T = 1$ and remains there thereafter. Now, let us choose $\tau_1 = 0.2$ which goes against the threshold value of τ_k (condition *(ii)* of Theorem 1). Then corresponding to the same initial value ($y_0 = 1$) one can have, from Fig. 6(b) that, $\Phi = \frac{V}{1+V} \equiv \frac{|y|}{1+|y|}$ is not only unable to reach and stay in the prescribed region $\Delta = \{y \in \mathbb{R}^n : \Phi(t) \le \varepsilon = 0.1\}$ but also unable to reach below the threshold Me^{-At} at the impulsive point $t_1 = s_1 + \tau_1$. Hence, the scenario of triggering the ETM (4) is not here (corresponding to the considered set of parameters and initial value) after the first triggering.

Example 3. The primary motive of this example is to justify the truthfulness of the proposed results of Section 3.2, numerically. In this context, let us examine the following one-dimensional impulsive

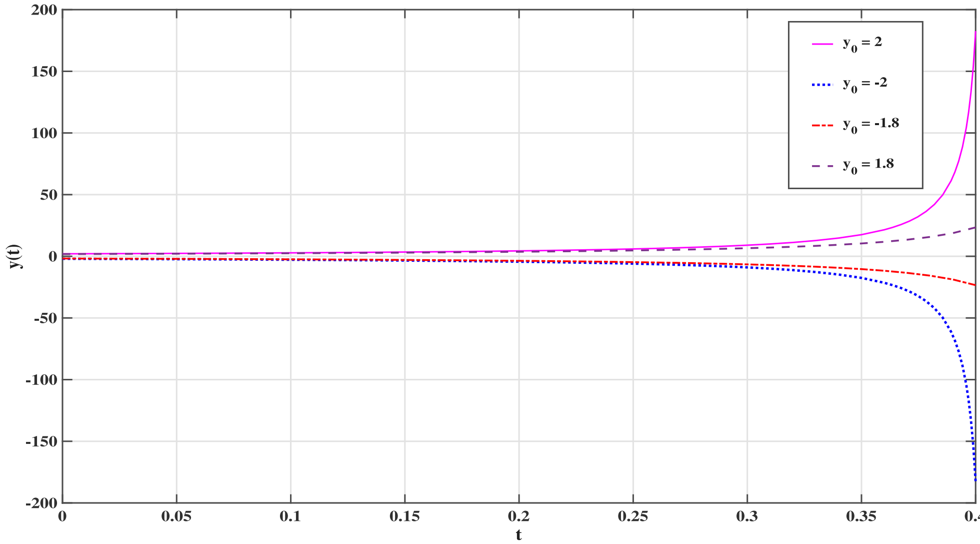


Fig. 4. Trajectories of the system (35) without impulsive control.

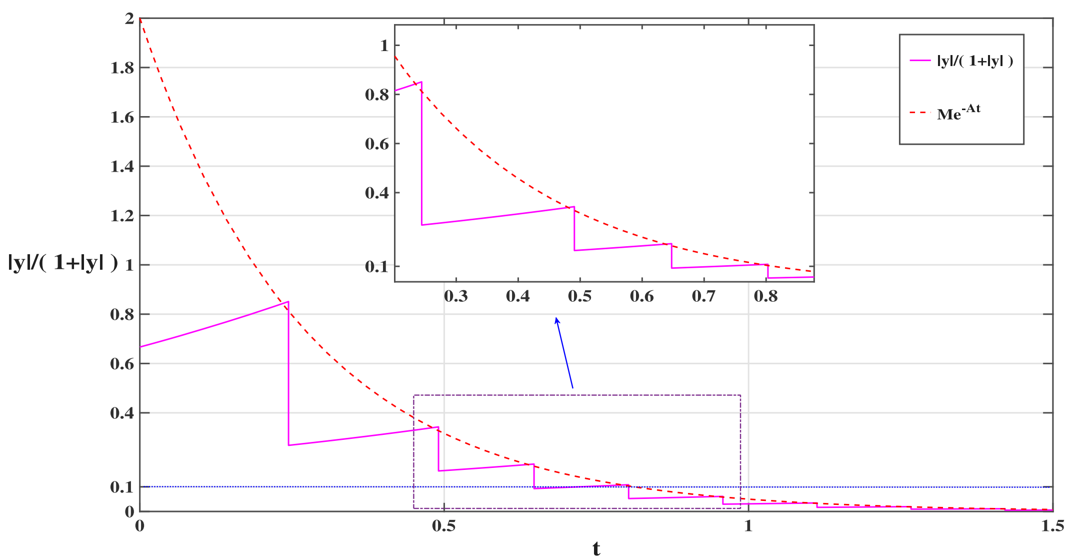
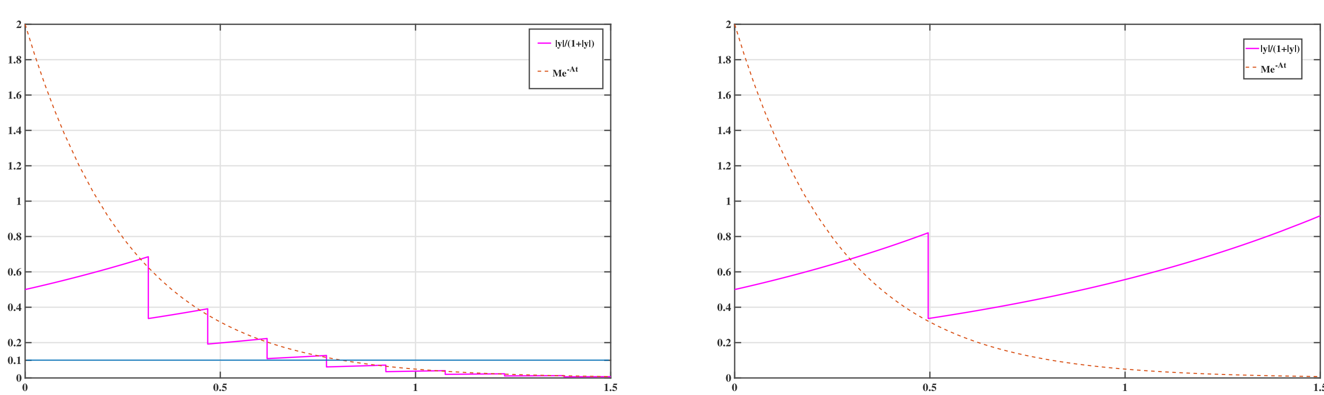


Fig. 5. Evaluation of $\frac{|y|}{1+|y|}$ corresponding to the system (35), with initial condition $y_0 = 2$, under the ETM (4).



(a) Simulation of the system (35) under the ETM (4) with delay 0.02 and $y_0 = 1$

(b) Evolution of the system (35) under the ETM (4) with delay 0.2 and $y_0 = 1$

Fig. 6. Simulations of Remark 8.

control system:

$$\begin{cases} \dot{y}(t) = -a_0 \mid y(t) \mid^{\frac{1}{2}} sgn(y(t)) + b_0 y(t), & t \neq t_k \\ y(t_k) = y(t_k^-) + c_k \frac{|y(s_k)|}{1+(1-c_k)|y(s_k)|} & \\ \qquad - \frac{1}{b_0} \left\{ a_0 + (b_0 \mid y(s_k) \mid - a_0) e^{\frac{b_0 \tau_k}{2}} \right\} sgn(y(s_k)), & k \in \mathbb{Z}_+ \end{cases}; \tag{36}$$

where a_0, b_0 and c_k $(k \in \mathbb{Z}_+)$ are positive real numbers with $0 < c_k < 1$. Further, $\{s_k\}_{k \in \mathbb{Z}_+}$ is the event-instances determined by the ETM (20) and, $\{t_k\}_{k \in \mathbb{Z}_+}$ denotes the impulse instances with $t_k = s_k + \tau_k$. Where, $\tau_k (\geq 0)$ denotes the actuation delay at the kth instant. For numerical verification, let us consider $a_0 = b_0 = 1$. From Fig. 7, one can observe that the trajectories of the system (36), without impulsive control, are unstable. Now, let us analyze the system (36) in the presence of the proposed impulsive controller. For this, consider the prescribed-time $T = 1$, $L = 2$ and $q = \frac{1}{2}$. Then, from Theorem 2, one can calculate $B = \frac{4}{5\sqrt{5}}$ and $\varepsilon_0 = \frac{1}{5}$. Moreover, consider $V(y) = \mid y \mid$ as the Lyapunov function, and $\Phi = \frac{V}{1+V} \equiv \frac{|y|}{1+|y|}$ as the κ^1 Function. Then, $\Psi = \frac{\Phi}{1-\Phi} = V$. Then, the condition *(i)* of Theorem 2 is fulfilled with $p = \frac{1}{2}$, $\varphi_1(y) = \varphi_2(y) = \mid y \mid$ and $h_k(y(s_k)) = c_k \frac{|y(s_k)|}{1+(1-c_k)|y(s_k)|} - \frac{1}{b_0} \left\{ a_0 + (b_0 \mid y(s_k) \mid - a_0) e^{\frac{b\tau_k}{2}} \right\} sgn(y(s_k))$. Now, if we choose $\mu_1 = \frac{1}{2}$, then, $\varepsilon_1 = \frac{1}{125} \leq \varepsilon_0 = \frac{1}{5}$ holds. Further, without loss of generality, let us consider $Q = 1.25$, $y_0 = 5$, $c_{k+1} = \frac{1}{2} \in (0,1)$ as long as $0 < s_{k+1} < \frac{\mu T}{Q}$ and $c_{k+1} = \frac{\varepsilon_1}{L} = \frac{1}{250} \leq \frac{\varepsilon_1}{L e^{\delta_k(\tau_{k+1}+t_k)}}$ for $\frac{\mu T}{Q} \leq s_{k+1} < \mu T$. Now, using the notion $t_0 = 0$ and $\delta_0 = \frac{1}{\mu T - t_0} \ln \left(\frac{L\Psi(t_0)}{\varepsilon_1 \left(1+\Psi(t_0) e^{b_0(\mu T - t_0)}\right)} \right) = 9.8139$, one can obtain $s_1 = 0.0645 < 0.4 = \frac{\mu T}{Q}$. And thus, $\tau_1 = 0.01$ can be assumed such that condition *(ii)* of Theorem 2 is satisfied. Similarly, one can obtain from Fig. 8 that, $s_2 = 0.1387, s_3 = 0.2149, s_4 = 0.2956, s_5 = 0.3840, s_6 = 0.4753$. Where, $\tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = 0.01$ is chosen such that they fulfill conditions *(ii)* and *(iii)* of Theorem 2. Also, one can obtain, $\delta_1 = 10.4741, \delta_2 = 11.0288, \delta_3 = 11.6085, \delta_4 = 12.5275, \delta_5 = 15.3520, \delta_6 = 0.5267$. Moreover, it can be observed from Fig. 8 that, the trajectory of the system (36) reaches the prescribed region $\Omega_1 = \left\{ y \in \mathbb{R} : \Phi(t) \leq \varepsilon_1 = \frac{1}{125} \right\}$ i.e, $\Omega_1 = \left\{ y \in \mathbb{R} : \mid y(t) \mid \leq \frac{1}{124} \approx 0.0081 \right\}$ within the time $\mu T = \frac{1}{2}$ and reaches the origin within the prescribed-time $T = 1$. This validates the truthfulness of the results proposed in Theorem 2. Moreover, there exist only one event-instance, $s_6 = 0.4753$, such that $\frac{\mu T}{Q} = 0.4 \leq s_6 < \mu T = 0.5$ holds. This validates the claim of Theorem 2. Further, $s_2 - s_1 = 0.0742 \geq \frac{\ln(L)}{b_0+\delta_1} + \tau_1 \approx 0.0704$, $s_3 - s_2 = 0.0762 \geq \frac{\ln(L)}{b_0+\delta_2} + \tau_2 \approx 0.0676$, $s_4 - s_3 = 0.0807 \geq \frac{\ln(L)}{b_0+\delta_3} + \tau_3 \approx 0.065$, $s_5 - s_4 = 0.0884 \geq \frac{\ln(L)}{b_0+\delta_4} + \tau_4 \approx 0.0612$, $s_6 - s_5 = 0.0913 \geq \frac{\ln(L)}{b_0+\delta_5} + \tau_5 \approx 0.0524$. Thus, the efficacy of Theorem 2 is further illustrated 2 numerically via Fig. 8.

Remark 9. It should be noted that the considered initial conditions in Fig. 7, of Example 3, are from outside the stable region, $\Omega = \left\{ y \in \mathbb{R} : \Phi(t) \leq \varepsilon_0 = \frac{1}{5} \right\}$. Thus, the corresponding Lyapunov function may or may not be negative definite. In our case, it is positive definite. Therefore, the trajectories are unstable (can be observed from Fig. 7). So, the impulsive controller has two jobs to do: first, it has to make sure that the trajectories become stable. The second is to ensure that the trajectories not only become stable but also reaches the equilibrium point within the provided prescribed-time. In addition, if we had chosen the initial conditions from the stable region $\Omega = \left\{ y \in \mathbb{R} : \Phi(t) \leq \varepsilon_0 = \frac{1}{5} \right\}$, the impulsive controller then would have only one task that is to reduce the settling-time according to the requirement of the user.

Remark 10. Most of the existing articles (for example, [26]) use the notion like $y(t_k) = c_k y(t_k^-), 0 < c_k < 1$, as the impulsive controller. However, the design of the impulsive controller used in Examples 2 and 3 is a bit complicated. The forms of the impulsive controller used to be based on the considered Lyapunov inequality. Since the Lyapunov-like sufficient conditions used in the present manuscript are different from the existing Lyapunov-inequality, the impulsive controllers are different. Further, the presence of actuation delays makes the designing of the impulsive controller more challenging, as one has to make sure that $y(t_k)$ not only depends on $y(t_k^-)$ but also on the value of the state $y(t)$ at the event- instance, $y(s_k)$. Consequently, the proposed impulsive controllers are designed by considering all the aforementioned points and are capable enough to provide the desired results.

5. Conclusion

The present manuscript studied the problem of practical prescribed and prescribed time stabilizability of nonlinear systems by means of event-triggered impulsive control. We have also considered the effects of actuation delays in the impulsive controller. Two different ETM and corresponding Lyapunov-like sufficient conditions are proposed in this regard to obtain practical prescribed and prescribed time stability of controlled impulsive systems. Moreover, Lyapunov functions with nonlinear rates (can be observed from Assumptions 1 and 2) are considered here and thus can be applied to a broad range of nonlinear systems. Exclusion of Zeno behavior is very important, while designing ETM, for practical implementations of the theoretical results. Therefore, the exclusion of Zeno-behavior has also been taken care of in the manuscript. A promising future work will be to obtain such results for time-delayed systems, following the direction of [27]. Another possible direction of future work is to consider external disturbances into account in the line of work of [28]. Further, our future studies will also consider applying the derived results in practical systems.

CRediT authorship contribution statement

Arnab Mapui: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Santwana Mukhopadhyay:** Writing – review & editing, Visualization, Supervision, Methodology, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

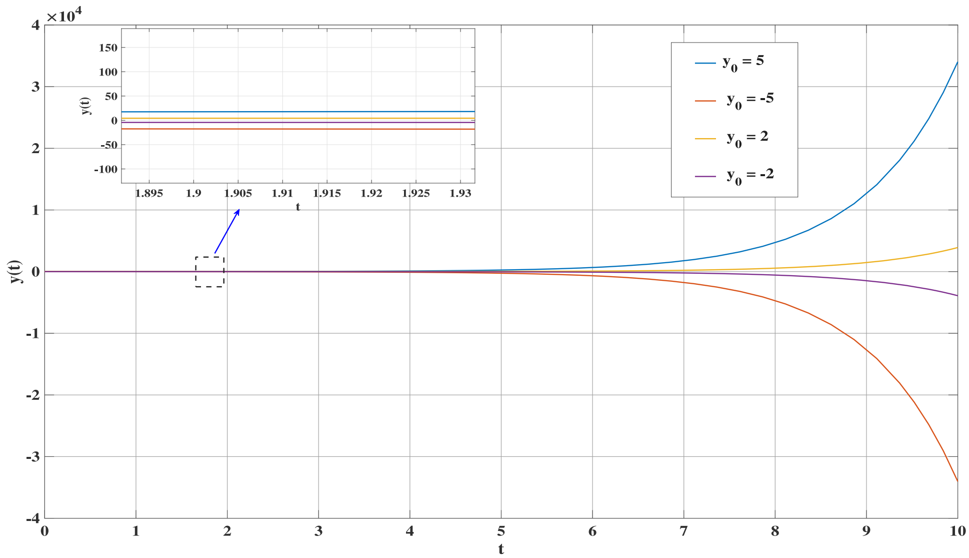

Fig. 7. Trajectories of the system (36) without impulsive control.

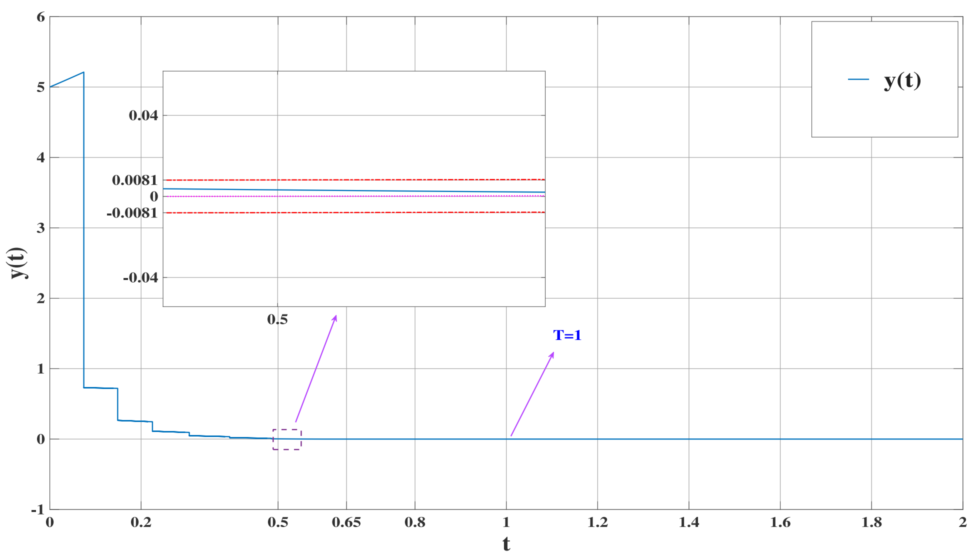

Fig. 8. Evaluation of the state trajectory $y(t)$ of the system (36), with initial condition $y_0 = 5$, under the ETM (20).

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