CERTIFICATE

It is certified that the work contained in the thesis titled **"Applications of Wavelets and Operational Matrices for Partial Differential Equations and Singular Integral Equations "** by **Mr. Vijay Kumar Patel** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.

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DECLARATION BY THE CANDIDATE

I,**Vijay Kumar Patel**, certify that the work embodied in this thesis is my own bona fide work and carried out by me under the supervision of **Dr. Vineet Kumar Singh** from **July 2013** to **November 2017** at the **Department of Mathematical Sciences**, Indian Institute of Technology (Banaras Hindu University), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not include them in this thesis and have not cited as my own work.

Date: ....
Place: Varanasi

(Vijay Kumar Patel)

CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my knowledge.

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Acknowledgement

With the grace of Almighty and blessings of my parents and pedagogues, I have reached to the completion of this thesis. It is a matter of great privilege and immense pleasure for me to express my deep regards to my respected supervisor, **Dr. Vineet Kumar Singh**, Associate Professor, Indian Institute of Technology (Banaras Hindu University), Varanasi. His wide experience, unrivalled knowledge of numerical methods, learned guidance, stimulating discussion and moral support helped me immensely in completing this work. Particularly, in the initial stages, he has sustained me with paternal care and thoughtful advised as like a true friend. His careful guidance shaped my understanding on research life as well as my chosen field. He lets me know the importance of understanding the physical meaning of the mathematical equations.

It is an honour for me to thank **Smt. Priya Singh (Ma'am)** (w/o supervisor) for their love, blessing, encouragement and morel support.

I would like to express my deep sense of gratitude to **Dr. E. B. Postnikov**, Professor, Theoretical Physics Department, Kursk State University, Kursk, Russia, and **Dr. Emran Tohidi**, Professor, Department of Mathematics, Kosar University of Bojnord, Bojnord, Iran, for his valuable help and discussions at the initial stages of my research work.

My sincere thanks are due to Late **Prof. O. P. Singh** (Former Head) and **Prof. L. P. Singh**, Head, Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University) Varanasi, India, for providing necessary facilities during my research period.

I express my deep sense of gratitude to my external RPEC member Prof. D. Singh, Department of Electrical Engineering, Indian Institute of Technology (BHU) for their valuable suggestions, guidance, appreciation and encouragement.

I owe a lot to all my teachers who taught me Mathematics. Also, my sincere thanks to all faculty members, Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University) Varanasi, India, for there valuable suggestions and help me in the sense of directly or indirectly. Also, special thanks to **Dr. R. K. Pandey** for recommendation and moral support.

I have been very happy and privileged to be associated with **Mr. Somveer Singh**, **Mr. Rahul Kumar Maurya** and **Ms. Vineeta** of my research group as it was a wonderful experience of working together and I hope that our collaboration continues.

I would like to thank my close friends, **Ms. Payal Gupta**, **Mr. Shiva Sharma**, **Ms. Deepika Pathak**, **Mr. Kamlesh Kumar Pandey**, **Mr. Shivam Shreevastava**, **Ms. Priti**, **Dr. Reshma Singh**, **Ms. Suverna Trivedi**, **Mr. Anup Tiwari**, **Mr. Sandeep Mishra**, **Mr. Sandeep Tiwari**, **Mr. Dikshant Singh**, **Mr. Vinod Kumar Singh**, **Mr. Gopal Gupta**, **Dr. Harendra Singh**, **Mr. Ishan Gupta**, **Dr. Arvind Kumar**, **Mr. Anup Singh**, **Mr. Debashish**, **Mr. Jitendra Pal Chaudhary** and many more (directly or indirectly) for their continued encouragement, moral support, appreciation, criticism and making me to stay at BHU.

Further, I would like to extend my thanks to my seniors **Dr. Naveen Jha**, **Dr. Chandra shekher Singh**, **Dr. Shailendra Tiwar**i, **Dr. Ghanshyam Prasad** and **Dr. Pappu Kumar** for help me a lot like little brother.

I would like to thank all the Ph.D. colleague of the department.

I gratefully acknowledge Ministry of Human Resource and Development, India for providing the scholarship in form of Junior Research Fellowship and Senior Research Fellowship.

I gratefully acknowledge Indian Institute Technology (BHU) Varanasi for financial support to present a paper in **Euler International Mathematical Institute, Saint Petersburg, Russia**.

I gratefully acknowledge DST SERB, India for financial support to present a paper in **Chuo University, Tokyo, Japan**.

I gratefully acknowledge CSIR, India for financial support to present a paper in **Euler International Mathematical Institute, Saint Petersburg, Russia**.

I gratefully acknowledge Indian Institute Technology (BHU) Varanasi and Leibniz university Honnover, Germany for financial support to present a paper in **Leibniz university Honnover, Germany**.

This acknowledgement would be incomplete if the name of great visionary "**Pt. Madan Mohan Malaviya**" is not mentioned, who made this divine center of knowledge. Deepest regards to him.

I would like to thanks my **Bhaiya** (Mr. Rajiv Kumar Patel) and **Bhabhi** (Mrs. Rekha Patel). They are supported me at all the condition and love too much without any expectation.

Finally, I owe **my family (specially Maa)** all my thanks for their deepest love, endless patience and continued support in the pursuit of my aspirations through all of the years of my study and I bow down my head **Kashi Vishwanath, Sankat Mochan and Maa Durga** for showers of blessings throughout my research work to complete it successfully.

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Place: Varanasi **(Vijay Kumar Patel)**

Contents

List of Figures

List of Tables

Preface

The main focus of this thesis is to analyze domain decomposition method for partial differential equations, fractional partial differential equations and partial integro-differential equations based on operational matrices.

The objective of this thesis consist by following five points:

- One purpose of this thesis is to provide a mathematical treatment of the topic of wavelets operational matrices that can be understood by someone who has an understanding the topic from engineering and science point of view.
- A second objective of this thesis is to introduce several new algorithms that efficiently and accurately as possible as solve the two variables problem over certain finite dimensional approximations.
- A third objective of this thesis is to provide convergence analysis of each proposed problem via approximated basis in l_2- norm.
- A fourth objective of this thesis is to provide error analysis of each proposed problems in l_2- norm.
- Finally, we will see that efficiency and accuracy of proposed operational matrix method via examples and error graphs (the graph is the absolute difference of analytical solution of proposed problem and numerical solution via proposed method).

The thesis consists of five chapters. In Chapter 1, we introduced history of wavelet and why we used wavelet. We introduced Legendre polynomials, Chebyshev polynomials, Bernoulli polynomials, Legendre wavelet, Chebyshev wavelets and Bernoulli wavelets. We have introduced fractional calculus also for solving fractional partial differential equation and review of fractional partial differential equations. Wavelet function generates significant interest from both theoretical and applied research given in the last sixteen years. In the present work, the family of wavelets will be considered due to their useful properties and the contribution of wavelets by Legendre, Chebyshev and Bernoulli wavelet based solution of partial differential equations, fractional partial differential equations and integral equations gained momentum in attractive way. Advantages of Wavelet methods over operational matrix method have led to tremendous application in science and engineering. The thesis describes a number of algorithms that can be used to quickly evaluate solution of partial differential equations (PIDEs), fractional partial differential equations (FPDEs), partial integro-differential equations (PIDEs) and fractional partial integro-differential equations (FPIDEs) using wavelets operational matrices and collocation method.

In chapter 2, a numerical solution of fractional partial differential equations (FPDEs) for electromagnetic waves in dielectric media will be discussed. For the solution of FPDEs, we developed a numerical collocation method using an algorithm based on two-dimensional shifted Legendre polynomials approximation, which is proposed for electromagnetic waves in dielectric media. By implementing the partial *Riemann − Liouville* fractional derivative operators, two-dimensional shifted Legendre polynomials approximation and its operational matrix along with collocation method are used to convert FPDEs first into weakly singular fractional partial integro-differential equations and then converted weakly singular fractional partial integro-differential equations into system of algebraic equation. Some results concerning the convergence analysis and error analysis are obtained. Illustrative examples are included to demonstrate the validity and applicability of the technique. The entire chapter in the form of a paper has been published in the Mathematical Methods in the Applied Sciences with vol 40, page $3698 - 3717$, (2017).

In chapter 3, we deal with a numerical wavelet collocation method (NWCM) using a technique based on two-dimensional wavelets (TDWs) approximation proposed for the fractional partial differential equations (FPDEs) for electromagnetic waves in dielectric media (EWDM). By implementing the *Riemann − Liouville* fractional derivative, TDWs approximation and its operational matrix along with collocation method are utilized to reduce FPDEs firstly into weakly singular fractional partial integro-differential equations (FPIDEs) and then reduced weakly singular FPIDEs into system of algebraic equation. Using Legendre wavelet approximation (LWA) and Chebyshev wavelet approximation (CWA), we investigated the convergence analysis and error analysis of the proposed problem. Finally, some examples are included for demonstrating the efficiency of the proposed method via LWA and CWA respectively. The entire chapter in the form of a paper has been published in the Journal of Computational and Applied Mathematics with vol 317, page $307 - 330$, (2017).

In chapter 4, an approximation method based on wavelets operational matrices for solving a class of linear and nonlinear weakly singular Volterra partial integro-differential equation (PIDE) are discussed. By implementing operational matrix method and also the associated two dimensional Legendre wavelet approximation (LWA) and Chebyshev wavelet approximation (CWA) as well, the considered PIDE will be reduced to the corresponding system of linear and nonlinear algebraic system of equations. Linear systems are solved by collocation method and nonlinear system solved by well-known Newton-Raphson method. Also, the convergence analysis of the suggested numerical scheme and error estimation are provided under several mild conditions. A variety of numerical examples (like: exponential functions, trigonometric functions,...) are considered to show the efficiency and accuracy of the proposed numerical approach using piecewise basis of both the wavelet.

In chapter 5, a new approximation scheme based on wavelets operational matrices have been discussed for the solution of proposed initial value complex partial differential equations(CPDE) which is derived from continuous wavelet transform. We developed an algorithm using transformation to obtain numerical solution of CPDE with a numerical wavelet collocation method (NWCM) based on two-dimensional Legendre wavelet (TDLW) and two-dimensional Bernoulli wavelet (TDBW) operational matrices. We converted CPDE into system of partial differential equations(PDEs) and this system of PDEs are converted into couple PDEs in terms of real and imaginary component. Finally, we obtained system of partial integro-differential equations(PIDEs) arising with nonlocal boundary conditions due to transformation. After implementation of wavelets scheme on PIDEs, PIDEs reduced into system of algebraic equations. In sequence of this, we also investigated the convergence analysis of TDLW, TDBW and error analysis respectively. Finally, Illustrative examples are included to demonstrate the validity and applicability of the presented wavelet scheme for proposed problem.

This thesis tries to be self-contained, however if you find a part which is not perfectly clear, all answers are surely included in one of the books/papers listed in the Bibliography.