CHAPTER 4
TOLERANCE BASED MODERATOR GUIDED HETEROGENEOUS GROUP DECISION MAKING INVOLVING EXPERTS AND END-USERS

This chapter focuses on two issues of group decision-making same as discussed in the first chapter. First, the multiple rounds of feedback recommendations in the consensus-reaching process (CRP) make GDM inefficient. Second, there is no involvement of heterogeneous decision-makers (DMs), possibly end-users, as stakeholders apart from the experts. To address the first issue, a novel threshold-based feedback mechanism is introduced to improve the efficiency of the CRP, which in turn helps experts reach a consensus in, at most, one round of feedback. To address the second issue, invite end-users to participate in decision-making where their majority group opinion is generated. After dealing with the opinions of experts and end-users exclusively, the concern is to obtain a global solution that is accepted by all the DMs. In this regard, a moderator with a tolerance degree is designed who promises to obtain the global decision considering the consensual opinion of experts and the cumulative opinion of end-users based on its tolerance value. The effectiveness of the proposed method is demonstrated through a case of healthcare service selection. Further, various experiments are conducted to show how the proposed work outperforms the existing works.

4.1 Background

Traditionally, the experts with the structured knowledge are considered to be the sole DMs who come together to derive the decision-making process. However, with the rapid advancement of information and communication technology, the social media platforms like Twitter, Facebook and WeChat, helped individuals become more informed and aware. They promptly share their opinions and experiences through these platforms [94], [95] and are willing to participate in the decision-making process. Consequently, these individuals being novice ones are the potential participants in the decision-making process.

With the explosion of information technologies, new challenges have been brought in decision-making. The decision-making events like service selection, in general, are important for society or a specific community of people [96]. Therefore, instead of only involving the representative stakeholders in decision-making, one should also involve the potentially affected individuals too. The important reasons are:

- Related information provided by the providers may not be sufficient to make the decision.
- Services could have been designed to address the need of a large group or to fulfill the specific need of a small group of customers.
- Rapid growth in the market is encouraging service providers to create different types of services, resulting in increasing the difficulty in service selection.

For example, in the healthcare sector, various developments like Telemedicine, AI-enabled medical devices, and blockchain-based electronic health records have taken place. These developments made it challenging to find the best healthcare service that satisfies the requirements of all stakeholders. Though system experts can be invited to select the best service alternative, they may only be able to incorporate some possible requirements of the end service user. Therefore, to better interconnect and perceive the service's usefulness, service users and the leading experts must evaluate the alternatives. The advantage of inviting the end service users is the actual practical standpoints. As far as we know, the GDM problems that simultaneously contain the experts and the users (the novice experts) called heterogeneous DMs in this thesis have not been thoroughly considered in the literature. In this direction, to fill the aforementioned research gap, the heterogeneity discussed in this study is about the types of DMs having structured and unstructured knowledge.

In general, it is challenging to reach an agreement with the DMs owing to different interests, motivations and backgrounds. Thus, CRP is essential to obtain the solution that is to be agreed upon by all the experts. However, it is assumed that for some application the resources are limited for building consensus. The GDM being a time-consuming process, one such resource that we consider is the number of rounds required to reach consensus [52]. It is observed that there are situations when decisions need to be made in a short time, especially in the emergency events [67]. These events occur suddenly, and decisions must be made in a limited time. Thus, an important issue is designing an effective and efficient feedback process for building CRP. To fill this research gap, this thesis considers the heterogeneity in the context of types of DMs, i.e., the experts and the end-users, while minimizing the number of rounds to reach a consensus.

In this direction, one of the objectives of this chapter is to propose a feedback mechanism that achieves consensus at once. However, in some scenarios, involving all the experts and end-users in the interactive process would be inconvenient, where they modify their preferences for several rounds. Also, the users may not be interested in getting feedback advice. All they can do is to provide their initial preferences sincerely. Thus, an alternative approach is necessary to obtain the optimal decision solution in the presence of both the experts and the end-users. For this, we here employ the concept of tolerance behavior of the moderator to obtain the final decision. This moderator's behavior is characterized by defining the tolerance degree of the moderator. The tolerance-based moderator controls the actual decision deviation from the expert's consensual solution in the presence of the user's opinion while generating the final decision. Therefore, this approach is conducive to modifying the expert's consensual opinion in the final decision generation.

4.2 Proposed Model

Though end-users can be invited to GDM, engaging them (as novice experts) in the CRP to modify their preferences would be difficult. Furthermore, giving up end-user’s opinions may lead to the loss of information. Focusing on the role of end-users in decision-making, this chapter makes the moderator responsible for obtaining the solution from the expert’s consensus opinion and end-users' cumulative opinion. The moderator decides the influence of the end-user’s opinion on the final solution.

The proposed GDM model consists of four phases. The first phase is the construction of an expert consensus solution. In this phase, the moderator decides on a consensus threshold for experts and gives feedback to experts to reach a consensus. We call the solution obtained in this phase 'Expert Consensus Solution (ExCS).' Here, we propose a novel CRP for the first phase in which, at most, one-time feedback is given to the experts to reach a consensus, whereas the second phase is the construction of end-users' cumulative solutions. In this phase, the moderator creates a cumulative solution based on the preferences given by the end-users in the second phase that represents the majority group opinion while preserving the extreme opinions of end-users. In this phase, we call the obtained solution' End-users Cumulative Solution (EuCS)'. The third phase is the construction of the final solution. In this phase, the moderator has to create a Global Consensus Solution (GCS) based on obtained solutions from phases first and second, creating a solution that considers the opinions of experts and end-users. We propose a novel method in which the moderator makes a GCS based on its tolerance degree. The tolerance degree associated with the moderator decides the degree to which the moderator can tolerate the deviation from experts' opinions while incorporating the users' opinions. It controls the impact of users' opinions in generating the final decision GCS. The fourth phase is the Selection Phase. The resolution framework of the proposed model based on experts and end-users' opinions and a tolerance-based moderator is presented in Fig. 4.1. The step of the proposed framework is summarized in Algorithm 1. The main difference between the proposed CRP (given in Fig. 4.1) and the CRP in existing works (shown in Fig. 2.1) is the tolerance-based moderator that considers end-users' opinion to generate the final solution GCS and the model defined CRP which minimizes the round to reach consensus.

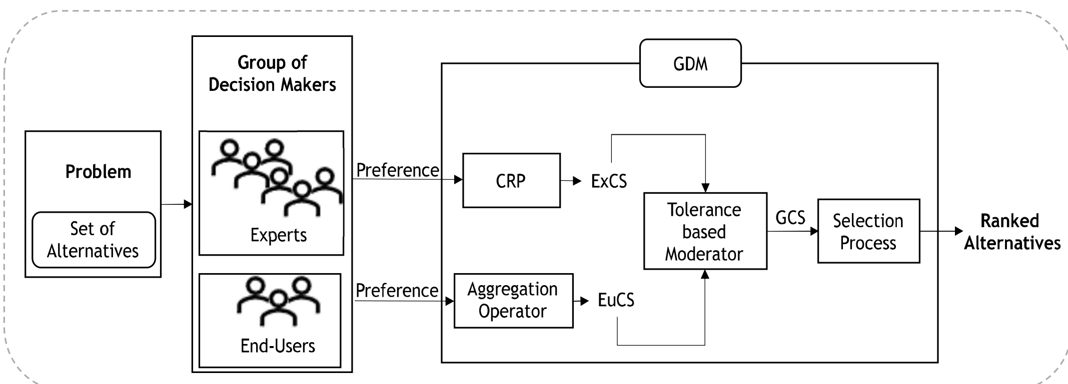

Fig. 4.1: Framework of the Proposed Tolerance Based Model

In the proposed GDM model, there is a problem to solve for which the opinions of experts and end-users are required. For the specified problem, there is a set of feasible alternatives, $X = \{x_1, x_2, \ldots. x_n\}$ where $n \geq 2$, and the group of two or more experts, $E = \{e_1, e_2, \ldots. e_m\}$ where $m \geq 2$. Experts are characterized by their own experience, ideas, backgrounds, and knowledge. They express their opinion about the provided set of alternatives. For incorporating the opinion of end-users as input to decision making, the moderator requests them to provide their opinion for the given set of alternatives. Their opinions would either be collected by requesting them directly or by collecting from the publicly available information on the Internet. Let l be the number of end-users represented as $U = \{u_1, u_2, \dots . u_l\}$, where $l \geq 2$. Each expert and end-user provide a preference matrix V and T, respectively, of size $n \times n$ in which each entry v_{ij} represents preference given to the alternative i over alternative j. Notably, we consider the cumulative preference of the end-users' so that we have the general opinion of users.

4.2.1 Expert Consensus Solution (ExCS)

Let $V^p = (v_{ij}^p)_{n \times n}$, for $i < j$ be the preference vector of the expert $e_p \in E$ located in the Euclidean space. Let $w_e = (w_e^1, w_e^2, \dots, w_e^m)$ be the weight vector of the experts, where $w_e^p \geq 0$ is the weight associated with the expert e_p and $\sum_{p=1}^{m} w_e^p = 1$. Then the group decision matrix of the experts $V^{GD} = (v_{ij}^{GD})_{n \times n}$ can be obtained by the weighted average operator, given in Eq. (2.1). To obtain the expert consensus solution (ExCS), consensus reaching process involves two stages: Consensus Measure and Threshold-based Feedback Mechanism. Both stages are discussed as follows:

(i) Consensus Measure

In GDM, the experts are required to participate in the discussion process to reach a consensus solution. In the consensus process, a method for obtaining the consensus solution for experts is introduced based on the degree of deviation between the individual expert's and group decision matrix as per [15]. The degree of deviation between V^p of expert $e_p \in E$ and V^{GD} defined using Eq. (4.1) is given as follows:

$$D(V^p, V^{GD}) = (\frac{1}{\sqrt{k}}(\sum\nolimits_{i,j \in \{1,2,\dots,n\} and \ i<j} (v_{ij}^p - v_{ij}^{GD})^2)^{1/2}) \tag{4.1}$$

where $k = \frac{n \times (n-1)}{2}$. Based on the distance between the individual and group decision matrix, the consensus degree of expert e_p is obtained as given in Eq. (4.2).

$$CD(V^p, V^{GD}) = 1 - D(V^p, V^{GD}) \tag{4.2}$$

Let $\lambda \in [0, 1]$ be the consensus threshold defined as the minimum expected consensus among the experts. If the obtained consensus level of e_p is greater than or equal to the threshold, i.e., $CD(V^p, V^{GD}) \geq \lambda$, the expert e_p reaches the consensus. Whereas if $CD(V^p, V^{GD}) < \lambda$, then expert e_p will be given the feedback to modify the preference matrix in order to achieve the consensus. The feedback process to guide experts below the consensus level is discussed below.

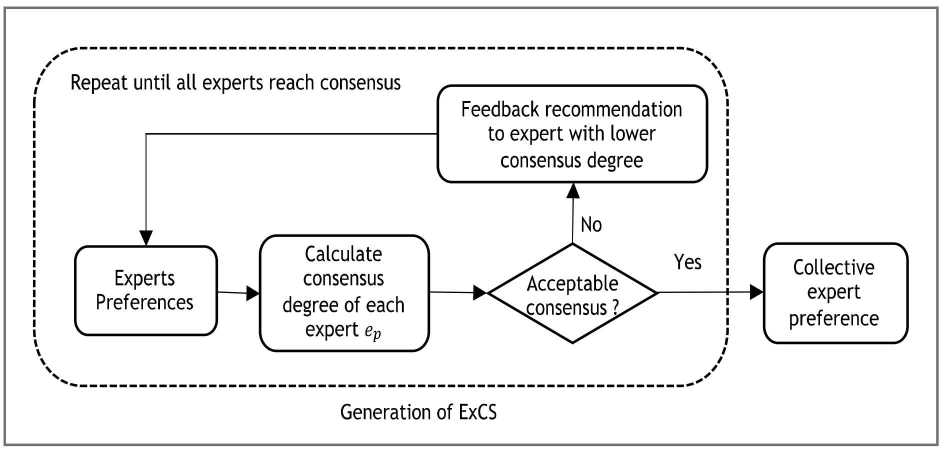

Fig. 4.2: A Framework of CRP with Threshold-based Feedback Recommendation

(ii) Threshold-based Feedback Mechanism

In the general CRP framework described in Chapter 2, the group preference matrix V^{GD} is directly employed to produce the advice given in the feedback using Eq. (2.4). The experts then modify their preferences accordingly to achieve the consensus threshold. In this study, we assume that experts accept the feedback recommendations and do not present non-cooperative behaviors. However, reaching a consensus threshold may take several feedbacks round due to the uncertain range of advice provided, which might be a time-consuming process and thus impact the consensus reaching process. To overcome this limitation, a constant and personalized feedback mechanism based on the consensus threshold is proposed, shown in Fig. 4.2, which is contended to favor the expert in accepting the personalized advice, thus achieving the group consensus at once.

In general, the minimum required consensus level, also called the consensus threshold, is the minimum degree of agreement ($\lambda \in [0,1]$) among the experts. That means, if the value of threshold $\lambda = 0$, it implies no agreement among the experts and if $\lambda = 1$, it implies full agreement among them over the decision, which is ideally not possible. Therefore, there will always be the possibility of some degree of disagreement among experts, which is allowed in every decision-making situation. Hence, provided the minimum degree of agreement, we can obtain the maximum degree of disagreement $(1 - \lambda)$, a which will be used in this work to formulate the feedback process. Thus, the proposed feedback mechanism provides personalized advice to experts so that the consensus level can be achieved in a single round. The feedback mechanism consists of two rules: (i) the identification rule, which is identical to the rule discussed in Chapter 2, and (ii) the direction rule, which aims to provide advice to the inconsistent experts to achieve the group consensus level based on the predefined threshold. The second rule is elaborated below.

If for $e_p \in E$, $D(V^p, V^{GD}) > (1 - \lambda)$, then e_p is an inconsistent expert, and the feedback mechanism should be carried out for the expert e_p. The advice provided to the expert e_p will be different from the V^p in order to satisfy $D(V^p, V^{GD}) \leq (1 - \lambda)$. We propose to compute personalized advice for the experts that varies according to the group preference matrix and the degree of disagreement that respects the initial preference of the experts while achieving consensus. To do so, a feedback matrix AV^p is computed for an inconsistent expert e_p at a maximum disagreement degree from the initial group preference matrix V^{GD}. The method for obtaining the feedback matrix in the feedback process is introduced based on two factors: the degree of deviation between an individual expert decision matrix V^p and the group decision matrix V^{GD}, i.e., $D(V^p, V^{GD})$ and the predefined degree of disagreement $(1-\lambda)$. Based on these two factors, a feedback matrix $AV^p = (av_{ij}^p)_{n\times n}$ will be generated to advice the expert e_p. This received advice if properly considered; the consensus would be reached without undergoing any other round of negotiation. The feedback matrix AV^p is calculated using Eq. (4.3).

$$AV^p = V^{GD} + \frac{(1-\lambda)/2}{D(V^p, V^{GD})}(V^p - V^{GD}) \tag{4.3}$$

AV^p satisfies the following.

$$D(AV^p, V^{GD}) = \left(\frac{1}{\sqrt{k}}\left(\sum_{i,j\,\in\{1,2,\dots,n\} and\ i<j}(av_{ij}^p - v_{ij}^{GD})^2\right)^{\frac{1}{2}}\right) = \frac{(1-\lambda)}{2} \tag{4.4}$$

Specifically, replacing $\frac{(1-\lambda)/2}{D(V^p,V^{GD})}$ by δ_p in Eq. (4.3), we have

$$av_{ij}^p = (1-\delta_p).v_{ij}^{GD} + \delta_p.v_{ij}^p \tag{4.5}$$

which guarantees that $av_{ij}^p + av_{ji}^p = 1$ and $av_{ij}^p \in [min(v_{ij}^{GD}, v_{ij}^p), max(v_{ij}^{GD}, v_{ij}^p)] \in [0,1]$. The value range of feedback parameter δ_p can be derived as follows. An expert e_p would only undergo the feedback mechanism only if its degree of deviation from the group opinion is greater than $(1-\lambda)$. As $\delta_p = \frac{(1-\lambda)/2}{D(V^p,V^{GD})}$, it will always be less than 1 since $D(V^p, V^{GD}) > (1-\lambda)$. As we already know that the value of the consensus threshold (λ) lies in the range [0,1] where 0 means no consensus and 1 means full consensus. When the consensus threshold is 1, we obtain the minimum value of δ_p, which is 0. This implies that the value of δ_p lies in the range [0,1). Further, when the value of δ_p is small (i.e., δ_p is closer to 0), the feedback mechanism focuses more attention on the group opinion than the individual opinion. When the value of δ_p is large (i.e., δ_p is closer to 1) the feedback mechanism focuses more attention on the individual opinion than the group opinion. Thus, the feedback mechanism is designed in such a way that whatever be the value of obtained δ_p, the group opinion V^{GD} will always be considered along with the individual opinion V^p. Thus, to improve the consensus level of the identified expert, the advised adjustment direction is:

$$\begin{cases} \overline{v}_{ij}^p \in [min(v_{ij}^{GD}, av_{ij}^p), max(v_{ij}^{GD}, av_{ij}^p)], i \geq j \\ \overline{v}_{ji}^p = 1 - \overline{v}_{ij}^p, \qquad i < j \end{cases} \tag{4.6}$$

Notice that because of $av_{ij}^p \in [min(v_{ij}^{GD}, v_{ij}^p), max(v_{ij}^{GD}, v_{ij}^p)]$, this adjustment direction ensures that the adjusted preference $\overline{v}_{ij}^p$ will be closer to the group preference than the preference v_{ij}^p that gets replaced and that too in the range of consensus threshold. After the experts modify their preference in the advised range, they certainly will agree with each other. Then the updated group decision matrix of the experts, $\overline{V}^{GD} = (\overline{v}_{ij}^{GD})_{n\times n}$ can be obtained using the weighted average operator given in Eq. (2.1). The aim of the feedback mechanism based on the consensus threshold is to carry out different adjustment suggestions, which respects the experts' initial preferences too.

From the above discussion, the following property can be derived.

Lemma 4.1: Let $V = \{V^1, V^2, \ldots . V^m\}$ and $w_e = \{w_e^1, w_e^2, \ldots, w_e^m\}$ be the m preference relations and the weight vector of the m experts expressed as $E = \{e_1, e_2, e_3, \ldots ., e_m\}$, respectively. $\overline{V}^p$ and $\overline{V}^l$ be the possible modified preference relation of inconsistent expert $e_p \in E$ and $e_l \in E$ consequent upon receiving feedback. $\overline{V}^{GD}$ be the modified group decision matrix utilizing the weighted average operator. Then,

$$D\left(\overline{V}^p,\overline{V}^{GD}\right) \le \max_{e_l\in E}\left\{D\left(\overline{V}^l,\overline{V}^p\right)\right\}$$

Proof: $\overline{V}^p$ and $\overline{V}^l$ are the modified preference relation of inconsistent expert e_p and e_l. From Eq. (4.1),

$$D\left(\overline{V}^p,\overline{V}^{GD}\right) = (\frac{1}{\sqrt{k}}(\sum_{i,j\in\{1,2,\ldots,n\} and\ i<j}(\overline{v}_{ij}^{GD} - \overline{v}_{ij}^{p})^2)^{1/2})$$

$$D\left(\overline{V}^l,\overline{V}^p\right) = (\frac{1}{\sqrt{k}}(\sum_{i,j\in\{1,2,\ldots,n\} and\ i<j}(\overline{v}_{ij}^{p} - \overline{v}_{ij}^{l})^2)^{1/2})$$

Then,

$$D\left(\overline{V}^p,\overline{V}^{GD}\right) = (\frac{1}{\sqrt{k}}(\sum_{i,j\in\{1,2,\ldots,n\} and\ i<j}(\overline{v}_{ij}^{GD} - \overline{v}_{ij}^{p})^2)^{1/2})$$

$$= (\frac{1}{\sqrt{k}}(\sum_{i,j\in\{1,2,\ldots,n\} and\ i<j}(\sum_{l=1}^{m} w_l\overline{v}_{ij}^{l} - \sum_{l=1}^{m} w_l\overline{v}_{ij}^{p})^2)^{1/2})$$

$$= (\frac{1}{\sqrt{k}}(\sum_{i,j\in\{1,2,\ldots,n\} and\ i<j}(\sum_{l=1}^{m} w_l(\overline{v}_{ij}^{l} - \overline{v}_{ij}^{p}))^2)^{1/2})$$

$$= \sum_{l=1}^{m} w_l\,(\frac{1}{\sqrt{k}}(\sum_{i,j\in\{1,2,\ldots,n\} and\ i<j}(\overline{v}_{ij}^{l} - \overline{v}_{ij}^{p})^2)^{1/2})$$

$$= \sum_{l=1}^{m} w_l D\left(\overline{V}^l,\overline{V}^p\right) \le \sum_{l=1}^{m} w_l \max_l\left\{D\left(\overline{V}^l,\overline{V}^p\right)\right\} = \max_l\left\{D\left(\overline{V}^l,\overline{V}^p\right)\right\}$$

This completes the Proof of Lemma 4.1.

Lemma 4.1 shows that the deviation degree between individual preference relation $\overline{V}^p$ of expert e_p and the associated group preference relation $\overline{V}^{GD}$ is not greater than the largest deviation degree between any two of the preference relations in $\overline{V}^p$ and $\overline{V}^l$, $e_l \in E$.

Lemma 4.2: Let $V = \{V^1, V^2, \ldots . V^m\}$ and $w_e = \{w_e^1, w_e^2, \ldots, w_e^m\}$ be the m preference relations and the weight vector of the experts expressed as $E = \{e_1, e_2, e_3, \ldots., e_m\}$, respectively. $\overline{V}^p$ and $\overline{V}^l$ be the possible modified preference relation of experts e_p and e_l consequent upon receiving the feedbacks. Then,

$$D\left(\overline{V}^l,\overline{V}^p\right) \le (1-\lambda)$$

Proof: Suppose V^{GD} be the group preference matrix associated with the original preferences of the experts. Then,

$$D\left(\overline{V}^{l}, \overline{V}^{p}\right) = \left(\frac{1}{\sqrt{k}}\left(\sum_{i,j\, \in\{1,2,\ldots,n\} and\ i<j} (\overline{v}_{ij}^{p} - \overline{v}_{ij}^{l})^{2}\right)^{\frac{1}{2}}\right)$$

Using Eq. (4.6), the advice generated for the inconsistent expert e_p will be:

$$\begin{cases} \overline{v}_{ij}^{p} \in [min(v_{ij}^{GD}, av_{ij}^{p}), max(v_{ij}^{GD}, av_{ij}^{p})], i \geq j \\ \overline{v}_{ji}^{p} = 1 - \overline{v}_{ij}^{p}, \qquad i < j \end{cases}$$

Let us consider that the modified preference relation of expert e_p and e_l are av_{ij}^{p} and av_{ij}^{l}, i.e., $\overline{v}_{ij}^{p} = av_{ij}^{p}$ and $\overline{v}_{ij}^{l} = av_{ij}^{l}$ respectively. Then,

$$D\left(\overline{V}^{l}, \overline{V}^{p}\right) = \left(\frac{1}{\sqrt{k}}\left(\sum_{i,j\, \in\{1,2,\ldots,n\} and\ i<j} (av_{ij}^{p} - av_{ij}^{l})^{2}\right)^{\frac{1}{2}}\right)$$

$$= \left(\frac{1}{\sqrt{k}}\left(\sum_{i,j\, \in\{1,2,\ldots,n\} and\ i<j} (av_{ij}^{p} - av_{ij}^{l} + v_{ij}^{GD} - v_{ij}^{GD})^{2}\right)^{\frac{1}{2}}\right)$$

$$= \left(\frac{1}{\sqrt{k}}\left(\sum_{i,j\, \in\{1,2,\ldots,n\} and\ i<j} ((av_{ij}^{p} - v_{ij}^{GD}) + (v_{ij}^{GD} - av_{ij}^{l}))^{2}\right)^{\frac{1}{2}}\right)$$

$$\leq \left(\frac{1}{\sqrt{k}}\left(\sum_{i,j\, \in\{1,2,\ldots,n\} and\ i<j} (av_{ij}^{p} - v_{ij}^{GD})^{2}\right)^{\frac{1}{2}}\right) +$$

$$\left(\frac{1}{\sqrt{k}}\left(\sum_{i,j\, \in\{1,2,\ldots,n\} and\ i<j} (v_{ij}^{GD} - av_{ij}^{l})^{2}\right)^{\frac{1}{2}}\right)$$

$$D\left(\overline{V}^{l}, \overline{V}^{p}\right) \leq D(AV^{p}, V^{GD}) + D(AV^{l}, V^{GD})$$

From Eq. (4.4) $D\left(\overline{V}^{l}, \overline{V}^{p}\right) \leq \frac{(1-\lambda)}{2} + \frac{(1-\lambda)}{2}$

Consequently, $D\left(\overline{V}^{l}, \overline{V}^{p}\right) \leq (1-\lambda)$

This completes the Proof of Lemma 4.2.

Lemma 4.2 shows that the largest deviation degree between the modified opinion $\overline{V}^{p}$ and $\overline{V}^{l}$ of the expert e_p and e_l, respectively, is not greater than the maximum disagreement degree $(1-\lambda)$.

Theorem 4.1: Using the proposed threshold-based feedback mechanism, experts reach consensus in at most one round of feedback, i.e., $CD\left(\overline{V}^{p}, \overline{V}^{GD}\right) \geq \lambda$

Proof: From lemma 4.1 and lemma 4.2, it can be inferred that

$$D\left(\overline{V}^{p}, \overline{V}^{GD}\right) \leq \max_{l}\left\{D\left(\overline{V}^{l}, \overline{V}^{p}\right)\right\} \leq (1-\lambda)$$

This implies that $D\left(\overline{V}^{p}, \overline{V}^{GD}\right) \leq (1-\lambda)$

Therefore, $CD\left(\overline{V}^{p}, \overline{V}^{GD}\right) \geq \lambda$

Theorem 4.1 shows that the deviation degree between the modified opinion $\overline{V}^{p}$ of the expert $e_p \in E$ and the updated group preference matrix $\overline{V}^{GD}$ obtained after experts modify opinion according to advice recommended is not greater than the maximum disagreement degree $(1-\lambda)$. That means each expert e_p reaches consensus just after accepting the feedback advice given to them at once.

4.2.2 End-Users' Cumulative Solution (EuCS)

The end-users, being naïve, are invited as resource persons to GDM. It is inconvenient to ask them to participate in the interactive process where the moderator produces feedback, and they modify their preferences accordingly [46], [97]. It is a time-consuming process and requires participants to be aware of the decision-making process. But in some situations, they may not be aware of their participation in the case where their opinion is being made use of from one or other sources. Moreover, the moderator's issue when relying on opinions of the end-user data is similar to reading the online customer reviews. Most people who give reviews for any service or product either hate it or love it. But most valuable feedbacks are from those in the middle who are not responding emotionally. They could be utilized as a resource person. Furthermore, if we discard the extreme decision information, we may lose some information. The end-user with extreme preferences is more likely to provide different opinions from the group. Thus, we need to produce the majority group opinion while preserving the extreme opinions in the decision making. For that, we prioritize the users based on the similarity of opinions.

First, we calculate the distance between each pair of users. Let $T^q = (t_{ij}^q)_{n\times n}$ be the preference relation of a user $u_q \in U$, and $T^s = (t_{ij}^s)_{n\times n}$ be the preference relation of a user $u_s \in U$. Here, we use the Euclidean distance function to measure the distance between the users u_q and u_s, given in Eq. (4.7).

$$D_{qs} = (\frac{1}{\sqrt{k}}(\sum\nolimits_{i,j\in\{1,2,\dots,l\}\ and\ i<j}(t_{ij}^q - t_{ij}^s)^2)^{1/2}) \tag{4.7}$$

Using this pairwise distance, we can obtain the relative distance of a user from other users. In this way, the weights of the users can be determined based on the similarity of opinions. The relative distance of user u_q can be calculated using Eq. (4.8).

$$D_q = \sum_{s=1}^{l} D_{qs} \tag{4.8}$$

Let w_q be the weight of the user u_q, and it can be computed using Eq. (4.9).

$$w_q = \frac{(1 - D_q)}{\sum_{s=1}^{l}(1 - D_s)} \tag{4.9}$$

The cumulative end-user's opinion EuCS, i.e., $T^C = (t_{ij}^C)_{n\times n}$, can be obtained using the weighted average (WA) operator as given in Eq. (4.10).

$$t_{ij}^{C} = WA\left(t_{ij}^{1}, t_{ij}^{2}, \dots \dots, t_{ij}^{l}\right) = \sum_{s=1}^{l} w_s . t_{ij}^{s} , for\ i, j\ =\ 1, \dots, n \quad (4.10)$$

In this way, we can preserve the impact of all the users' information in the cumulative opinion.

4.2.3 Global Consensus Solution

In decision-making with heterogeneous DMs, the opinions of experts and end-users can be dispersive since both groups can have different knowledge and experience backgrounds. Therefore, it is required to obtain the final solution considering ExCS and EuCS both. Different from existing GDM models, our proposed model allows the opinion of experts and end-users. The moderator is responsible for generating the global solution called Global Consensus Solution (GCS) to get the final decision result. The moderator considers ExCS as a reference solution and operates to incorporate the end-users opinions to get the GCS. The GCS may come across some deviation from the reference solution ExCS to incorporate the opinions of the end-users EuCS. To handle the degree of deviation caused in the reference solution, i.e., ExCS, the moderator is furnished with a parameter called tolerance degree θ that defines how much deviation from the reference solution can be tolerated to obtain the final solution.

The value of the tolerance degree θ can be determined in the same way we determine the consensus threshold for CRP based on the nature of the problem. As already discussed in the manuscript, an appropriate value of θ can control the degree of deviation of experts toward users. So, depending upon the nature of the application, one can decide the value of θ. If the value of θ is closer to 0, the moderator shows no tolerance, i.e., the moderator is reluctant to converge towards the user's opinion. It reduces to expert only for decision making. If the tolerance degree θ is less than but closer to 1, too much importance is given to users' opinions. Thus, the tolerance degree θ controls the impact of users' opinions in generating the final decision GCS. This indicates that although the subjectivity of the consensus solution gets increased by adding the tolerance degree, adding the tolerance degree provides the moderator with a greater degree of freedom to control the decision process.

To harness the valuable and important opinions of experts and end-users both, the moderator generates the final decision depending on tolerance degree θ. The process for obtaining the GCS gets initiated once ExCS $\overline{V}^{GD}$ and EuCS T^C is obtained (shown in Fig. 4.3). The GCS determination depends on the degree of deviation between the end-user's cumulative opinion and the expert's consensus opinion that works as follows.

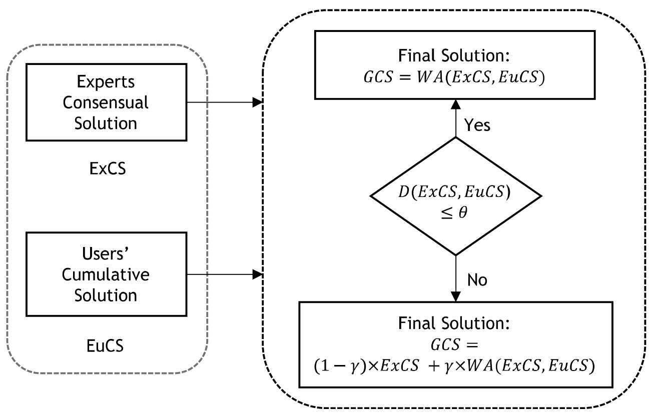

Fig. 4.3: Obtaining the Final Solution using Moderators' Tolerance

Let w_e and w_u be the weights associated with the ExCS and EuCS, respectively. To obtain the final decision, the moderator performs the evaluation operation by comparing the degree of deviation, using Eq. (4.1), between the ExCS and EuCS denoted as $D(\overline{V}^{GD}, T^C)$ with its tolerance degree. If the obtained degree of deviation is less than or equivalent to θ, then the moderator is ready to perform operations over the reference solution ExCS with respect to EuCS to obtain GCS. If the differences in the opinions of the experts and users are greater than the moderators' tolerance, i.e., $D(\overline{V}^{GD}, T^C) > \theta$, then the moderator generates a final solution subject to its maximum tolerance degree θ.

To better manage the experts' and users' opinions in the final solution, conditional GCS generation is defined in Eqs. (4.11) - (4.12).

$$GCS = WA(\overline{V}^{GD}, T^C) = (w_e \times \bar{v}_{ij}^{GD}) + (w_u \times t_{ij}^C), \;\; if\ D(\overline{V}^{GD}, T^C) \leq \theta \quad (4.11)$$

$$GCS = (1-\gamma) \times \overline{V}^{GD} + \gamma \times WA(\overline{V}^{GD}, T^C), \quad if\ D(\overline{V}^{GD}, T^C) > \theta \quad (4.12)$$

$$\text{where } \gamma = \frac{\theta}{D(\overline{V}^{GD}, WA(\overline{V}^{GD}, T^C))}$$

If the user opinion lies within the tolerance degree θ, i.e., $D(\overline{V}^{GD}, T^C) \leq \theta$ then the final decision will be the weighted average (WA) of the expert and user opinion, defined in Eq. (4.11). The GCS in Eq. (4.12) combines two reference points and is used to generate specific solution. This defines the case when users' opinion lies beyond the tolerance degree, i.e., $D(\overline{V}^{GD}, T^C) > \theta$, the final decision GCS can be formulated exactly on the tolerance boundary.

The selection of tolerance value θ can be determined voluntarily based on the specific decision problem that controls the emphasis on experts' and end-users' opinions. An appropriate θ can control the degree of deviation of experts toward users. In this study, $\theta = 0$ means the moderator has no tolerance behavior, i.e., the moderator is reluctant in converging towards the user's opinion, i.e., only experts play the role in decision making. If the tolerance degree θ is less than but closer to 1, then too much importance is given to the user's opinion. If $\theta = D(\overline{V}^{GD}, WA(\overline{V}^{GD}, T^C))$, the weighted average is the final solution which reduces to the case discussed in Eq. (4.11), and if $\theta = 1$, then Eq. (4.11) holds true as from Eq. (4.1), we can say that the maximum degree of deviation between ExCS and EuCS is always 1. In such a case, the user's opinion will always be within the tolerance of the moderator, and so the solution in Eq. (4.11). Furthermore, the weights provided to the experts and users play an integral role in producing the GCS as it reflects the relevance of the users in the final decision. In this way, the opinions of both the experts and the end-users are reserved in the final decision, depending on the weights provided to them. The proposed GDM is summarized in Algorithm 1.

Algorithm 1. The Proposed GDM Model

Input: Preference $V^p = (v_{ij}^p)_{n\times n}$ of each expert $e_p \in E$ and Preference $T^q = (t_{ij}^q)_{n\times n}$ of each end-user $u_q \in U$, the consensus threshold λ and the tolerance degree θ.

Output: The final ranking of alternatives $X = \{x_1, x_2, x_3, \ldots, x_n\}$

Step 1: Compute the experts' group preference matrix, $V^{GD} = (v_{ij}^{GD})_{n\times n}$ using Eq. (2.1)

Step 2: Determine the consensus degree of each expert $e_p \in E$, $CD(V^p, V^{GD})$ using Eq. (4.2).

If $CD(V^p, V^{GD}) < \lambda$, go to Step 3; otherwise $\overline{V}^p = V^p$.

Step 3: Adopt feedback mechanism to generate recommendations for inconsistent experts obtained as follows:

Compute feedback matrix $AV^p = (av_{ij}^p)_{n\times n}$ using Eq. (4.3)

Utilize Eq. (4.6) to generate the adjustment direction for each inconsistent expert e_p.

Step 4: Compute ExCS, $\overline{V}^{GD}$ using Eq. (2.1)

Step 5: Use Eq. (13)-(15) to obtain the weight w_q of end-user u_q.

Step 6: Compute EuCS, $T^C = (t_{ij}^C)_{n\times n}$ using Eq. (4.10).

Step 7: Calculate $D(\overline{V}^{GD}, T^C)$ using Eq. (4.1)

If $((\overline{V}^{GD}, T^C) \leq \theta)$
Calculate GCS using Eq. (4.11)

Else

Calculate GCS using Eq. (4.12)

Step 8: Obtain the final ranking of alternatives by applying Eq. (2.5) on GCS.

Step 9: End

4.3 An Illustrative Example

Digital transformation and automation in information systems for healthcare have a positive impact of technology, leading to smart healthcare services. Telemedicine, AI-enabled medical devices and blockchain electronic health records are just a few concrete examples of digital transformation in healthcare that completely reshape how one interacts with health professionals, how our data is shared among providers and how decisions are made about our treatment plans and health outcomes. The market of Internet of Things (IoT)-based healthcare is predicted to reach USD 260.75 Billion in 2027 [98]. Many IoT-based healthcare solution providers are present in the market, such as CONTUS [99], MINDBOWSER [100] and Biz4Intellia [101]. They are offering various healthcare services. Finding the best healthcare service for the requirements of all stakeholders is a challenging problem. We discussed the possible reasons behind this problem in Section 4.1: insufficient information, availability of many services and the inappropriate number of targeted end-users. Therefore, experts and end-users both should be invited to evaluate the healthcare services. Without consent and satisfaction of end-users, the selected service will fail no matter how much good experts were involved in GDM. The advantage of inviting the end-users is that the moderator can use the untapped potential of the end-users (who may or may not be recognized as experts) to make a meaningful decision. Thus, the ineffective service and its bad outcomes could be eliminated by the decision made collaboratively by the experts and end-users, resulting in higher quality service.

Suppose a hospital management wants to select a healthcare service provider providing healthcare IoT solutions. Through a process of pre-evaluation, we identify four healthcare IoT solutions providers, say x_1:Service Provider 1; x_2: Service Provide 2; x_3: Service Provide 3; x_4: Service Provide 4. To evaluate the services provided by these service providers, we also assume that there are 6 experts $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and eight related stakeholders who are experienced users of these services $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ are invited to form a selection committee. Note that more than eight users can also be involved in the proposed method, but for the demonstration purpose, only eight users' opinions are considered here in this example. All these six experts and eight users use fuzzy preference relation (also called additive fuzzy preference relation) to express their preference information, which is listed as follows:

$$V^1 = \begin{pmatrix} 0.50 & 0.45 & 0.06 & 0.21 \\ 0.55 & 0.50 & 0.52 & 0.03 \\ 0.94 & 0.48 & 0.50 & 0.76 \\ 0.79 & 0.97 & 0.24 & 0.50 \end{pmatrix} \quad V^2 = \begin{pmatrix} 0.50 & 0.21 & 0.08 & 0.57 \\ 0.79 & 0.50 & 0.19 & 0.07 \\ 0.92 & 0.81 & 0.50 & 0.91 \\ 0.43 & 0.93 & 0.09 & 0.50 \end{pmatrix}$$

$$V^3 = \begin{pmatrix} 0.50 & 0.54 & 0.35 & 0.87 \\ 0.46 & 0.50 & 0.95 & 0.78 \\ 0.65 & 0.05 & 0.50 & 0.69 \\ 0.13 & 0.22 & 0.31 & 0.50 \end{pmatrix} \quad V^4 = \begin{pmatrix} 0.50 & 0.30 & 0.55 & 0.38 \\ 0.70 & 0.50 & 0.69 & 0.57 \\ 0.45 & 0.31 & 0.50 & 0.61 \\ 0.62 & 0.43 & 0.39 & 0.50 \end{pmatrix}$$

$$V^5 = \begin{pmatrix} 0.50 & 0.02 & 0.95 & 0.43 \\ 0.98 & 0.50 & 0.82 & 0.48 \\ 0.05 & 0.18 & 0.50 & 0.88 \\ 0.57 & 0.52 & 0.12 & 0.50 \end{pmatrix} \quad V^6 = \begin{pmatrix} 0.50 & 0.25 & 0.17 & 0.89 \\ 0.75 & 0.50 & 0.49 & 0.23 \\ 0.83 & 0.51 & 0.50 & 0.88 \\ 0.11 & 0.77 & 0.12 & 0.50 \end{pmatrix}$$

$$T^1 = \begin{pmatrix} 0.50 & 0.72 & 0.40 & 0.35 \\ 0.28 & 0.50 & 0.26 & 0.56 \\ 0.60 & 0.74 & 0.50 & 0.30 \\ 0.65 & 0.44 & 0.70 & 0.50 \end{pmatrix} \quad T^2 = \begin{pmatrix} 0.50 & 0.91 & 0.96 & 0.89 \\ 0.09 & 0.50 & 0.21 & 0.44 \\ 0.04 & 0.79 & 0.50 & 0.39 \\ 0.11 & 0.56 & 0.61 & 0.50 \end{pmatrix}$$

$$T^3 = \begin{pmatrix} 0.50 & 0.18 & 0.87 & 0.33 \\ 0.82 & 0.50 & 0.05 & 0.57 \\ 0.13 & 0.95 & 0.50 & 0.54 \\ 0.67 & 0.43 & 0.46 & 0.50 \end{pmatrix} \quad T^4 = \begin{pmatrix} 0.50 & 0.61 & 0.90 & 0.53 \\ 0.39 & 0.50 & 0.53 & 0.94 \\ 0.11 & 0.47 & 0.50 & 0.27 \\ 0.47 & 0.06 & 0.73 & 0.50 \end{pmatrix}$$

$$T^5 = \begin{pmatrix} 0.50 & 0.10 & 1.00 & 0.32 \\ 0.89 & 0.50 & 0.46 & 0.69 \\ 0.00 & 0.53 & 0.50 & 0.44 \\ 0.68 & 0.31 & 0.56 & 0.50 \end{pmatrix} \quad T^6 = \begin{pmatrix} 0.50 & 0.01 & 0.75 & 0.73 \\ 0.99 & 0.50 & 0.07 & 0.99 \\ 0.25 & 0.93 & 0.50 & 0.21 \\ 0.27 & 0.01 & 0.79 & 0.50 \end{pmatrix}$$

$$T^7 = \begin{pmatrix} 0.50 & 0.11 & 0.84 & 0.82 \\ 0.89 & 0.50 & 0.78 & 0.75 \\ 0.16 & 0.22 & 0.50 & 0.30 \\ 0.18 & 0.25 & 0.70 & 0.50 \end{pmatrix} \quad T^8 = \begin{pmatrix} 0.50 & 0.34 & 0.97 & 0.62 \\ 0.66 & 0.50 & 0.88 & 0.88 \\ 0.03 & 0.12 & 0.50 & 0.37 \\ 0.38 & 0.12 & 0.63 & 0.50 \end{pmatrix}$$

The following sub-sections illustrate the use of the proposed GDM model. The first sub-section demonstrates the Expert Consensus Solution (ExCS). The second sub-section demonstrates the calculation of End-user Cumulative Solution (EuCS). Lastly, the Global Consensus Solution (GCS) is obtained.

4.3.1 Illustration of Expert Consensus Solution (ExCS)

Step 1: Value of λ depends on the decision-making problem [34] and λ can be between 0 to 1, where value of $\lambda = 0$ implies no agreement among the experts and $\lambda = 1$ implies full agreement among them over the decision, which is ideally not possible. Let $\lambda = 0.80$ and all six experts have equal weights. Using Eq. (2.1), we calculate the initial group decision matrix V^{GD} of the experts.

$$V^{GD} = \begin{pmatrix} 0.50 & 0.29 & 0.36 & 0.56 \\ 0.71 & 0.50 & 0.61 & 0.36 \\ 0.64 & 0.39 & 0.50 & 0.79 \\ 0.44 & 0.64 & 0.21 & 0.50 \end{pmatrix}$$

Step 2: We calculate the degree of consensus between each expert matrix and the group decision matrix using Eq. (4.2).

$$CD(V^1, V^{GD}) = 0.7558, CD(V^2, V^{GD}) = 0.7545, CD(V^3, V^{GD}) = 0.7215, CD(V^4, V^{GD}) = 0.8420, CD(V^5, V^{GD}) = 0.7103, CD(V^6, V^{GD}) = 0.8234$$

Step 3: Since $\lambda = 0.80$, we find that only experts e_4 and e_6 achieve the consensus from the obtained results in step 2. The feedback mechanism is initiated for other experts.

Step 4: We use the feedback mechanism and compute feedback matrices AV^1, AV^2, AV^3, and AV^5 for inconsistent experts e_1, e_2, e_3, and e_5, respectively using Eq. (4.5).

$$AV^1 = \begin{pmatrix} 0.50 & 0.36 & 0.24 & 0.41 \\ 0.64 & 0.50 & 0.57 & 0.22 \\ 0.76 & 0.43 & 0.50 & 0.77 \\ 0.59 & 0.78 & 0.23 & 0.50 \end{pmatrix} \qquad AV^2 = \begin{pmatrix} 0.50 & 0.26 & 0.25 & 0.56 \\ 0.74 & 0.50 & 0.44 & 0.24 \\ 0.75 & 0.56 & 0.50 & 0.84 \\ 0.44 & 0.76 & 0.16 & 0.50 \end{pmatrix}$$

$$AV^3 = \begin{pmatrix} 0.50 & 0.38 & 0.36 & 0.67 \\ 0.62 & 0.50 & 0.73 & 0.51 \\ 0.64 & 0.27 & 0.50 & 0.75 \\ 0.33 & 0.49 & 0.25 & 0.50 \end{pmatrix} \qquad AV^5 = \begin{pmatrix} 0.50 & 0.20 & 0.56 & 0.51 \\ 0.80 & 0.50 & 0.68 & 0.40 \\ 0.44 & 0.32 & 0.50 & 0.82 \\ 0.49 & 0.60 & 0.18 & 0.50 \end{pmatrix}$$

Step 5: We generate advice using Eq. (4.6), which are as follows:

i). For e_1, it should decrease the values with respect to the v_{12}^1 in the range [0.29, 0.36], and increase values with respect to position v_{13}^1, v_{14}^1, v_{23}^1, v_{24}^1 and v_{34}^1 in the range [0.24, 0.36], [0.41, 0.56], [0.57, 0.61], [0.22, 0.36] and [0.76, 0.78], respectively.

ii). For e_2, it should increase values with respect to position v_{12}^2, v_{13}^2 , v_{23}^2 and v_{24}^2 in the range [0.26, 0.29], [0.25, 0.36], [0.44, 0.60], [0.24, 0.36] respectively and decrease values with respect to position v_{14}^2 and v_{34}^2 in the range [0.55, 0.56], [0.79, 0.84] respectively.

iii). For e_3, it should increase values with respect to position v_{13}^3 and v_{34}^3 in the range [0.35,0.36] and [0.75, 0.78], respectively, and decrease values with respect to position v_{12}^3, v_{14}^3, v_{23}^3 and v_{24}^3 in the range [0.29, 0.38], [0.55, 0.67], [0.61, 0.73] and [0.36, 0.51], respectively.

iv). For e_5, it should increase values with respect to position v_{12}^5, v_{14}^5 and v_{34}^5 in the range [0.20, 0.29], [0.51, 0.55] and [0.78, 0.81], respectively, and decrease values with respect to position v_{13}^5, v_{23}^5 and v_{24}^5 in the range [0.36, 0.56], [0.61, 0.67] and [0.36, 0.40], respectively.

Suppose that all experts accept the suggestions provided and make the corresponding modifications. our new preferences are given as follows:

$$\bar{V}^1 = \begin{pmatrix} 0.50 & 0.36 & 0.24 & 0.41 \\ 0.64 & 0.50 & 0.57 & 0.22 \\ 0.76 & 0.43 & 0.50 & 0.77 \\ 0.59 & 0.78 & 0.23 & 0.50 \end{pmatrix} \quad \bar{V}^2 = \begin{pmatrix} 0.50 & 0.27 & 0.25 & 0.56 \\ 0.73 & 0.50 & 0.59 & 0.35 \\ 0.75 & 0.41 & 0.50 & 0.82 \\ 0.44 & 0.65 & 0.18 & 0.50 \end{pmatrix}$$

$$\bar{V}^3 = \begin{pmatrix} 0.50 & 0.30 & 0.36 & 0.65 \\ 0.70 & 0.50 & 0.65 & 0.41 \\ 0.64 & 0.35 & 0.50 & 0.75 \\ 0.35 & 0.59 & 0.25 & 0.50 \end{pmatrix} \quad \bar{V}^5 = \begin{pmatrix} 0.50 & 0.25 & 0.41 & 0.54 \\ 0.75 & 0.50 & 0.62 & 0.37 \\ 0.59 & 0.38 & 0.50 & 0.79 \\ 0.46 & 0.63 & 0.21 & 0.50 \end{pmatrix}$$

Step 6: We calculate the updated group decision matrix $\bar{V}^{GD}$ of the experts using Eq. (2.1).

$$\bar{V}^{GD} = \begin{pmatrix} 0.50 & 0.29 & 0.33 & 0.57 \\ 0.71 & 0.50 & 0.60 & 0.36 \\ 0.67 & 0.40 & 0.50 & 0.77 \\ 0.43 & 0.64 & 0.23 & 0.50 \end{pmatrix}$$

Then we calculate the degree of consensus between each expert matrix and the updated group decision matrix using Eq. (4.2). Obtained results are as follows:

$$CD(\bar{V}^1, V^{GD}) = 0.9022, CD(\bar{V}^2, V^{GD}) = 0.9605, CD(\bar{V}^3, V^{GD}) = 0.9542,$$

$$CD(\bar{V}^4, V^{GD}) = 0.8344, CD(\bar{V}^5, V^{GD}) = 0.9583 \text{ and } CD(\bar{V}^6, V^{GD}) = 0.8327$$

We can observe that all experts achieved the consensus in at most one feedback round and, therefore, $\bar{V}^{GD}$ becomes the ExCS.

4.3.2 Illustration of End-user Cumulative Solution (EuCS)

Step 1: We calculate the relative distance of each end-user from other users using Eq. (4.8). Obtained results are as follows:

$$D_1 = 0.3657, D_2 = 0.3928, D_3 = 0.3285, D_4 = 0.2839,$$

$$D_5 = 0.2944, D_6 = 0.3516, D_7 = 0.33185, D_8 = 0.3147.$$

Step 2: We calculate the weight of each end-user using Eq. (4.9). Obtained results are as follows:

$$w_1 = 0.1186, w_2 = 0.1135, w_3 = 0.1255, w_4 = 0.1338,$$

$$w_5 = 0.1319, w_6 = 0.1212, w_7 = 0.1274, w_8 = 0.1281.$$

Step 3: We calculate the end-user cumulative solution (EuCS) T^C using Eq. (4.10).

$$T^C = \begin{pmatrix} 0.50 & 0.37 & 0.84 & 0.57 \\ 0.63 & 0.50 & 0.42 & 0.73 \\ 0.16 & 0.58 & 0.50 & 0.35 \\ 0.43 & 0.27 & 0.65 & 0.50 \end{pmatrix}$$

4.3.3 Illustration of the Global Consensus Solution (GCS)

Let $\theta = 0.3$, i.e., tolerance degree of moderator is 0.3 to modify experts' opinion ExCS and let equal weight is provided to the experts and users solution, i.e., $w_e = w_u = 0.5$

Step 1: We calculate the degree of deviation between the ExCS and EuCS, i.e., $D(\overline{V}^{GD}, T^C)$ using Eq. (4.1), and we get $D\left(\overline{V}^{GD}, T^C\right) = 0.3201$ which is greater than tolerance degree of moderator. In such a case Eq. (4.12) will get executed and the obtained GCS is:

$$GCS = \begin{pmatrix} 0.50 & 0.36 & 0.81 & 0.57 \\ 0.64 & 0.50 & 0.43 & 0.71 \\ 0.19 & 0.57 & 0.50 & 0.38 \\ 0.43 & 0.29 & 0.62 & 0.50 \end{pmatrix}$$

Finally, the selection process activated, and the following ranking of alternatives is obtained using Eq. (4.1): $x_2 \succ x_1 \succ x_4 \succ x_3$

4.4 Performance Evaluation and Discussions

This section first presents the visualization of the consensus process based on the multidimensional scaling (MDS) [102] technique used to project high dimensional data into the 2-dimensional map. Then, some comparisons are given to demonstrate the advantage of our proposed CRP model.

4.4.1 CRP Visualization

To visualize the consensus evolution in 2-D maps, we use the MDS technique. Fig. 4.4 presents the visual representation of experts' preferences. It is to note that MDS does not represent the absolute positions of the preferences, but it represents the relative nearness of the preferences among the experts. Fig. 4.4(a), 4.4(b), 4.4(c), and 4.4(d) represent the initial states (IS) and final state (FS) of 6 experts, represented as e_m^{IS} and e_m^{FS}, respectively for different thresholds. At consensus threshold $\lambda = 0.80$, $\lambda = 0.85$, $\lambda = 0.90$ and

$\lambda = 0.95$ respectively, for the example discussed in section 4, the experts reach the consensus level after one feedback. We can observe from Fig. 4.4 that the closeness in the preferences of experts increases with the increase in threshold in the CRP. We should note that the efficiency of CRP relies on the number of feedback rounds to achieve the consensus. In the proposed feedback mechanism, we achieve the consensus in only one round, irrespective of the threshold value. Therefore, the proposed CRP is an efficient consensus reaching process in terms of the number of feedback rounds.

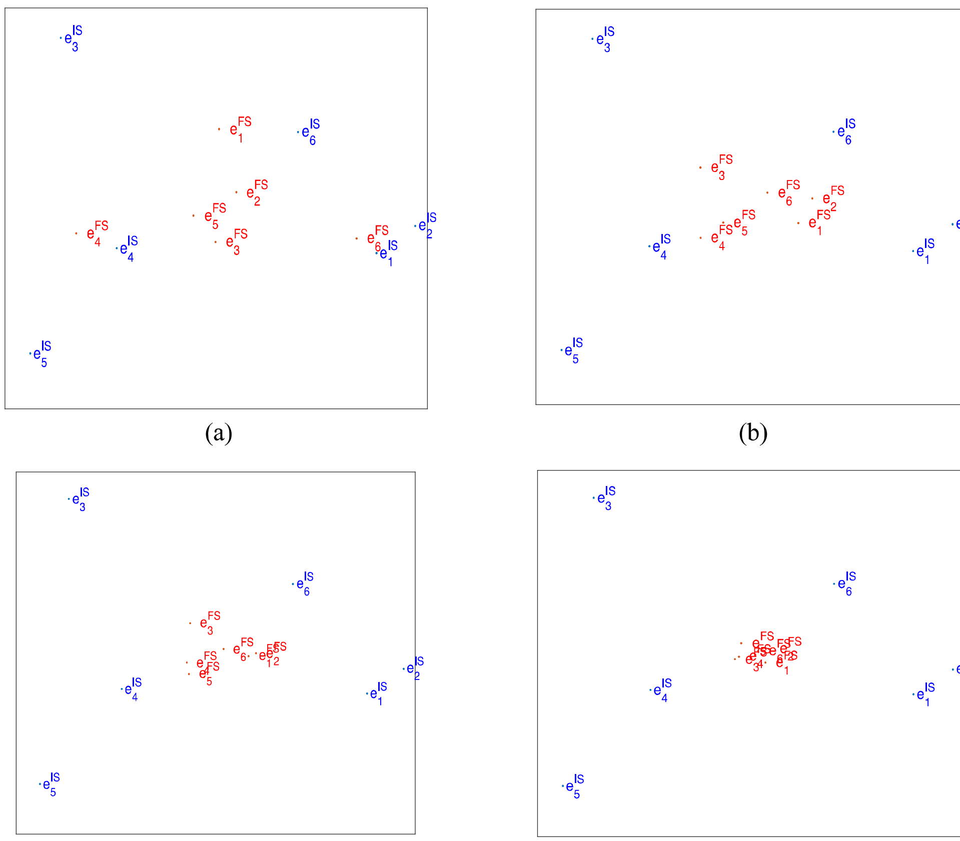

Fig. 4.4: Visualization of Preferences at Different Thresholds (a) $\lambda = 0.8$*; (b)* $\lambda = 0.85$*; (c)* $\lambda = 0.9$*; (d)* $\lambda = 0.95$

4.4.2 Comparison with Existing Consensus Reaching Models

In this section, we compare the proposed model with the existing CRP [103], [104] to show the novelty of the article and obtained results are given in Table 4.1. Adjustment process in existing CRP is given in Eq. (4.13).

$$\begin{cases} av_{ij}^{p} = \delta . v_{ij}^{GD} + (1-\delta) . v_{ij}^{p}, & i \geq j \\ av_{ji}^{p} = 1 - av_{ij}^{p}, & i < j \end{cases} \tag{4.13}$$

where $\delta \in [0,1]$ is a feedback parameter to control the degree of advice given to inconsistent experts. In existing CRP, feedback parameter is same for all experts, unlike the proposed CRP where feedback parameter δ_p is calculated using the threshold λ and degree of deviation $D(V^p, V^{GD})$ in Eq. (4.5) for each inconsistent expert e_p. Since threshold λ is the same for all the experts, we conclude that feedback parameter δ_p for expert e_p depends on $D(V^p, V^{GD})$ for a given threshold. Without altering the essence of the CRP, we replace the Eq. (4.5) with Eq. (4.13) to compare the proposed CRP with the existing CRP [103], [104].

We also compare the consensus improvements provided in Li and Dong [105] with the proposed consensus model in this paper. Obtained results are given in the Table 4.1. To design the work in Li and Dong [105], we replace the Step 3(b) in Algorithm 1 with following step to modify the opinion of the expert.

Modified Step 3(b) in Algorithm 1: Let $AV^p = (av_{ij}^p)_{n\times n}$ be the preference relation of the corresponding moderator e_y to help e_p to improve its consensus. Also, e_y has an acceptable consensus degree and has the closest distance to e_p. Therefore, the direction rule provided to e_p is as follows:

i). If $v_{ij}^{p} \leq av_{ij}^{p} \leq v_{ij}^{GD}$, then e_p should increase the preference for pairwise (x_i, x_j) to be close to AV^p, i.e., $\bar{v}_{ij}^{p} \subseteq (v_{ij}^{p}, av_{ij}^{p}]$.

ii). If $v_{ij}^{p} \leq v_{ij}^{GD} \leq av_{ij}^{p}$, then e_p should increase the preference for pairwise (x_i, x_j) to be close to AV^p, i.e., $\bar{v}_{ij}^{p} \subseteq (v_{ij}^{p}, v_{ij}^{GD}]$.

iii). If $av_{ij}^{p} \leq v_{ij}^{GD} \leq v_{ij}^{p}$, then e_p should decrease the preference for pairwise (x_i, x_j) to be close to AV^p, i.e., $\bar{v}_{ij}^{p} \subseteq [v_{ij}^{GD}, v_{ij}^{p})$.

iv). If $v_{ij}^{GD} \leq av_{ij}^{p} \leq v_{ij}^{p}$, then e_p should decrease the preference for pairwise (x_i, x_j) to be close to AV^p, i.e., $\bar{v}_{ij}^{p} \subseteq [av_{ij}^{p}, v_{ij}^{p})$.

v). Otherwise, e_p should change the preference for pairwise (x_i, x_j), i.e., $\bar{v}_{ij}^{p} = v_{ij}^{p}$.

Table 4.1. Evaluation of Proposed CRP for different values of δ_p

At $\lambda = 0.8$	Feedback Parameter		Initial Consensus	Final Consensus	No. of Iterations	No. of inconsistent experts	Ranking result
Proposed CRP	Depends on $D(V^p, V^{GD})$	Different for different experts	0.7679	0.9070	1	4	$x_2 \succ x_1 \succ x_4 \succ x_3$
CRP using Eq. (4.13)	Voluntarily selected	$\delta = 0.4$	0.7679	0.8305	2	4	$x_2 \succ x_4 \succ x_1 \succ x_3$
		$\delta = 0.65$	0.7679	0.8736	1	4	$x_2 \succ x_4 \succ x_1 \succ x_3$
Li et al. [105]	-	-	0.7679	0.8606	3	8	$x_2 \succ x_1 \succ x_4 \succ x_3$

Observation 1: In most CRPs [15], [103], [104], the adjustment advice produced by the feedback mechanism is used as reference advice for the experts to adjust their preferences. But these CRPs do not consider the predefined consensus threshold that has to be ultimately achieved. For the case solved in Section 4.3, compared to these CRPs, the proposed model provides the adjustment suggestions based on the individual's degree of deviation from the group preference matrix and the consensus threshold. The results in

Table 4.1 show that the proposed CRP converges to the consensus faster than the CRP that selects the feedback parameter voluntarily using (4.13), where the feedback parameter is same for all inconsistent experts.

Observation 2: The number of iterations to achieve the consensus is three, with eight inconsistent experts in Li and Dong [105] method. This shows that the proposed method has better performance in terms of the number of iterations to achieve the consensus than the Li and Dong [105] method.

Observation 3: Also, in the existing CRP, when δ is greater than 0.5, the feedback generation using Eq. (4.13) leads experts to the group decision rather than towards the experts' initial opinion. If δ is smaller than 0.5, then Eq. (4.13) will show a higher impact of expert's initial opinion than the group preference. Different values of δ will show different results, and accordingly, the number of iterations may increase or decrease to reach the consensus.

Observation 4: It is difficult even for experts to repeatedly accept feedback advice and change preferences accordingly in a real-world decision-making situation. They may have limited time and understanding, or sometimes the system itself has time constraints to generate decisions like in emergency group decision-making [8], [106]. In all such cases, the proposed model respecting the experts' initial opinion generates feedback recommendations such that experts reach the consensus at once, which is more reasonable than the works in [26], [27].

4.4.3 Impact of Threshold degree θ

The proposed GDM model is a flexible GDM model which takes care of the experts' opinions. It is easier to implement and understand. Here, the moderator is responsible for giving the decision result while preserving both experts' and users' opinions. Moreover, the model is well suited for situations where experts' opinion is more important than users and vice versa. The value of tolerance controls the influence of either of the opinions in the final decision result. We can understand this for extreme values of tolerance defined in two cases.

Case 1: When the tolerance degree θ is minimum, the moderator is reluctant to change the expert's consensual opinion, implying that the final decision (GCS) will purely rely on experts' opinions. Factually, it favors the situation where consensual opinions from the experts are given more priority than the users.

Case 2: When the tolerance degree θ is maximum, it indicates the maximum deviation from experts' consensual solution that the moderator can tolerate. Even if the moderator is allowed to converge toward the users' opinion, the final decision considers the experts' opinion in the form of the weighted average of experts' and users' opinions.

Thus, our model is sufficiently flexible in preserving the experts' and user' preferences in the final decision. One can argue that the experts have higher knowledge than the users assumed as novice experts; the expert's opinion in either situation is not ignored fully. The final decision result always gives thought to the experts' opinion.

4.4.4 Impact of Presence of Experts and End-Users

The proposed model resembles the classical GDM situations under certain conditions. Table 4.2 illustrates the behavior of the proposed GDM under three cases. Case 1 discusses the situation when there are only experts and no users for decision-making. In Case 1, our model behaves like a classical GDM with CRP but with improved efficiency in terms of the number of feedback rounds to reach consensus. In Case 2, where there are no experts and only users are there for decision-making process, our model resembles the classical GDM employing aggregation and exploitation phase [47]. Lastly, our proposed model works well when both the experts and users are invited for decision-making.

Table 4.2. Analyzing the Behavior of Proposed Model

	No. of Experts	No. of Users	GDM
Case 1	m	0	Classical GDM with improved CRP efficiency
Case 2	0	n	Classical GDM without CRP [47]
Case 3	m	n	Proposed GDM

4.4.5 Impact of End-Users' Opinion

In the proposed model, if we do not consider the opinion of users, ExCS becomes the final decision. GCS is the final decision when we consider the opinion of users. Therefore, to observe the impact of users' opinions on the final decision, first, we calculate the correlation between the ranks obtained from the final decision when users' opinion is not considered, i.e., ExCS and cumulative opinion of end-users, i.e., EuCS. Further, we calculate the correlation between the ranks obtained from the final decision when users' opinion is considered, i.e., GCS, and end-users' cumulative opinion, i.e., ExCS. The Spearman's Rank Correlation Coefficient is the most frequently used tool to measure the correlation and is defined as:

$$CC = 1 - \frac{6\sum_{i=1}^{n} dif_i^2}{n^3 - n}, \quad j = 1,2, \dots n \tag{4.14}$$

where dif_i is the difference between the ranks of i^{th} alternative obtained from two different decisions, n is the number of the alternatives, and CC has a value between -1 and $+1$. CC close to $+1$ implies no significant difference between the obtained ranks in two different decisions. Thus, it implies a great correlation between the two decisions. CC close to -1 implies almost a reverse ordering of the two decisions. CC close to 0 implies a great difference between the orderings of the two decisions. This means that there is no correlation between the two decisions.

Using Eq. (2.5) the rank of alternatives obtained for ExCS is $x_3 \succ x_2 \succ x_4 \succ x_1$, EuCS is $x_1 \succ x_2 \succ x_4 \succ x_3$, and GCS is $x_2 \succ x_1 \succ x_4 \succ x_3$. We calculate CC between ExCS and EuCS when users' opinion is not considered. The obtained value is -0.4. Further, we calculate CC between GCS and EuCS when users' opinion is considered in the final decision. The obtained value of CC is 1. In case of considering the opinions of end-users in GDM, the obtained CC value is 1, which is greater than the CC value when end-users opinion is not considered. This means the final decision would be more accurate and possibly better accepted than when users' opinion is not incorporated.

4.4.6 Impact of ExCS and EuCS on Final Decision Result

We analyze the effect of the closeness measure of ExCS and EuCS on the final decision result for different values of tolerance degree for the example discussed above. Obtained results can be seen in Fig. 4.5. The tolerance degree of the moderator will affect the closeness measure of ExCS and EuCS with the final decision, calculated in terms of similarity of the experts' and users' opinion, respectively, with the final decision. As the tolerance degree of the moderator increases gradually, the closeness measure of ExCS with the final decision, i.e., $CM(GCS, ExCS)$, decreases progressively, while the closeness measure of users with the final decision, i.e., $CM(GCS, EuCS)$, increases gradually. Then both become stable. This result is caused due to the moderator's tolerance for the convergence of experts' opinions towards users' opinions. The more the tolerance degree is, the more users' closeness to the final decision will be but only upto certain value of tolerance after which it becomes constant. Specifically, when the tolerance degree exceeds the maximum possible deviation between the experts and the user's opinion, the weighted aggregation result will always be considered the final decision and the constant value of the closeness measure. As a whole, the overall closeness measure of the experts and users with respect to final solution becomes constant after certain value of tolerance.

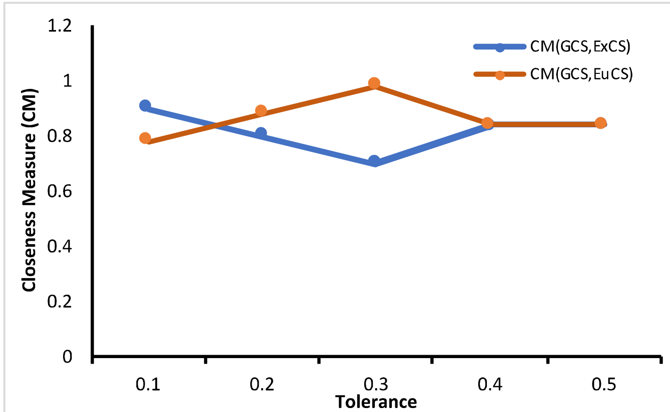

Fig. 4.5: Effect of θ on the Closeness Measure

4.5 Summary

This work developed a novel GDM model to address the two issues of existing GDM models. The first issue is that multiple rounds of feedback recommendations in the consensus reaching process (CRP) makes GDM inefficient. The second issue is no involvement of heterogeneous decision-makers (DMs), possibly end-users as the stakeholders apart from the experts. To address these issues, the contributions of this paper are as follows:

1. A novel threshold-based feedback mechanism is introduced to improve the efficiency of the CRP. Experts reach consensus in at most one round of feedback using the proposed mechanism.

2. A method is designed to get the cumulative opinion of end-users that represents the majority group opinion while preserving the extreme opinions of end-users.

3. A novel concept of moderator's tolerance is introduced to obtain the final decision. The final decision considers the consensual opinion of experts and the cumulative opinion of end-users. The tolerance threshold defines the acceptable deviation of the

final decision from the consensual opinion of experts in order to preserve the opinions of the experts.

In the proposal, the preference structures of the experts and the users are considered fuzzy preference relations. However, other more complex preference structures such as hesitant fuzzy linguistic preference relation (HFLPR) or hesitant fuzzy preference relation (HFPR) can be used directly in the proposed GDM model. Also, we verified the effectiveness of the proposed model using an example of the healthcare system. A few important observations obtained while analyzing the performance of the proposed GDM are as follows:

(a) The feedback parameter is important and depends on threshold λ and degree of deviation $D(V^p, V^{GD})$.

(b) Experts' opinion is not entirely ignored in the proposed method.

(c) The whole process reduces to the traditional GDM with experts when the number of service users is zero and the role of tolerance degree of the moderator goes off.

(d) The proposed method behaves like classical GDM that works through aggregation and comprehension of the users' experience for decision making when the number of experts becomes zero.

The proposed model relies on the additional arrangement of working on users' opinions apart from the traditional models based solely on experts' opinions. We understand that the additionality may be performed when the users' opinions based on their own experiences are perceived to be useful in decision making. One limitation of the proposed work is that it does not address the non-cooperation of the experts. However, there may be experts who may exhibit non-cooperation for convergence because of reluctance to accept the feedback provided [75]. Compared to the traditional GDM models, the proposed CRP model is not much interactive as it reduces the number of rounds of discussions.