

Conclusion and Future Scope

We generalized the class of $C1$ modules to pure $C1$ modules, the class of $C3$ modules to pure $C3$ modules, and the class of $C2$ modules to Finite $C3$ modules. Also, we generalized the class of $D2$ modules to copure $D2$ modules.

This thesis provides a comprehensive understanding of different types of generalizations of $C1$, $C2$, $C3$, and $D2$ modules and their characterizations over several rings. In Chapter 2, we studied the generalization of $C1$ modules as Pure $C1$ modules and characterized semi-simple rings, von Neumann regular rings, local rings, and PDS rings in terms of pure $C1$ modules. Chapter 3 deals with the new notion of pure $C3$ modules which is the generalization of $C3$ modules, and characterizes many rings in terms of pure $C3$ modules. Also, we introduced pure $C3$ envelopes and pure $C3$ covers of a module. Chapter 4 explores the new notion of Finite $C3$ modules which is the generalization of $C2$ modules and characterized hereditary rings, von Neumann regular rings, and SI-rings, etc. Also, we introduced finite $C3$ envelopes and finite $C3$ covers of a module. Chapter 5 deals with the new notion of copure $D2$ modules and explores direct summands and direct sums of copure $D2$ modules. We gave new characterizations of copure semi-simple rings, and copure hereditary rings with respect to copure $D2$ modules.

Future Scope

During the study, we found a number of research problems that are unavailable in the literature on which work can be done in the future. We want to enlist a few of them as follows:

1. Generalize the class of $C3$ modules in terms of copure modules, which can be another generalization of $C2$ modules.
2. Generalize the class of pure $D2$ modules to pure $D3$ modules, which can be another generalization of $D2$ modules.
3. Generalize the class of finite $D2$ modules to Finite $D3$ modules, which can be another generalization of $D2$ modules.
4. Generalize the class of $D3$ modules in terms of copure modules, which can be another generalization of $D2$ modules.