CHAPTER 7

STATISTICAL MODELLING OF TRIBOLOGICAL PARAMETERS USING RSM AND ANN

7.1 INTRODUCTION

The present chapter focuses on predictions of tribological properties (wear and COF) in both dry and lubricating sliding conditions at any given set of input variables through a statistical modelling tool. Experiments are conducted based on central composite design (CCD) matrix through Response surface methodology using design expert 13 software package. A significant model is generated based on a best-fit curve to obtain a master regression equation for both wear and COF.

We use this regression equation for predicting wear and COF at any given set of input based on applications. Regression equation helps reducing large number of experiments, eventually we save overall resources and time.

Artificial neural network using Python is also being used to do the statistical modelling to predict the tribological properties of the present material system for validation. Comparison of experimental with predicted wear and COF by ANN has been done. The run number is taken based on CCD.

7.2 STATISTICAL MODELLING OF WEAR AND COF IN DRY SLIDING CONDITION

7.2.1 Prediction of Wear and COF using Response Surface Methodology (RSM)

7.2.1.1 Central composite design (CCD)

Tables 7.1 and 7.2 present the design matrix of 20 iterations for process parameters at three different levels employing the central composite design (CCD) technique, which is used to determine wear and coefficient of friction (COF) of alloy and composites. ANOVA (Analysis of variance) analysis suggested by response surface methodology (RSM) using design of expert 13 software package is employed which suggested a quadratic model to examine the wear and COF response.

Table 7.1 Central composite design (CCD) table with experimental value of wear

Std.	**Run**	**1st Factor A: Load (N)**	**2nd Factor B: Sliding Distance (m)**	**3rd Factor C: ZrB_2 (vol.%)**	**Response Wear (mm^3)**
7	1	10	5000	9	1.3
14	2	30	3000	9	3
15	3	30	3000	4.5	3.8
4	4	50	5000	0	24
10	5	50	3000	4.5	6
19	6	30	3000	4.5	4
17	7	30	3000	4.5	4.2
13	8	30	3000	0	13
16	9	30	3000	4.5	3
9	10	10	3000	4.5	0.8
11	11	30	1000	4.5	1.1
12	12	30	5000	4.5	7
20	13	30	3000	4.5	3.3
3	14	10	5000	0	9
18	15	30	3000	4.5	4
8	16	50	5000	9	5.5
1	17	10	1000	0	0.9
2	18	50	1000	0	11
5	19	10	1000	9	0.22
6	20	50	1000	9	1.1

Table 7.2 Central composite design (CCD) table with experimental value of COF

Std.	**Run**	**1st Factor** **A: Load (N)**	**2nd Factor** **B: Sliding Distance (m)**	**3rd Factor** **C: ZrB_2 (Vol.%)**	**Response** **COF**
9	1	10	3000	4.5	0.4635
5	2	10	1000	9	0.430
12	3	30	5000	4.5	0.5635
18	4	30	3000	4.5	0.5430
10	5	50	3000	4.5	0.5970
11	6	30	1000	4.5	0.4850
15	7	30	3000	4.5	0.550
8	8	50	5000	9	0.648
20	9	30	3000	4.5	0.540
16	10	30	3000	4.5	0.555
6	11	50	1000	9	0.550
4	12	50	5000	0	0.569
7	13	10	5000	9	0.526
3	14	10	5000	0	0.470
2	15	50	1000	0	0.486
14	16	30	3000	9	0.580
17	17	30	3000	4.5	0.541
19	18	30	3000	4.5	0.548
13	19	30	3000	0	0.510
1	20	10	1000	0	0.326

7.2.1.2 Quadratic model and analysis of variance for wear and COF

The quadratic coefficients were determined through ANOVA and analyzed each factor of wear and COF for alloy and composites. Tables 7.3 and 7.4 present model summary statistics and suggest quadratic model. **R^2 = 99.19%** & adjusted **R^2 = 98.46%** for wear and **R^2 = 98.86%** & adjusted **R^2 = 97.84%** for COF suggest that model is consistent with total variance. Tables 7.5 and 7.6 present the ANOVA for both wear and COF of quadratic model **P-values less than 0.0001** and lack of fit "**not significant**" clearly suggest that the model is statistically fit for predictions and optimization of wear and COF.

Table 7.3 Statistics model summary for wear

Source	**Std. Deviation**	**R^2**	**Adj. R^2**	**Pred. R^2**	**Press**	
Linear	3.04	0.7513	0.7046	0.4960	303.53	
2FI	2.12	0.9022	0.8570	0.5781	251.39	
Quadratic	**0.6960**	**0.9919**	**0.9846**	**0.9463**	**31.97**	**Suggested**
Cubic	0.5654	0.9968	0.9898	-0.7138	1021.24	Aliased

Table 7.4 Statistics model summary for COF

Source	**Std. Deviation**	**R^2**	**Adj. R^2**	**Pred. R^2**	**Press**	
Linear	0.0249	0.8890	0.8682	0.4960	0.0183	
2FI	0.0267	0.8958	0.8477	0.5781	0.0706	
Quadratic	**0.0101**	**0.9886**	**0.9784**	**0.8378**	**0.0144**	**Suggested**
Cubic	0.0056	0.9979	0.9933	-0.7397	0.0232	Aliased

Table 7.5 Quadratic model of ANOVA for wear

Source	Sum of Square	d_f	Mean square	F-Value	p-Value	
Model	**591.04**	**9**	**65.67**	**135.56**	**< 0.0001**	**significant**
A-Load	124.61	1	124.61	257.23	< 0.0001	
B-Sliding distance	104.98	1	104.98	216.70	< 0.0001	
C-Composition	218.09	1	218.09	450.19	< 0.0001	
AB	8.61	1	8.61	17.78	0.0018	
AC	50.50	1	50.50	104.25	< 0.0001	
BC	30.81	1	30.81	63.60	< 0.0001	
A^2	1.84	1	1.84	3.80	0.0798	
B^2	0.0778	1	0.0778	0.1606	0.6971	
C^2	39.33	1	39.33	81.19	< 0.0001	
Residual	4.84	10	0.4844			
Lack of Fit	**3.76**	**5**	**0.7512**	**3.45**	**0.1001**	**Not significant**
Pure Error	1.09	5	0.2177			
Cor Total	595.89	19				

Table 7.6 Quadratic model of ANOVA for COF

Source	Sum of square	d_f	Mean square	F-Value	p-Value	
Model	**0.0880**	**9**	**0.0098**	**96.58**	**< 0.0001**	**significant**
A-Load	0.0403	1	0.0403	397.70	< 0.0001	
B-Sliding distance	0.0250	1	0.0250	246.47	< 0.0001	
C-Composition	0.0139	1	0.0139	137.44	< 0.0001	
AB	0.0004	1	0.0004	4.30	0.0649	
AC	0.0000	1	0.0000	0.3569	0.5635	
BC	0.0001	1	0.0001	1.34	0.2731	
A^2	0.0009	1	0.0009	9.09	0.0130	
B^2	0.0016	1	0.0016	16.04	0.0025	
C^2	0.0000	1	0.0000	0.3415	0.5719	
Residual	0.0010	10	0.0001			
Lack of Fit	**0.0008**	**5**	**0.0002**	**4.93**	**0.0524**	**Not significant**
Pure Error	0.0002	5	0.0000			
Cor Total	0.0890	19				

7.2.1.3 Regression equation obtained from RSM

Normal probability diagram given in Figs. 7.1 and 7.2 for wear and COF indicate no scattering and all the residues lie in a straight line with errors being placed normally. Importance of each parameter was assessed. The composition of ZrB_2 was found to be the most influencing parameters followed by load and sliding distance for wear, while the most contributing factor for COF was found to be load followed by sliding distance and composition. Statistical model summary for wear and COF with **R^2 = 99. 20 %** and **R^2 = 98. 86% respectively** suggest that the model is compatible and the **P- value less than 0.05** for both wear and COF reveals that the model is statistically significant for predictions.

The obtained regression equations for both wear and COF which can be utilized to predict wear and COF at any given inputs are given below:

Wear *= -4.4104 + 0.348235 * A + 0.00208735 * B + -1.23375 * C + 2.56875e-05 * AB + -0.0278056 * AC + -0.000216944 * BC + -0.00205455 * A^2 + -4.29545e-08 * B^2 + 0.186577 * C^2*

$$\mathbf{COF} = 0.214374 + 0.00657619 * A + 6.90119e\text{-}05 * B + 0.011948 * C + -1.84375e\text{-}07 * AB + -2.36111e\text{-}05 * AC + -4.58333e\text{-}07 * BC + -4.57386e\text{-}05 * A^{\wedge}2 + -6.07386e\text{-}09 * B^{\wedge}2 + -0.000175084 * C^{\wedge}2$$

where, A denotes- Load (N), B denotes- Sliding distance and C denotes- Composition

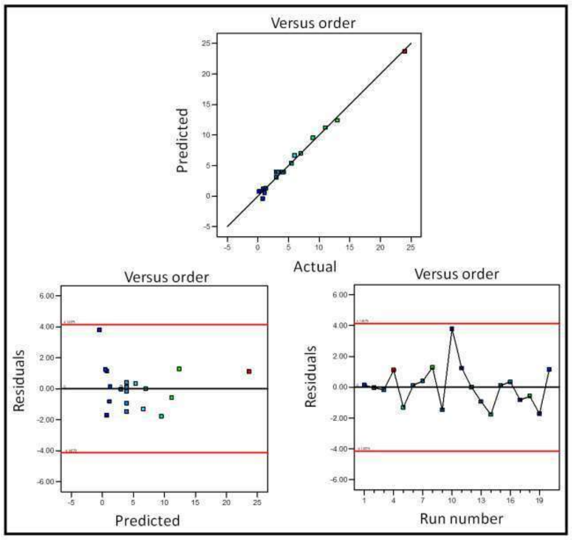


Fig. 7.1 Normal probability for wear

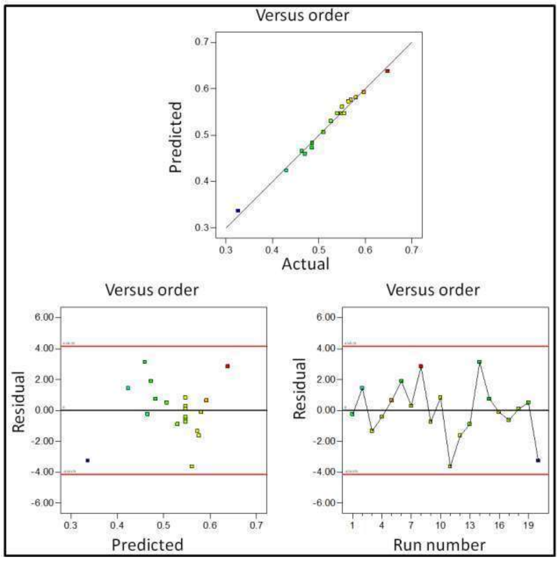


Fig. 7.2 Normal probability for residual of COF

Figures 7.3 and 7.4 present the 2D and 3D surface contour plots of change in wear and COF with sliding distance, load and vol.% of ZrB_2. It can be seen from the contours that sliding distance, load and vol.% of ZrB_2 play a key role in both wear and COF behaviour. Both wear and COF rise with rise in sliding distance and load. In contrast, this can be seen in Fig. 7.3(a-c) that with rise in ZrB_2 vol.%, the wear decreases while the COF value increases (Fig. 7.4(a-c)). Obtained contour plots for wear and COF are in agreement with the results obtained in Chapter 4 [Vineet et al., 2022(b)].

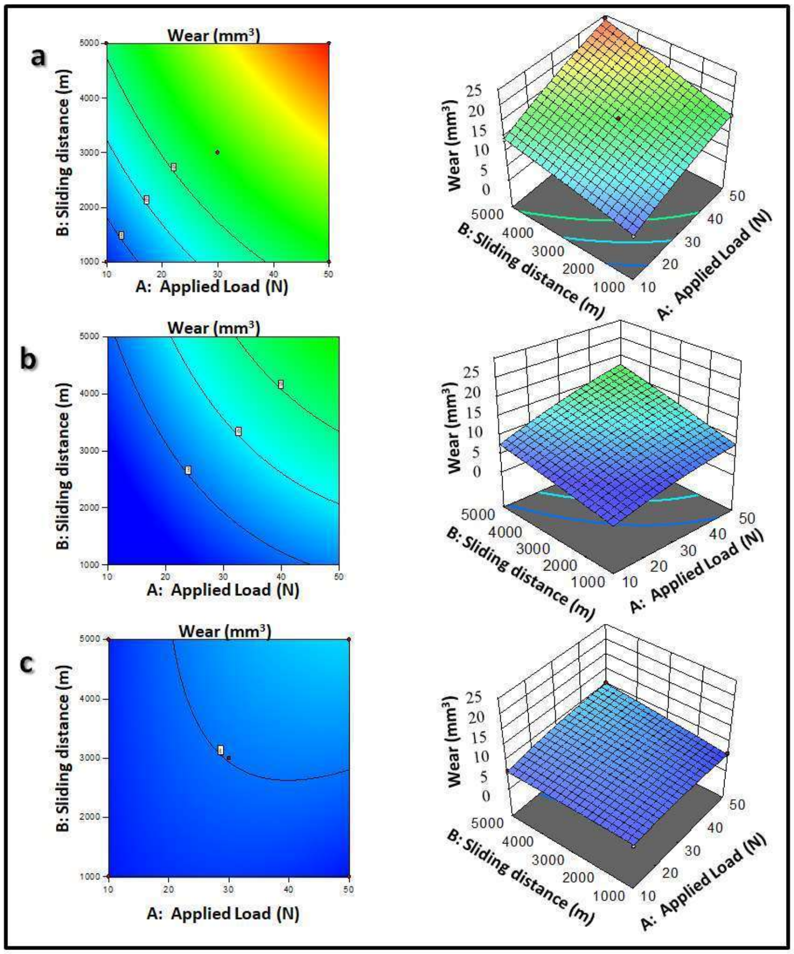


Fig. 7.3 2D and 3D- surface contour plot showing wear variation with sliding distance and applied load for (a) C 0.0 (b) C4.5 (c) C9.0

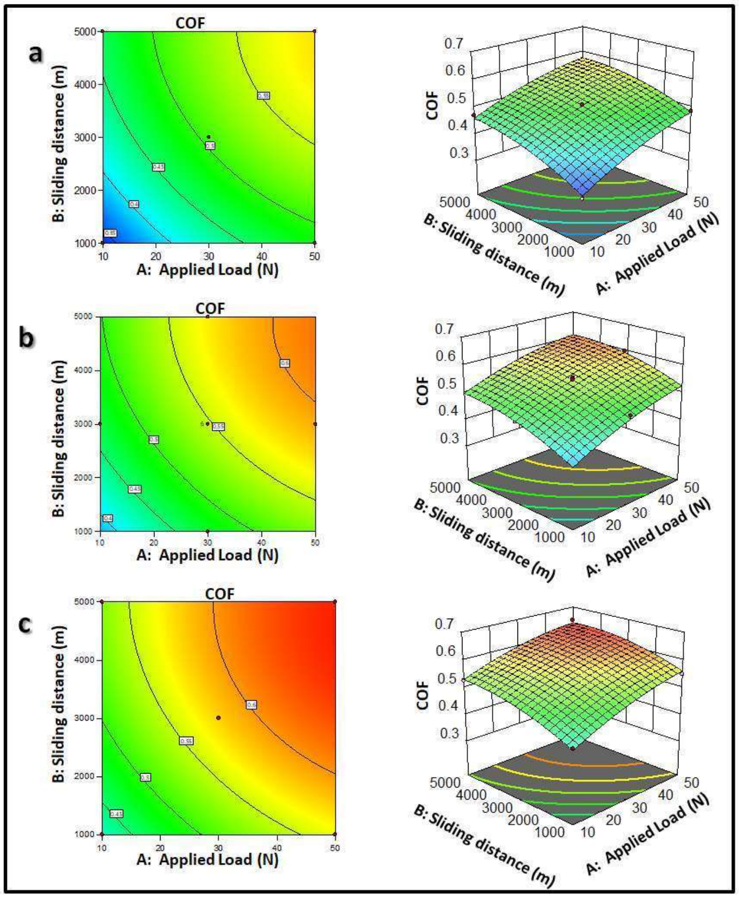


Fig. 7.4 2D and 3D- surface contour plot showing COF variation with sliding distance and applied load for (a) C0.0 b) C4.5 (c) C9.0

7.2.2 Prediction of Wear and COF using Artificial Neural Network (ANN)

All data has been categorized in two parts: (i) training State (data set ~80% and (ii) testing (remaining data set ~20%). During training, every neuron gets the input signal from other neuron, and assigns weight of the variables to each of these inputs. The weight of the variables is adjusted and multiple iterations are carried out to minimize error. Table 7.7 presents the training parameters used for current study for both wear and COF. After training the network, testing data set is used to check the accuracy of network model and its error percentage. Figures. 7.5 and 7.6 present the ANN training and test parts for wear and

COF and clearly reveal that R2- value is very close to one (1) which confirms the validity of ANN model for experimental sets of data [Vineet et al., 2022(c)].

Table 7.7 Parameters taken for ANN training

Training Parameters	**Values**
No. of Iterations	10000
Learning Parameters	0.001
No. of layers	3
No. of neurons in input layer	3
No. of neurons in hidden layer	20
No. of neurons in output layer	1

A well-defined ANN can be used to forecast new outcomes within the identical knowledge domain. Figures 7.7 and 7.8 present the anticipation level of ANN structures based on mean square error and selected data based on CCD obtained from RSM to show the experimental vs. predicted graph. It has been found that the mean error for experimental sets is ~6% for wear and ~5% for COF. This clearly suggests the significance of model to predict the tribological properties.

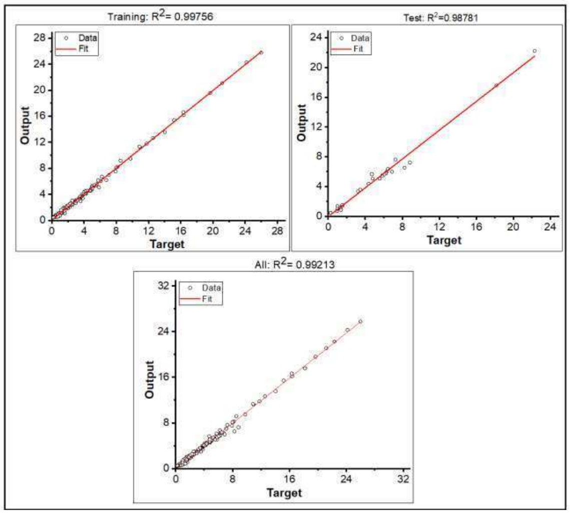


Fig. 7.5 ANN training and test results for wear

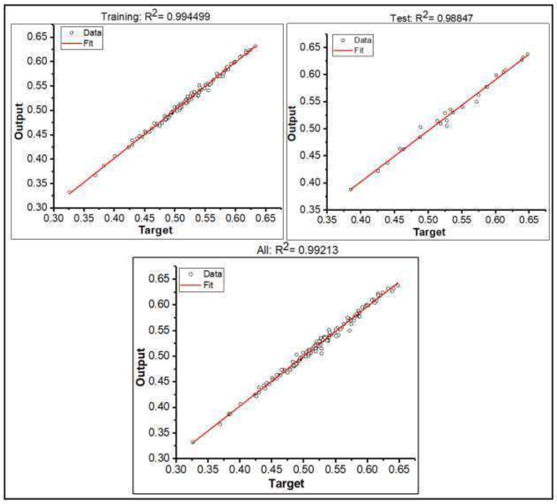


Fig. 7.6 ANN training and test results for COF

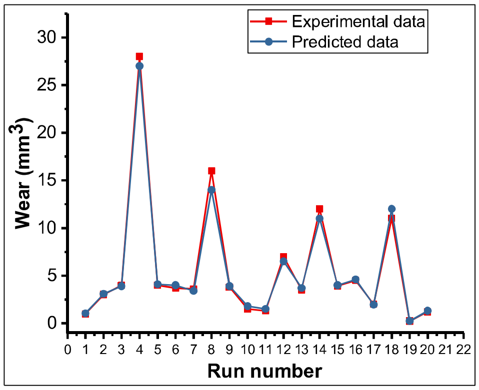


Fig. 7.7 Wear comparison between experimental and predicted value of ANN

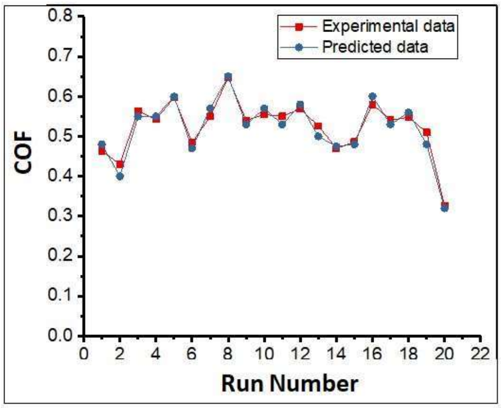
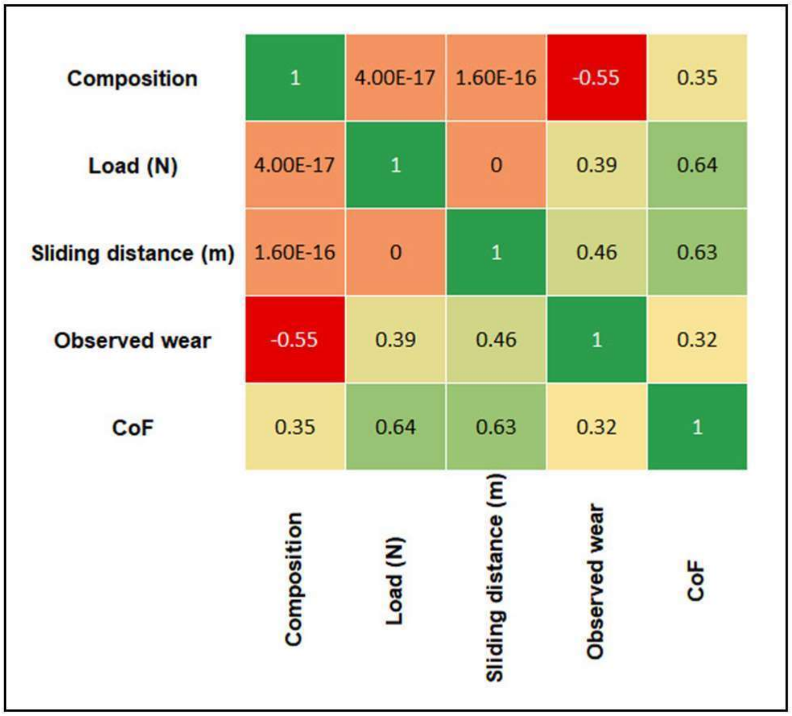


Fig. 7.8 COF comparison between experimental and predicted value of ANN

Figure 7.9 presents the correlation matrix obtained using ANN and shows the correlation coefficient i.e. wear and COF with applied load, sliding distance, and composition. The correlation matrix has been presented here to clearly demonstrate the effect and sensitivity of these input variables on wear and COF. This can be seen that the wear is negatively influenced by the composition which suggests that wear is decreasing with an increase in vol.% of ZrB_2. Sliding distance and applied load positively influence the wear which suggests an increase in wear with increasing sliding distance and load. Amount of ZrB_2 is the most contributing factor in influencing the wear. However, COF is positively influenced by composition, load, and sliding distance and load is most contributing parameter for increase in COF followed by sliding distance and composition.



	Composition	Load (N)	Sliding distance (m)	Observed wear	CoF
Composition	1	4.00E-17	1.60E-16	-0.55	0.35
Load (N)	4.00E-17	1	0	0.39	0.64
Sliding distance (m)	1.60E-16	0	1	0.46	0.63
Observed wear	-0.55	0.39	0.46	1	0.32
CoF	0.35	0.64	0.63	0.32	1

Fig.7.9 Correlation matrix of tribological properties with varying parameters

Figure 7.10 presents the 3D surface contour plots for wear and COF predicted using ANN. This can be clearly observed that with increasing load and sliding distance, both wear and COF values increase, while with increasing composition from 0 vol.% to 9 vol.%, the wear value decreases but COF value increases. This is also in agreement with correlation matrix (Fig. 7.9).

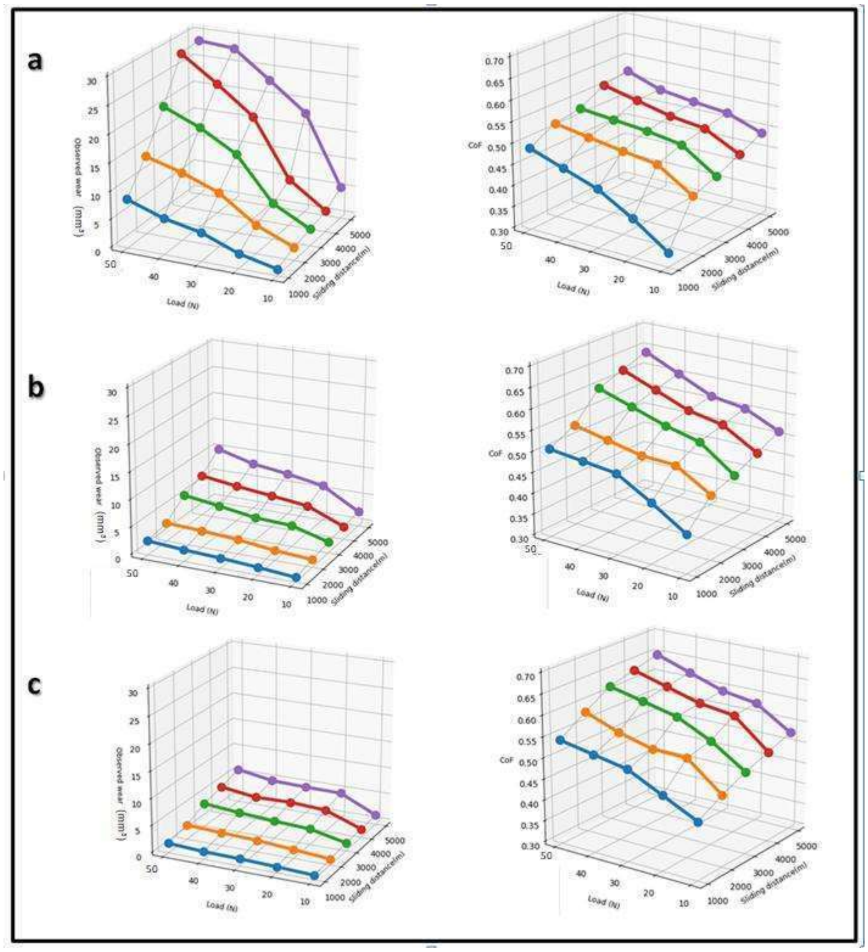


Fig. 7.10 3D- surface contour plot showing wear and COF variation with sliding distance and applied load for (a) C0.0 (b) C4.5 (c) C9.0

This can be observed that the predicted value of wear and COF by ANN is showing similar trends and in close tolerance with values predicted by the RSM model (Figs. 7.3, 7.4, and 7.10). Also, the correlation matrix shows the similar contribution factor as suggested by ANOVA model using RSM (Tables 7.5 and 7.6 and Fig. 7.10). This can be concluded that both the models are significant and efficient to predict the tribological properties of the present material system.

Further, to verify the efficacy of both the models, a set of random input variables (20 N load, 4000 m sliding distance and C3.0 composition) has been selected and at these input variables wear and COF values were determined using regression equations obtained from RSM. To check the accuracy of the predictions, further, the tribological test is conducted at 20 N load and 4000 m sliding distance for C3.0 composition (Three tests were conducted to verify the repeatability of data). The obtained experimental values (also given in chapter 4) and predicted values are given in Table 7.8. This can be observed that experimental and predicted values by RSM of wear and COF are very close and having **less than 5% error.** This further confirms that both the models can be used for the predictions of tribological properties at any given set of inputs.

Table 7.8 Predicted and experimental values of the wear and COF

	Predicted value by RSM		**Experimental value**	
Input parameters	Wear (mm^3)	COF	Wear (mm^3)	COF
Load -20 N Sliding distance - 4000 m Composition - C3.0	5.15	0.5191	5.25 ± 0.3	0.5190 ± 0.025

7.2.3 Optimization and Verification of the Model

The objective of verifying the model is achieved by analyzing the predicted outcomes of the model with experimentally obtained results. RSM was utilized to get the maximum amount of information with minimum time and experiments. To examine the influence of load, sliding distance, and ZrB_2 vol.% on wear and COF, a 3D contour surface is plotted. 3D contour helps in determining correlation among parameters and to gain their optimum values for least wear and COF. The effect of input parameters as well as maximum and minimum points for wear are discussed in detail in chapter 4 [Vineet et al., 2022(a)]. The correlation between input parameters at lowest load, lower sliding distance, and highest vol.% of ZrB_2 are the most optimized parameters for which least wear was predicted, while the lowest load & sliding distance and ZA alloy are the most optimized parameters for which least COF was predicted. The ANN shows the correlation between input and output parameters. ANN investigation potential is dependent on the comparison of experimental values and predicted values. It was essential to check the neural model capabilities to analyze and forecast input parameters' collective influence on composites' tribological properties. Based on neural network examination, ANN was found to have good performance in predicting wear and COF. The optimization is carried out using the Design Expert 13 (DOE) software. It is observed that with the rise in load and sliding distance, the wear value also increases. However, with an increase in ZrB_2 vol.%, the wear value decreases. Table 7.8 clearly indicates the errors evaluated are very small, thus, the model is satisfactory and fit for modelling and predictions of tribological properties [Vineet et al., 2022(c)].

7.3 STATISTICAL MODELLING OF WEAR AND COF IN LUBRICATING SLIDING CONDITION

7.3.1 Prediction of Wear and COF using Response Surface Methodology (RSM)

7.3.1.1 Central composite design (CCD)

Tables 7.9 and 7.10 present the design matrix of 20 iterations for process parameters at three different levels. This has been done using CCD technique to determine wear and COF of alloy and composites. Design of expert 13 software package is used for RSM for analysis of variance (ANOVA) which suggests quadratic models as best fit curve to investigate the wear and COF (response).

Table 7.9 Central composite design (CCD) table with experimental value of wear

Standard	**Run**	**1st Factor A:Load (N)**	**2nd Factor B: Sliding Distance (m)**	**3rd Factor C:ZrB$_2$ (vol.%)**	**Response Wear ×10^{-2} (mm^3)**
19	1	30	3000	4.5	17
15	2	30	3000	4.5	19
10	3	50	3000	4.5	25
6	4	50	1000	9	8
5	5	10	1000	9	2
7	6	10	5000	9	7
18	7	30	3000	4.5	18
20	8	30	3000	4.5	16.5
14	9	30	3000	9	11
1	10	10	1000	0	12
12	11	30	5000	4.5	24
13	12	30	3000	0	45
8	13	50	5000	9	20
11	14	30	1000	4.5	11
3	15	10	5000	0	37
17	16	30	3000	4.5	18
16	17	30	3000	4.5	16.5
2	18	50	1000	0	52
9	19	10	3000	4.5	5
4	20	50	5000	0	80

Table 7.10 Central composite design (CCD) table with experimental value of COF

Standard	**Run**	**1st Factor A:Load (N)**	**2nd Factor B:Sliding Distance (m)**	**3rd Factor C:ZrB_2 (vol.%)**	**COF $\times 10^{-3}$**
7	1	10	5000	9	17
15	2	30	3000	4.5	27
1	3	10	1000	0	18
4	4	50	5000	0	50
19	5	30	3000	4.5	28
17	6	30	3000	4.5	26
20	7	30	3000	4.5	27.5
6	8	50	1000	9	23
16	9	30	3000	4.5	26.5
13	10	30	3000	0	39
18	11	30	3000	4.5	25
2	12	50	1000	0	37
9	13	10	3000	4.5	17
12	14	30	5000	4.5	30
3	15	10	5000	0	23
5	16	10	1000	9	9
14	17	30	3000	9	22
10	18	50	3000	4.5	34
11	19	30	1000	4.5	24
8	20	50	5000	9	32

7.3.1.2 Quadratic model and analysis of variance for wear

The quadratic coefficients were determined through ANOVA and analyzed each factor of wear and COF for alloy and composites. Tables 7.11 and 7.12 present model summary statistics and suggest quadratic model and **R^2 = 99.54%** and **97.87%** respectively for wear and COF, which clearly suggest that model is consistent with total variance. Tables 7.13 and 7.14 present the ANOVA for quadratic model **P-values less than 0.0001** and lack of fit "**not significant**" for both wear and COF. This clearly suggests that the model is statistically fit for optimization.

Table 7.11 Statistics model summary for wear

Source	Std. Dev.	R^2	Adj. R^2	Pred. R^2	Press	
Linear	8.90	0.8084	0.7725	0.6073	2597.10	
FI	6.68	0.9122	0.8717	0.5076	3256.19	
Quadratic	**1.74**	**0.9954**	**0.9913**	**0.9655**	**227.99**	**Suggested**
Cubic	1.03	0.9990	0.9970	0.7507	1648.43	Aliased

Table 7.12 Statistics model summary for COF

Source	Std. Dev	R^2	Adj. R^2	Pred. R^2	Press	
Linear	2.60	0.9294	0.9162	0.8657	205.74	
2FI	2.18	0.9597	0.9411	0.8502	229.56	
Quadratic	**1.81**	**0.9787**	**0.9594**	**0.8094**	**292.12**	**Suggested**
Cubic	1.37	0.9926	0.9766	-3.3941	6732.81	Aliased

Table 7.13 Quadratic model of ANOVA for wear

Source	Sum of square	d_f	Mean square	F-Value	p-Value	
Model	**6582.56**	**9**	**731.40**	**242.70**	**< 0.0001**	**Significant**
A-Load	1488.40	1	1488.40	493.89	< 0.0001	
B-Sliding distance	688.90	1	688.90	228.59	< 0.0001	
C-ZrB_2	3168.40	1	3168.40	1051.35	< 0.0001	
AB	12.50	1	12.50	4.15	0.0690	
AC	512.00	1	512.00	169.89	< 0.0001	
BC	162.00	1	162.00	53.76	< 0.0001	
A^2	9.55	1	9.55	3.17	0.1054	
B^2	1.11	1	1.11	0.3695	0.5568	
C^2	341.05	1	341.05	113.17	< 0.0001	
Residual	30.14	10	3.01			
Lack of Fit	**25.14**	**5**	**5.03**	**5.03**	**0.0504**	**Not significant**
Pure Error	5.00	5	1.0000			
Corrected Total	6612.70	19				

Table 7.14 Quadratic model of ANOVA for COF

Source	Sum of square	d_f	Mean square	F-Value	p-Value	
Model	**1499.54**	**9**	**166.62**	**50.94**	**< 0.0001**	**Significant**
A-Load	846.40	1	846.40	258.78	< 0.0001	
B-Sliding Distance	168.10	1	168.10	51.40	< 0.0001	
C-ZrB2	409.60	1	409.60	125.23	< 0.0001	
AB	10.13	1	10.13	3.10	0.1090	
AC	36.13	1	36.13	11.05	0.0077	
BC	0.1250	1	0.1250	0.0382	0.8489	
A^2	16.57	1	16.57	5.07	0.0481	
B^2	2.51	1	2.51	0.7661	0.4020	
C^2	17.82	1	17.82	5.45	0.0418	
Residual	32.71	10	3.27			
Lack of Fit	**26.87**	**5**	**5.37**	**4.61**	**0.0595**	**not significant**
Pure Error	5.83	5	1.17			
Cor Total	1532.25	19				

7.3.1.3 Regression equation obtained from RSM

Normal probability for wear and COF given in Figs 7.11 and 7.12 indicates no scattering and all the residues lie in a straight line with errors being placed normally. From ANOVA analysis, importance of each parameter was measured. For wear, the content of ZrB_2 is found to be the most influencing parameters followed by applied load and sliding distance. While for COF, the most influencing factor is applied load followed by ZrB_2 content and sliding distance. Statistical model summary for wear and COF with **R^2 = 99. 54%** and **R^2 = 97. 87%** respectively suggest that the model is compatible and **P- value less than 0.05** for both wear and COF reveal that the model is statistically significant for predictions.

The obtained regression equations for both wear and COF which can be further utilized to predict the wear and COF at any given inputs are given below:

$$\textbf{Wear} = (-3.26705 + 1.1958 * A + 0.00450795 * B - 4.73838 * C + 3.125e\text{-}05 * A*B - 0.0888889 *A*C - 0.0005 *B*C -0.00465909 * A^\wedge 2 + 1.59091e\text{-}07 * B^\wedge 2 + C^\wedge 2)*10^{-2}$$

$$\textbf{COF} = (7.66307 + 0.850057 * A + 0.00270057 * B - 1.80354 * C + 2.8125e\text{-}05 * AB -0.0236111 * AC - 1.38889e\text{-}05 * BC -0.00613636 * A^\wedge 2 -2.38636e\text{-}07 * B^\wedge 2 + 0.125701 * C^\wedge 2) \times 10^{-3}$$

where, A- load, B-Sliding distance and C- ZrB_2 Vol.%.

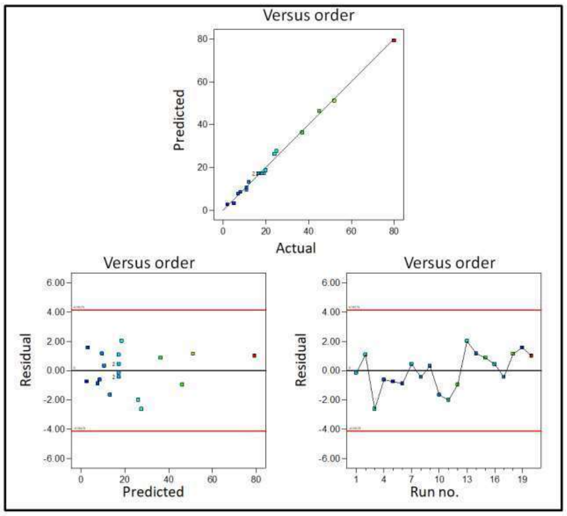


Fig. 7.11 Normal probability for residual of wear

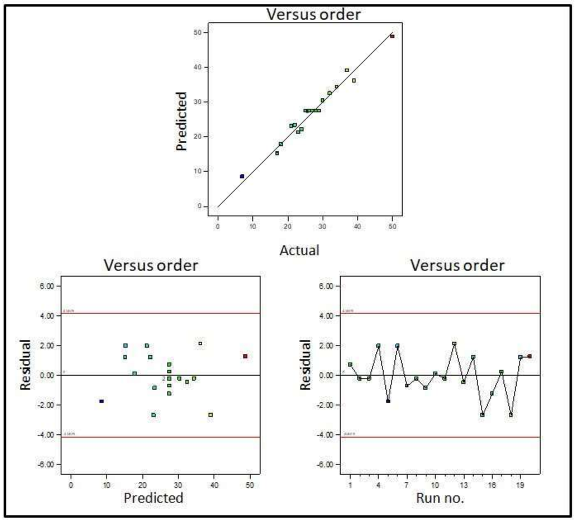


Fig.7.12 Normal probability for residual of COF

Figures 7.13 and 7.14 present 2D and 3D surface contour plots showing change in wear and COF with sliding distance, load and varying vol.% of ZrB_2. It can be seen from the contours that an increase in sliding distance, load, and reinforced ZrB_2 vol.% plays a key role in tribological behavior. With increase in load and sliding distance, the loss of material increases linearly; while with increase in reinforcement, both wear (volume loss) and COF decrease linearly. The observations from wear and COF properties in chapter 5 clearly validate tribological behavior obtained by the RSM model indicated in Figs 7.13 and 7.14.

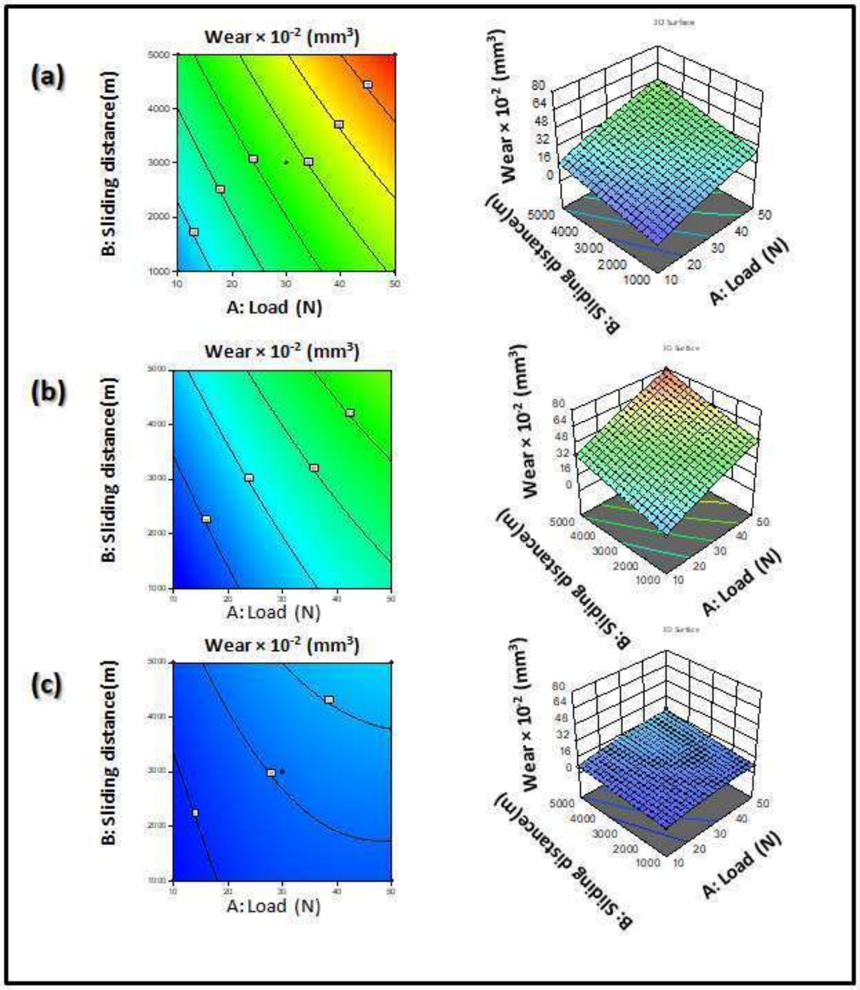


Fig. 7.13 2D and 3D- surface contour plot showing wear variation with sliding distance and applied load for (a) C0.0 (b) C4.5 (c) C9.0

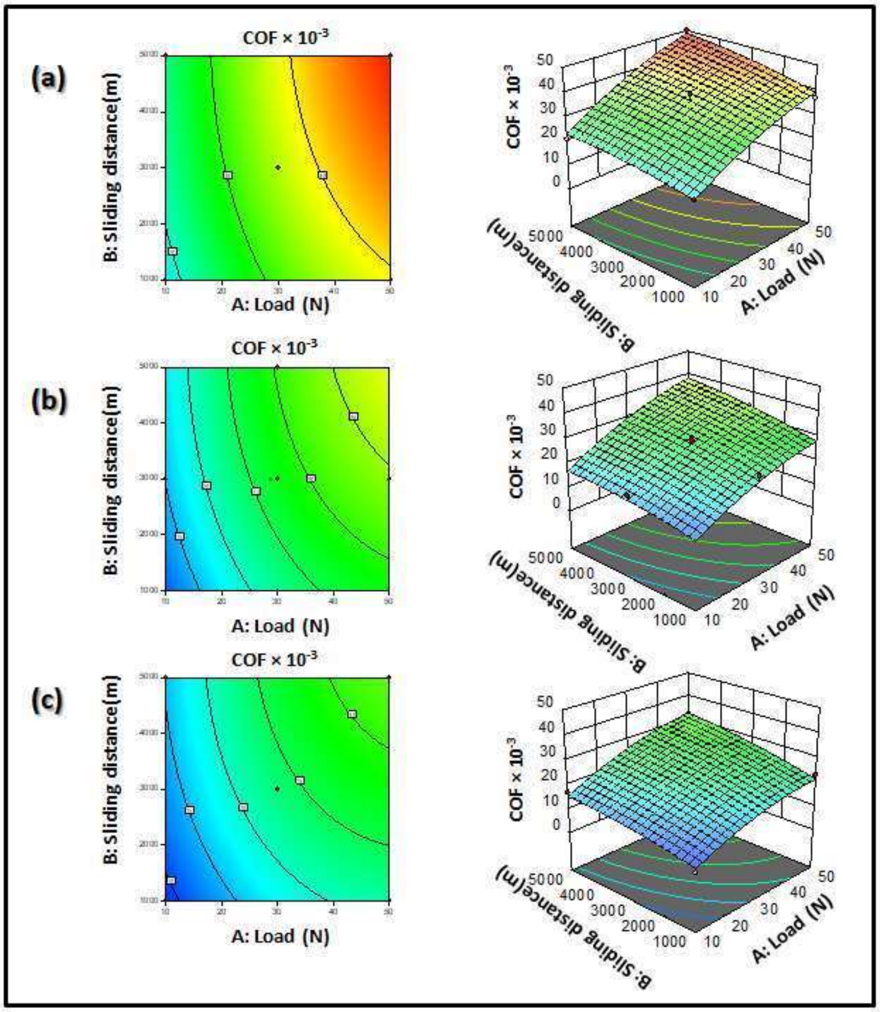


Fig. 7.14 2D and 3D- surface contour plot showing wear variation with sliding distance and applied load for (a) C0.0 (b) C4.5 (c) C9.0

7.3.2 Prediction of Wear and COF using ANN

All the data is categorized into two parts: (i) training State (data set ~80% and (ii) testing (Remaining data set ~20%). During training, every neuron gets the input signals from other neurons and assigns the weight of the variables to each of these inputs. The weight of the variables is adjusted and multiple iterations are carried out to minimize error. Table 7.15 presents the training parameters used for the current study for both wear and COF. After training, the network testing data set is used to check the accuracy of the network model and its error percentage. Figures 7.15 and 7.16 present the ANN training and test parts for wear and COF and reveal that R^2-value is very close to one (1) which confirms the validity of the ANN model for experimental sets of data. A well-defined ANN may be utilized to forecast new outcomes within the identical knowledge domain. Figures 7.17 and 7.18 present the anticipation level of ANN structures based on mean square error and selected data based on CCD obtained from RSM to show the experimental vs. predicted graph. It has been found that for wear, the error for experimental sets is in the range of 1 to 6.25% with an average %age error of 3% and the error for COF ranges from 1 to 8.33%, with an average %age error of 4%. This suggests the significance of the model to predict the tribological properties.

Table 7.15 Parameters taken for ANN training

Training Parameters	**Values**
No. of Iterations	10000
Learning Parameters	0.001
No. of layers	3
No. of neurons in input layer	3
No. of neurons in hidden layer	20
No. of neurons in output layer	1

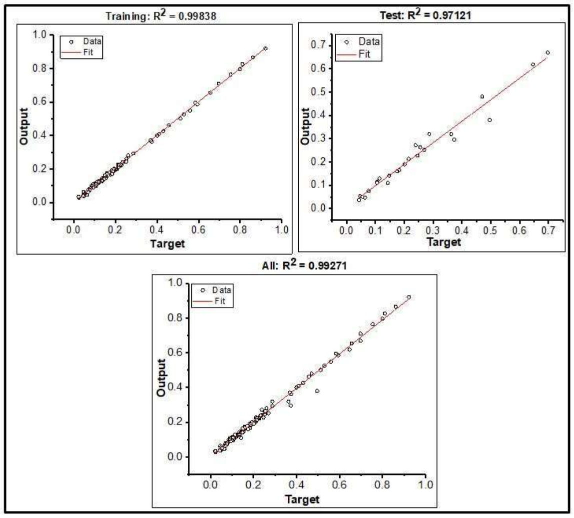


Fig. 7.15 ANN training and test results for wear

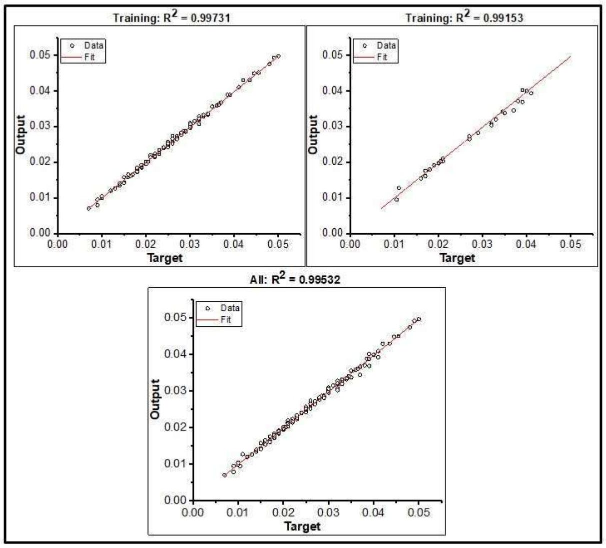


Fig. 7.16 ANN training and test results for COF

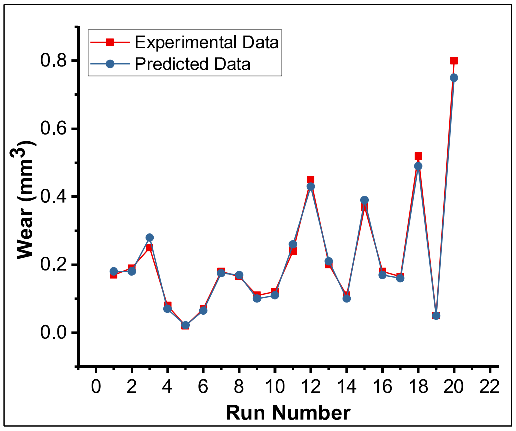


Fig. 7.17 Wear comparison between experimental and predicted value of ANN

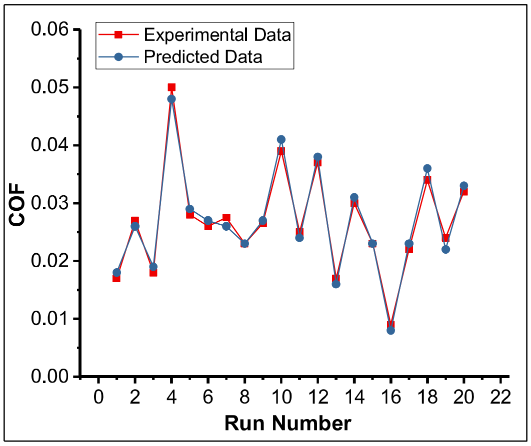
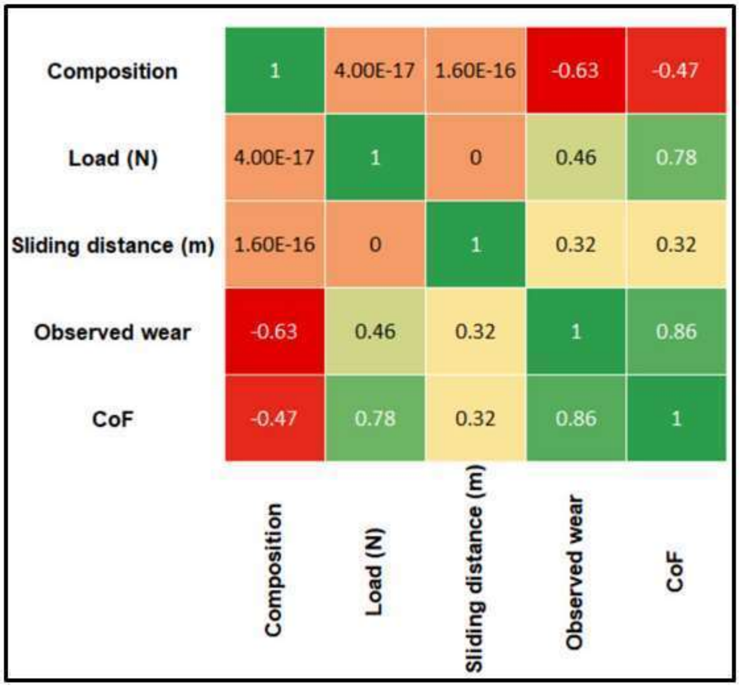


Fig. 7.18 COF comparison between experimental and predicted value of ANN

Figure 7.19 presents the correlation matrix obtained using ANN and shows the correlation coefficient i.e. wear and COF with applied load, sliding distance, and composition. A correlation matrix demonstrates the effect and sensitivity of these variables on wear and COF. This can be seen that wear is negatively influenced by the composition which suggests that wear is decreasing with an increasing vol.% of ZrB_2 and is the biggest contributing factor in influencing the wear. Sliding distance and applied load positively influence the wear which indicates an increase in wear with increasing sliding distance and load. In lubricating sliding condition, a contrast behavior of COF variation is observed (also discussed in chapter 5). COF is also negatively influenced by composition. Load is the biggest contributing parameter for increase in COF followed by sliding distance and composition.



	Composition	Load (N)	Sliding distance (m)	Observed wear	CoF
Composition	1	4.00E-17	1.60E-16	-0.63	-0.47
Load (N)	4.00E-17	1	0	0.46	0.78
Sliding distance (m)	1.60E-16	0	1	0.32	0.32
Observed wear	-0.63	0.46	0.32	1	0.86
CoF	-0.47	0.78	0.32	0.86	1

Fig. 7.19 Correlation matrix of tribological properties with varying parameters

Figure 7.20 presents the 3D surface contour plots for wear and COF, predicted by using ANN. This can be clearly observed that with increasing load and sliding distance, both wear and COF increase, while with increasing composition from 0 vol.% to 9 vol.%, both wear and COF decrease linearly. This is also in agreement with correlation matrix (Fig. 7.19) and the results are given in Chapter 5.

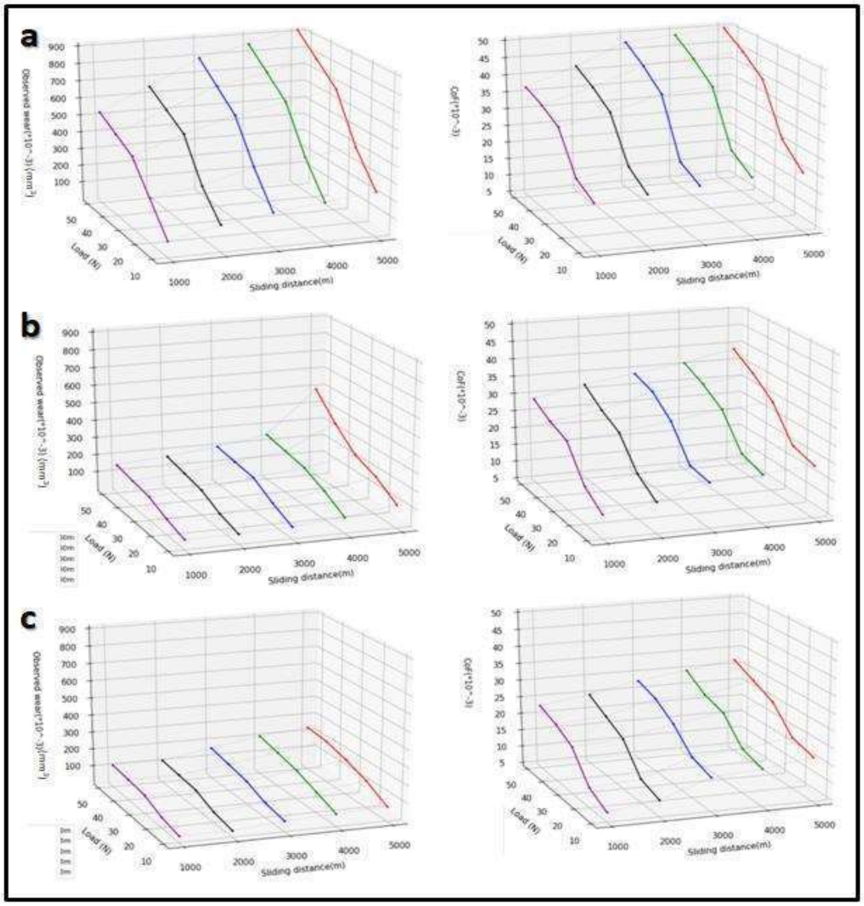


Fig. 7.20 3D- surface contour plot showing wear and COF variation with sliding distance and applied load for (a) C0.0 (b) C4.5 (c) C9.0

This can be observed that predicted values of wear and COF by ANN are showing similar trends and are in close tolerance with the values predicted by RSM model. The correlation matrix is also showing the similar contribution factors as suggested by ANOVA model using RSM. This can be concluded that both the models are significant and efficient to predict the tribological properties of present materials system.

Further, to verify the efficacy of both the models, a set of random input variables i.e. 20 N load, 4000 m sliding distance and C3.0 composition has been selected. The wear and

COF values are predicted using regression equation from RSM for the mentioned set of input variables. To check the accuracy of the predictions, the tribological test is further conducted at 20 N load, 4000 m sliding distance and C3.0 composition (Three tests were conducted to verify the repeatability of data). The obtained experimental values (also given in Chapter 5) and predicted values are given in Table 7.16. This can be observed that experimental values and the predicted ones by RSM for wear and COF are very close and having **less than 5 %** error. This further confirms that both the models can be used for the predictions of tribological properties at any given (with practical feasibility) set of inputs.

Table 7.16 Predicted and experimental values of the wear and COF

	Predicted value by RSM		**Experimental value**	
Input parameters	Wear (mm^3)	COF	Wear (mm^3)	COF
Load -20 N Sliding distance - 4000 m Composition - C3.0	0.155	0.0215	0.16 ± 0.02	0.022 ± 0.0015

7.3.3 Optimization and Verification of the Model

The motive of verifying the model is achieved by analyzing the predicted outcomes of the model with experimentally obtained results. RSM was utilized to get the maximum amount of information with minimum time and experiments. To examine the influence of load, sliding distance, and ZrB_2 vol.% on tribological properties, 2D and 3D contour surface are plotted. 3D contour helps in determining correlation among parameters and to gain their optimum values for least wear and COF. The correlation between input parameters at lowest load, lowest sliding distance and highest vol.% of ZrB_2 are the most optimized parameters for which minimum wear and COF were predicted. It is observed that with an increase in load and sliding distance, the wear and COF values also increase. However, with increasing ZrB_2 vol.%, the wear and COF values decrease. Table 7.16 clearly indicates very small error, thus, the model is satisfactory and fit for modelling and predictions of tribological properties.

7.4 CONCLUSIONS

To reduce the number of experiments, time and resource consumption, predictions and optimizations of tribological properties with dependent variables is crucial. Following conclusions can be made from the present studies:

- The results signify that vol.% of ZrB_2 is the main contributing factor and negatively influences (as the vol.% increases, wear decreases) wear in both dry and lubricating sliding conditions.
- Applied load is the main contributing factor in influencing the coefficient of friction (COF).
- COF increases with increase in vol.% of ZrB_2 in dry sliding condition, however, in lubricating condition it is negatively influenced, that shows self-lubricating properties of composite.
- The regression equation obtained for both wear and COF for present material system can be effectively utilized to predict tribological properties at any given set of input variables.
- Based on applications, the optimization of tribological parameters can be easily obtained using regression equation for both wear and COF.
- Conformity trial with optimum parameters using response surface methodology (RSM) with error value less than ~5% for wear and COF in both dry as well as lubricating

sliding condition, signifies that the model equation used for modelling and optimization is in accordance with the experimental values and significant.

- Conformity trial with Artificial neural network (ANN) shows that the mean error values for wear and COF are ~4.7% and ~ 3% respectively in dry sliding condition. And the mean error values in lubricating sliding condition for wear and COF are ~4% and ~5% respectively, which signify the efficacy of the model used in ANN and further validate the efficacy of the model.
- The results are in accordance with the established theory of minimum wear and COF at lower applied load and sliding distance and vice- versa.
- This can be concluded that the Response surface methodology and Artificial neural network are very significant statistical tools to optimize the tribological parameters and can further be used for similar kind of material systems. They reduce time and resource consumption which ultimately effect overall cost of fabrication of such material systems.