CHAPTER 2

Literature review

2.1 General

The structural shape optimization in order to be a fully automated computer-aided design process works in three stages, the first being geometrical modelling, followed by the second stage of structural analysis, and the third stage being optimization. These three stages work in collaboration with each other (ref. Figure 2.1) until a desired optimized shape is not obtained.

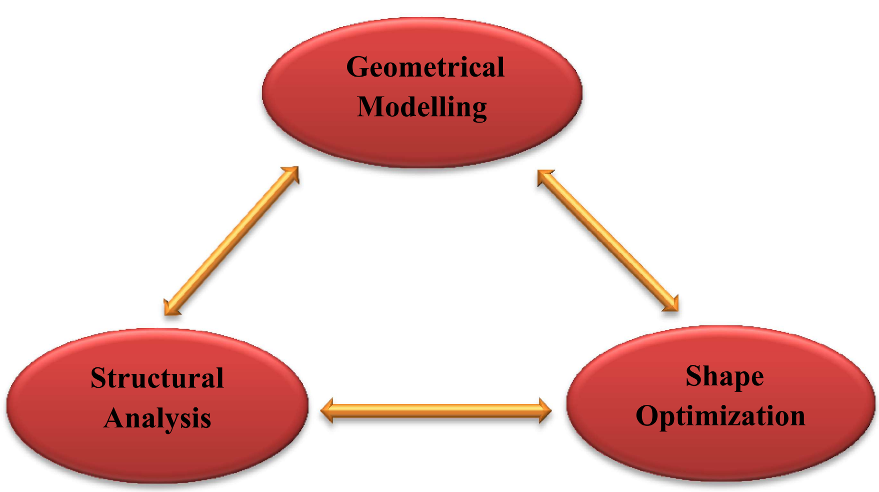

Figure 2.1: Basis stages of computer-aided shape optimization.

The outcome of the analysis is determined by the given limitations on geometry and structural responses like stress, displacements, natural frequencies, etc. Across the decades numerous numerical analytic approaches have been utilised to achieve the optimal shape of structural elements, but the shortcomings of numerical analysis somehow prevent structural shape optimization from being successful as they are responsible for solution accuracy and computing time for shape optimization. With the introduction of Finite elements methods (FEM), there has been widespread use of FEM in structural analysis and optimization problems. But the use of FEM has brought to the resulting framework the FEM-related problems, such as mesh distortion and subsequent re-meshing due to significant shape changes, non-continuous stress values across element boundaries as a result of the field function's linear approximation, and sensitivity of the solution to the distribution of nodes within the domain. The induction of the shape optimization procedure in addition to the structural analysis in FEM makes these problems even worse. The traditional shape optimization process that employs FEM is associated with several key concerns, including:

a) **Incongruent geometric representation of the design & analytical model:**

Studies which are based on parametric design have problems with shape representation (Tortorelli, 1993). In these investigations, a base design that includes a number of fixed topological characteristics and changeable geometric entities is produced. The response of the system is assessed using an analytical method that is developed from geometric entities. In FEM-based shape optimization variety of mathematical models has been used to describe the geometry of design and its analysis. The Computer-Aided Geometric Design (CAGD) elements used to depict the design model include Bezier elements (Silva et al., 2020) where they suggested that the relationship between the iso-geometric analytic model and the geometric model, based on a boundary representation, is simplified by the usage of Bezier elements. Lopez et al. (2020) suggested that by using a single patch and Bezier triangle discretization, it is possible to describe intricate geometrical regions, including holes.), B-spline (Yoely et al., 2018) suggested that the use of B-spline representation has made it possible to include explicit limitations on hole maximum and minimum areas as well as boundary curvatures. As a result, the optimization problem may incorporate real-world design issues including flexibility in the position and size of holes and avoiding stress concentrations at sharp corners. However, this was not the first use of B-spline for geometry representation and many researchers (Kumar and Parthasarathy, 2011; Lee et al., 2009; Edwards et al., 2007 have already used the B-spline representation.), Non-Uniform Rational B-spline (NURBS) curves, and surfaces (Cho and Ha, 2008; Najafi et al., 2017). The design model is made compatible for analysis by discretizing using elements and nodes. These incongruities in the process of optimization if not handled meticulously can instigate problems with accuracy while also lengthening computation times and necessitating additional efforts.

b) Selection of Design Variables

The term "design variable" refers to a numerical parameter that might vary while the design is being optimized. The selection of design variables for shape optimization plays a very pivotal role in the determination of the final outcome. However, one major challenge in the way of selection of design variables for optimization is to have a minimum overall number of design variables while allowing for the maximum geometric flexibility feasible. These two criteria are often at odds with one another (Imam, 1982). One of the most pronounced conventions in shape optimization based on FEM has been to choose design variables as the nodes present at the boundary of FE mesh (Zienkiewicz and Campbell, 1973). Node-based shape optimization is a widely used method in structural engineering for finding the optimal shape of a structure. In this method, the structure is represented by a set of nodes connected by elements, allowing for greater flexibility in the shape of the structure. One advantage of node-based shape optimization is that it allows for the optimization of complex structures with irregular geometry. The shape of the structure can be modified by moving the nodes, rather than being limited to a predefined set of shapes. This can lead to more efficient and effective designs. Another advantage of node-based shape optimization is that it allows for the optimization of individual structural elements or groups of elements. This can be useful for optimizing the size and shape of specific components in order to meet specific design objectives. While using individual node mobility as design variables in shape optimization can offer more design possibilities and avoid the need for shape design parametrization, this approach has faced several challenges, including an increase in the number of design variables that increases the computational load and produces impractical, jagged, and unintelligible geometric shapes. In order to address these problems, geometry parameterization techniques were created in the 1980s and 1990s. These techniques included the following: a) the widely used design element concept (Imam, 1982; Braibant and Fleury, 1984; Botkin, 1982; Wang et al., 1985), b) spline representation, such as Bezier, B–spline, and Cubic spline (Annicchiarico and Cerrolaza, 1999; Cerrolaza et al., 2000), c) polynomial equations (Bhavikatti and Ramakrishnan, 1979, 1980; Prasad and Emerson, 1984), and d) natural design variables—fictitious loads (Tortorelli, 1993; Belegundu and Ranjan, 1988).

Typically, these methods aid in fewer design variables and better geometric designs. Additional information on these parametrization strategies may be obtained in review papers (Ding, 1986; Haftka and Grandhi, 1986). As previously indicated, node-based shape optimization employing FE meshes results in geometric designs that are too jagged for practical use. The problem is caused by shape derivatives' lack of smoothness. For gradient-based optimization approaches, regularisation techniques and filter methods are also utilised to prevent numerical instabilities. To apply in-plane and out-of-plane regularisation, FE nodes' co-ordinates are divided into two groups depending on the geometry they define (the collection of potential optimization variables can alternatively be represented using normal co-ordinates) and the element's shape (internal co-ordinates and tangential co-ordinates). The regularization of internal and tangential co-ordinates in shape optimization typically involves in-plane techniques, which are commonly known as mesh regularization. Normal direction co-ordinates on the other hand, are subject to out-of-plane regularization methods, which help to ensure that the mesh remains smooth and well-behaved in all directions, thereby maintaining the structural integrity of the optimized design. In the case of 2D and 3D components, in-plane regularization ensures that the finite element (FE) mesh is consistent and that the edge and face angles, as well as the aspect ratio, have appropriate values. This helps to maintain the quality and accuracy of the FE mesh, which is critical for achieving reliable and precise results in shape optimization. This method guarantees smooth geometrical modifications and gradient fields. Mesh regularisation methods are typically divided into geometrical and mechanical methods. A widely used approach for mesh regularization is to employ a mechanical technique based on form-finding. This involves determining the free-form equilibrium shape of a membrane under a particular stress field (Stavropoulou et al., 2014; Firl et al., 2013; Linhard and Bletzinger, 2010). Inside a framework for node-based optimization, vertex morphing is another technique for shape control, which involves a combination of in-plane regularization and out-of-plane filtering. The idea behind it is to use a linear map to connect a control field with the geometry. The problem of optimization is then defined in the control space, and the geometry of the problem domain is controlled by the changed control space utilised after each iteration. Design changes may be made here without the need for extra mesh regularisation since the normal and tangential components of the co-ordinates are handled concurrently. The optimization process may be independently integrated using this approach, which is reliable (Hojjat, 2014).

Regarding non-parametric shape optimization, the Traction technique was created to account for non-smooth shape gradients by using the gradient method in Hilbert space (Azegami, 1994; Azegami et al., 1995; Azegami and Wu, 1996). According to this method, to obtain the domain variation that minimizes the objective function in a shape optimization problem, the design domain contains a linear elastic continuum, which is exposed to a distributed external force that matches the design boundary's negative shape gradient. Instead of the boundary being changed proportionately to the form gradient as is the case with the direct gradient technique, this method reshapes the design domain with a boundary that is smoother for one-time differentiability.

c) Domain re-meshing

Since shape optimization is an iterative process, and the shape changes at every iteration, the changed geometry causes the whole analytical model to change.

Hence, it becomes necessary to update the analytical model. It can be challenging to adapt conventional finite element methods since the original mesh might not be fundamentally suitable for producing acceptable solutions due to the altered domain as a result of mesh distortion. Therefore, in order to have an accurate analysis of the model, it becomes important to update the mesh in accordance with the newly updated model (Pathak, 2000). In this situation, re-meshing is necessitated, which considerably raises the computational effort needed for the iterative shape optimization process (Haftka and Grandhi, 1986; Jang et al., 2004).

Methods like adaptive mesh refinements have been used to tackle the problem of domain re-meshing. Shape optimization has used adaptive FEM having refinement by nodal modifications, refinement in polynomial order or refinements in element size since the 1980s (Bennett and Botkin, 1985; Kikuchi et al., 1986; Canales et al., 1993) and still does (Mohite and Upadhyay, 2015). The refinement in the element size method, which implements adaptivity through smaller-sized components while maintaining the constant order of the shape functions, is widely utilised in practise. In polynomial refinement, adaptivity is done via an increase in polynomial order, which employs a coarse mesh with big elements. The polynomial refinement approach has certain advantages from the standpoint of shape optimization: First, larger pieces may allow significant shape changes before deteriorating to a degenerate level, and second, the mesh topology does not alter during the adaptivity process (Salagame and Belegundu, 1995). The element size refinement and polynomial refinement methods generally coexist alongside the node modification approach. Utilizing the concept of lowest energy and the corresponding optimal mesh, variational node modification-dependent shape optimization was established by Thoutireddy and Ortiz (2004) in a novel manner. The variational node modification approach may be utilized to estimate equilibrium shapes that reduce system energy if geometric limitations on maintaining the initial geometric profile are lifted.

The two most crucial elements in the adaptive mesh refinement approach are proper mesh refinement strategy and dependable and precise error estimation (before and at a later stage). Error estimation is a method used to assess the accuracy of numerical solutions in computer simulations. There are two main types of error estimation: global and local. Global error estimation involves evaluating the overall error in a simulation for the entire domain being studied. Local error estimation, on the other hand, focuses on the accuracy of the numerical solution in a specific area or region. Generally, mesh refinement involves adjusting the mesh or grid of points used in the simulation to improve accuracy by identifying areas with errors through error estimation. Effective and often used for this purpose are global and local error estimators (Schleupen et al., 2000). However, this technique is not commonly used in industrial applications due to difficulties in integrating it with computer-aided design systems (Bazilevs et al., 2010).

d) Mesh topology

The calculation of design sensitivity is a crucial component of optimization techniques based on gradients, as it plays a significant role in determining the convergence of the optimization problem. In the context of shape optimization using the finite element method and gradient-based approach, it is beneficial to maintain the same mesh topology for both the initial and modified shapes. This can help reduce "numerical noise" and improve the accuracy of sensitivity computation, leading to better optimization results. This can be achieved through various methods such as mesh morphing and mesh deformation (Lacroix and Bouillard, 2003; Keulen et al., 2005; Kim and Choi, 2005). However, sensitivity calculations are not needed in non-gradient method-based optimization (Pathak and Sehgal, 2009).

e) Solution Accuracy

One of the challenges of non-gradient methods is achieving high solution accuracy. These methods do not have the guarantee of convergence to the global optimum, and they may produce suboptimal solutions or even fail to converge. However, there are several strategies that can be employed to improve solution accuracy in non-gradient methods. One strategy is the use of error estimators which assess the accuracy of the numerical solution and identify areas where the solution is less accurate. This information can be used to refine the mesh, or grid of points used in the simulation, in these areas as described in adaptive mesh refinement, which can significantly improve the accuracy of the solution. Another strategy is to carefully design the optimization algorithm and choose appropriate parameters. This can include selecting an appropriate population size in evolutionary algorithms or setting the appropriate tolerance levels in global optimization methods.

There have been efforts to address the challenges and limitations of using the finite element method (FEM) and shape optimization as discussed above, but there is still a need for further research and development to improve the robustness of these methods.

2.2 Zero order methods

Optimization is a crucial aspect of many fields, including engineering. The goal of optimization is to find the optimal solution to a problem, which is often represented by finding the minimum or maximum of an objective function. There are two main categories of optimization methods: gradient-based and non-gradient/zero-order based. Gradient-based optimization algorithms rely on the gradient of the objective function to guide the search for the optimal solution. The gradient is a vector that points in the direction of the steepest ascent, and thus, these algorithms iteratively move in the opposite direction to find the minimum. Some popular gradient-based optimization algorithms include gradient descent, stochastic gradient descent, and conjugate gradient. These algorithms are effective when the objective function is smooth and differentiable, and the gradient can be easily computed. However, in some cases, the objective function may not be differentiable or the gradient may be difficult or unreliable to compute. In these situations, non-gradient-based optimization methods are more suitable. Non-gradient-based methods do not rely on the gradient of the objective function and instead use other techniques such as direct search or evolutionary algorithms.

Several studies have compared the performance of gradient-based and non-gradient-based optimization methods on various problems. Mattheck and Burkhardt (1990) proposed a method for structural optimization based on the concept of "living structures," which involves adding material to areas of higher stress and removing material from areas of lower stress to achieve an optimal shape of the structure. This method does not rely on gradients and can be applied to various types of structures. A steady von Mises stress distribution is obtained over the entire surface of the optimized shape of the structure. To emulate the process of living structures they used an iterative procedure where they added an imaginary layer at the boundary and permitted it to contract or grow in accordance with the recently discovered boundary stress for the case without an adhesive layer. Another study by Bonyadi and Michalewicz (2017) compared the genetic algorithm to gradient descent on a set of benchmark optimization problems and found that the genetic algorithm was more robust to noise and able to find the global optimal solution more consistently. In addition, non-gradient-based methods have been shown to be more effective for handling complex constraints and multimodal objective functions. For example, a study by Hazi and Abdulazeez (2021) compared the performance of the genetic algorithm and gradient descent on an optimization problem with complex constraints and found that the genetic algorithm was able to find the optimal solution more consistently. Similarly, a study by Wang and Shoup (2011) compared the performance of the Nelder-Mead simplex algorithm and gradient descent on a multimodal objective function and found that the Nelder-Mead simplex algorithm was able to find the global optimal solution more consistently. At times non-gradient-based optimization methods have shown to be more efficient and effective than gradient-based methods in a variety of settings. These methods are more robust to noise, able to handle complex constraints and multimodal objective functions, and able to consistently find the global optimal solution. As such, they are an important tool in the optimization toolbox and should be considered when solving complex optimization problems.

There is a rich collection of literature which discusses non-gradient/zero-order-based optimization algorithms in the field of structural optimization. The non-gradient/zero order includes methods like heuristic methods (Lamberti and Pappalettere, 2011; Saka and Dogan, 2012), genetic algorithms (Jenkins, 1991), evolutionary algorithms (Lagaros et al., 2002), stochastic search methods (Saka, 2009), simulated annealing methods (Sonmez, 2008), particle swarm optimization methods (Poli, 2008), harmony search methods (Geem, 2009) etc. which are used frequently used by the researchers to obtain an optimized outcome.

2.2.1 Evolutionary algorithms

Evolutionary algorithms are a type of non-gradient-based optimization algorithm that mimics natural evolution to find the optimal solution to a problem. They are population-based algorithms that operate in four general steps: reproduction, mutation, recombination, and selection. These steps are used to generate new solutions that are evaluated using a fitness function, which determines which solutions are fit to survive and be included in the next generation. Evolutionary algorithms have been applied in various fields of engineering optimization, including structural optimization.

2.2.1.1 Genetic algorithm

A Genetic Algorithm (GA) is a type of evolutionary algorithm that is commonly used for optimization. It was first proposed by John Holland in 1975 (Holland, 1975) and operates by selecting an initial population of potential solutions, evaluating them using a fitness function, and then using stochastic transformations to generate new solutions (offspring). The fittest solutions from the current population and offspring are then selected to form a new population for the next iteration. This process is repeated until the algorithm converges to the optimal or suboptimal solution of the objective function. The structure of a GA typically involves the following steps (Gen and Cheng, 1999):

a) Initialization of the population
b) Evaluation of the fitness function
c) Selection of fit solutions
d) Crossover (recombination) to generate offspring
e) Mutation of offspring
f) Repeat steps 2-5 until convergence.

GAs have been applied to a wide range of optimization problems and are known for their ability to handle complex multimodal objective functions. Its application can be found in diverse structural engineering problems, including structural reliability (Deng et al., 2005; Wang and Ghosn, 2005), seismic design of lifeline systems (Li et al., 2008), bridge design, maintenance and repair (Furuta et al., 1995; Elbeltagi et al., 2005; Elbehairy et al., 2006), truss structure optimization (Dede et al., 2011; Manoharan and Shanmuganathan, 1999; Rahami, 2011), reinforced concrete flat slab buildings (Sahab et al., 2005), design optimization of steel structures (Fu et al., 2005; Burns, 2002; Hasançebi et al., 2010), viscous dampers (Bigdeli et al., 2012), seismic zoning (García-Pérez et al., 2003), and steel telecommunication poles (Khedr, 2007). Others can be found in Balamurugan et al. (2008, 2011), Madeira et al. (2010), Manan et al. (2010), Zhou (2010), Jain and Saxena (2010), Wang et al. (2006), Guest and Genut (2010), Bureerat and Limtragool (2006), Wang and Tai (2005). These algorithms have proven to be effective in finding optimal or suboptimal solutions for these complex structural problems.

2.2.1.2 Evolutionary Strategies

Evolutionary Strategies (ES) are a type of evolutionary algorithm that was initially developed by Rechenberg and Schwefel (Rechenberg, 1965, 1971; Schwefel, 1965;

1975). The algorithm starts by creating an initial population of potential solutions (μ), called the parent population. Each individual in the parent population is represented by a set of parameters, an objective value, and a set of strategy parameters that can be adjusted during the evolution process. The parent population then reproduces to form the offspring population (λ) by combining the strategy and object parameters of a randomly chosen group of individuals from the parent population and applying mutations to these new parameters. Finally, the selection step chooses the best individuals from the offspring population to become the new parent population. The process is repeated to evolve better solutions over time. Two main types of ESs exist: (μ + λ) – ES and (μ, λ) – ES, which differs in the number of parents and offspring used in the reproduction process. ESs follow this process of reproduction, mutation, and selection to improve solutions to a problem over time, guided by the goal of mimicking the process of natural selection.

ES have a broad application in the field of structural engineering, it has been used to solve a wide variety of problems. These include but are not limited to: Optimizing trusses (Thierauf and Cai, 1997; Hasançebi, 2007, 2008), and the design of a cantilever beam (Chen and Chen, 2009), optimizing the shape of a connection rod and minimizing the volume of a square plate with a central cut-out (Papadrakakis et al., 1998), steel frames and cylindrical shells (Hasançebi et al., 2010; Muc and Muc-Wierzgon, 2012). These optimization problems can be difficult to solve using traditional optimization methods because of their high dimensionality, non-linearity, and constraints. ES, as a global optimization method, is suitable for solving these types of problems. This is why it's often found in the literature and has been used in many research studies to optimize different types of structures.

Evolutionary algorithms being a class of heuristic techniques do not have a mathematical proof of convergence, meaning that the solution found may not be the global optimum. However, in practice, they have been shown to be very effective in finding good solutions quickly, particularly for large-scale problems. One of the benefits of evolutionary algorithms is that they are relatively insensitive to the dimensionality of the problem, allowing them to be used for large-scale problems. However, it can be a drawback when it comes to small-scale problems as it requires a large population of candidate solutions to be maintained. Comparisons between evolutionary algorithms and other non-gradient optimization methods are common in literature and evolutionary algorithms are often considered as a standard approach and reference to measure other methods.

2.2.2 Physical algorithms

Physical algorithms in structural optimization use physical principles to solve optimization problems related to the design and analysis of structures.

2.2.2.1 Harmony search

The Harmony Search (HS) algorithm is an optimization technique that was first proposed in 2001 by Geem, Kim, and Loganathan (Geem et al., 2001). It is based on the concept of finding harmony in music by combining sounds of different frequencies, with the goal of achieving an aesthetically pleasing outcome. In the context of optimization, the "best state" corresponds to the global optimum. The algorithm utilizes three key elements: random selection, memory retention, and pitch adjustment. These are used to find the optimal solution by adjusting two main parameters: the acceptance rate (r_{accept}) for new harmonies into the Harmony Memory (HM), and the rate of pitch adjustment (r_{pa}). The basic structure of the HS algorithm is outlined in (Geem, 2009). The algorithm utilizes the concept of harmony in music to optimize solutions in a more elegant manner.

The HS algorithm has been used to solve a variety of structural engineering problems, such as the optimization of truss structures (Lee and Geem, 2004; Kaveh and Talatahari, 2009), pin-connected structures (Li et al., 2007), steel frames (Degertekin and Hayalioglu, 2010) and reinforced concrete frames (Kaveh and Sabzi, 2011). Some examples of specific applications include the minimum-cost design of steel frames, the optimum design of sway frames, and the design of cellular beams. The HS algorithm has also been used in the pressure vessel and welded beam design (Lee and Geem, 2005). Additionally, a number of other structural design optimization problems have also been tackled using the HS algorithm, such as the sizing and configuration of truss structures as well.

2.2.2.2 Simulated annealing

The Simulated Annealing (SA) algorithm (Kirkpatrick et al., 1983; Cerny, 1985) is a method for solving optimization problems that mimic the annealing process used in materials science. The annealing process involves heating a material to high temperatures and allowing the atoms to move around and explore different configurations, then slowly cooling the material which causes the atoms to settle into a new configuration with lower internal energy. In optimization, SA uses this concept to find the optimal solution for a problem by starting with an initial state, which is considered a local minimum. The algorithm then generates new solutions by "heating" the current solution, similar to how the atoms are heated in the annealing process. The new solutions may then be accepted based on probability, which is calculated using the decrease in the objective function value and a "temperature" measure. As the algorithm progresses, the temperature gradually decreases, allowing the algorithm to find a global minimum. This also allows us to prevent getting stuck in local minima by accepting a solution that may have a higher objective value but can lead to a global minimum. The basic structure of an SA algorithm is outlined in (Eglese, 1990).

Simulated Annealing (SA) algorithms have been applied to various structural engineering design optimization problems. Some examples include the optimization of tensegrity systems (Xu and Luo, 2010), truss structures (Manoharan and Shanmuganathan, 1999), steel frames (Ohsaki et al., 2007; Hasançebi et al., 2010), laminated composite structures (Akbulut and Sonmez, 2008, 2011), concrete frames (Paya et al., 2008) and cross-sections (Serra, 2005). These applications demonstrated the effectiveness of the SA algorithm in solving complex and difficult optimization problems.

2.2.2.3 Ray optimization

Ray Optimization (RO) is an innovative optimization method that takes inspiration from the fundamental laws of light refraction. The algorithm utilizes a set of agents, which can be considered as light particles that possess their own location and direction. At each iteration, the agents calculate an "origin" point, which represents the average of the best solution found so far and the best solution known by each agent individually. By following the principles of light refraction, and making use of small random perturbations, the agents adjust their direction towards the origin, updating their position as they converge towards it. This process continues until the optimal solution is found. The basic structure of the Ray Optimization Algorithm is outlined in (Kaveh and Khayatazad, 2012). The Ray Optimization (RO) algorithm has been effectively applied to the design of welded beams, springs and trusses (Kaveh and Khayatazad, 2013).

2.2.2.4 Tabu search

The Tabu Search (TS) method is a local search heuristic that was first proposed by Glover in 1989. The algorithm is commonly used in combination with other algorithms to overcome the constraints of local optimality. It is mostly applied to combinatorial optimization problems that are discrete in nature and have constraints. In the TS method, an initial solution is chosen from a set of feasible solutions, X. The set of possible moves that can be made from the current solution is represented as S(x). A portion of these moves is selected as the tabu moves, denoted by T, which is chosen based on a function that utilizes past information from the search process up to the t iterations prior to the current iteration. The function used to determine membership in T may include a list of tabu conditions, such that:

$T(x) = \{s \in S: s \text{ violates the tabu conditions}\}$.

The basic structure of the TS method can be described in pseudocode as follows (Glover, 1989):

i. Choose an initial solution $x \in X$.
ii. Set $T = \{\}$, the empty set of tabu moves.
iii. Repeat until the stopping criterion is met:
    a) Select a move $s \in S(x)$ such that s is not in T.
    b) Update $x = x + s$.
    c) Update $T = T \cup \{s\}$.

d) Store information from the current iteration for use in determining future tabu moves.

iv. Return the final solution x.

The TS permits the algorithm to remember past information and apply it to improve its future steps. This can help prevent the algorithm from getting stuck in a locally optimal solution. In the field of structural engineering, TS has been used to optimize the weight of frames (Kargahi et al., 2006), the design of steel and truss structures (Ohsaki et al., 2007), and to assess the seismic performance of optimized frame structures (Kargahi and Anderson, 2006).

Physical and stochastic algorithms like evolutionary algorithms, do not have a mathematical proof of convergence. These algorithms are designed to avoid getting trapped in local optima, and have been known to find better global solutions compared to other methods. However, their inclination to move away from local optima can make convergence difficult. Therefore, physical and stochastic algorithms are often used in conjunction with these methods to complement their ability to identify local solutions. As stochastic-based methods, physical and stochastic algorithms can be difficult to reproduce. Running the same algorithm on the same problem can lead to vastly different results. This can be a problem for scientific research, as reproducibility is an important aspect of the scientific method. One way to overcome this issue is to carefully program the algorithm and keep track of the "random" strings that are used in the algorithm.

2.2.3 Swarm algorithms

Swarm algorithms are inspired by the behaviour of decentralized and self-organizing systems, whether they are found in nature or in artificial systems. In structural engineering, swarm algorithms are commonly used to model biological systems that follow simple rules, which leads to the emergence of "intelligent" system behaviour. Several swarm algorithms like ant colony optimization, artificial bee colony, shuffled frog-leaping, particle swarm optimization, and are often used in structural engineering.

2.2.3.1 Ant colony optimization

The Ant Colony Optimization (ACO) algorithm mimics the actions of an ant colony searching for food, and it is named after this behaviour. This stochastic combinatorial optimization method employs mathematical principles from graph theory to replicate ant foraging by creating paths through the use of pheromone communication. The algorithm was originally named "Ant System" by its creators Dorigo, Maniezzo and Colorni, and a more in-depth explanation of it can be found in their publication (Dorigo et al., 1996). It's mostly used as an optimization algorithm to solve the combinatorial optimization problem.

The algorithm's success is attributed to three key factors: utilizing positive feedback as a method for search and optimization, utilizing distributed computation to prevent premature convergence, and implementing a constructive greedy heuristic to quickly find reasonable solutions. The positive feedback technique involves selecting the most favourable option, based on prior positive results. Distributed computation mimics the increased efficiency of a group of ants working together, as opposed to individually. And the greedy heuristic approach only allows for locally optimal moves, ensuring early discovery of good solutions. The algorithm has been applied in various structural engineering applications, such as optimizing bridge deck rehabilitation (Elbeltagi et al., 2005), solving minimum weight and compliance problems in structural topology design (Luh and Lin, 2009) and optimizing designs for truss structures (Kaveh and Talatahari, 2009), concrete frames (Kaveh and Sabzi, 2011) and steel frames (Camp et al., 2005; Aydogdu and Saka, 2012).

2.2.3.2 Particle swarm optimization

Particle Swarm Optimization (PSO) is a method that utilizes the flocking behaviours of animals as a model for its algorithm. This approach, which was first introduced by Eberhart, Kennedy, and Shi, shares some similarities with genetic algorithms, as it evaluates a set of potential solutions and assigns them a "fitness" value. The algorithm functions by simulating particles that symbolize possible solutions, moving throughout the search space and adapting their fitness values based on a specific criterion. Each particle is affected by its neighbouring particles, and mathematical equations are employed to steer the particles through a d-dimensional hyperspace, guiding them toward more optimal solutions. More information on PSO can be found in Kennedy and Eberhart (1942), and for the general structure of a PSO algorithm, please refer to (Kennedy, 2006).

PSO algorithms have been used in various structural engineering applications, such as optimizing the design of transport aircraft wings (Venter and Sobieszczanski-Sobieski, 2004), cellular beams (Erdal et al., 2011), pin-connected structures (Li et al., 2007), bridge deck rehabilitation (Elbeltagi, 2005), topology design of continuum structures (Luh et al., 2011), identifying structural damage (Begambre and Laier, 2009), steel structures and truss-structures (Fourie and Groenwold, 2002; Luh and Lin, 2011), optimum design of reinforced concrete frames (Kaveh and Sabzi, 2011).

2.2.3.3 Shuffled frog-leaping

The Shuffled Frog-Leaping (SFL) method is a population-based approach that uses local search heuristics to mimic the behaviour of frogs in a swamp. It was proposed by Eusuff et al., (2006) and is considered a member of the recent group of evolutionary memetic algorithms. The algorithm enables the development of independent communities, which evolve independently of one another. The communities are then shuffled, facilitating the exchange of local search information between them. This exchange of information helps the algorithm converge towards a global optimum. The general, the SFL algorithm follows the structure as explained in Eusuff et al., (2006).

The SFL method has been applied to the optimization of pipe sizes for water distribution network design, as well as the design of bridge deck repairs in the literature references (Eusuff and Lansey, 2003; Elbeltagi et al., 2005; Elbehairy et al., 2006).

2.2.3.4 Artificial bee colony

The Artificial Bee Colony (ABC) algorithm, proposed by Karaboga, (2005), models the food-foraging behaviour of honey bee swarms. The algorithm is composed of three types of bees: scout bees, which explore the search space in a random manner; employed bees, which perturb a solution based on the neighbourhood of their food sources; and onlooker bees, which are assigned to food sources using a selection process that is based on probabilities. The selection of food sources by onlookers is based on the amount of nectar, with a preference for those with high values. If a new source has more nectar than a remembered source, the position is updated, and the old one is forgotten. When a solution shows no improvement after a set number of attempts, determined by a limit parameter, the food source is abandoned, and the employed bee becomes a scout bee. ABCs general structure is presented in Parpinelli et al., (2011).

The ABC algorithm has been applied to structural optimization problems in the literature, specifically to truss structures (Hadidi et al., 2010; Sonmez, 2011, 2011), inverse analysis of dam-foundation systems (Kang et al., 2009), laminated composite components (Omkar et al., 2011), and welded beam and coil spring design (Pham et al., 2009).

Swarm algorithms, like other heuristic methods, are not guaranteed to converge to a global optimum mathematically. They are typically designed with a specific problem in mind, and their performance on problems with different structures may not be as good. However, swarm algorithms have been observed to be highly effective when used for the particular problems they were designed for.

2.2.4 Direct search methods

Derivative-free optimization (DFO) is a field of optimization research that has grown in popularity in recent years. Unlike methods that rely on derivative information, DFO methods do not require the calculation of derivatives and have a mathematical convergence theory. Several DFO algorithms are commonly discussed in the literature, including:

i. Directional direct search methods (DDS): These methods generate a search direction based on the function evaluations, and perform a one-dimensional optimization along that direction (Audet and Dennis, 2006; Conn et al., 2009)

ii. Simplicial direct search methods (SDS): These methods use a simplex (a set of d+1 points in d-dimensional space) to generate a new set of points, and then search for the one that minimizes the objective function (Nelder and Mead, 1965).

iii. Simplex gradient methods (SGM): Similar to simplicial direct search, these methods use a simplex but in addition, they make use of gradient information (Conn et al., 2009).

iv. Trust region (TR) methods: These methods estimate the local behaviour of the function by using a model around the current point, and then use this information to determine the next step that minimizes the objective function (Chen et al., 2012).

One of the main advantages of DFO is its mathematical convergence theory which ensures the quality of the final solution obtained, making it a reliable method to achieve local optimality. However, one of the challenges of using these methods is that they typically scale poorly with dimension, which means that they may require a large number of function evaluations when the number of variables in the problem is very high. This could make it impractical or computationally expensive to use these methods for high-dimensional problems.

Since the development of computer-aided analysis and design methods, there has been a significant amount of academic interest in the use of the above-discussed techniques for the optimization of engineering structures. However, these methods have been little understood and infrequently used by practising civil engineers, who often regard them as not applicable to real-world design situations. This perception is likely due to the perceived mathematical complexity of these methods and the limited scope of their application. Most of these methods aim to minimize the weight or volume of a structure while satisfying certain design constraints such as limits on member stresses and structure deflection.

2.3 Literatures on shape optimization of beams

Shape optimization is an active area of research in the field of optimization of beams, where the goal is to find the optimal shape of a beam that satisfies certain constraints and maximizes or minimizes a given objective function. In the early years, Hasengawa, (1992) proposed two zero-order techniques: one for adjusting thickness and the other for modifying boundary co-ordinates. Mattheck and Burkhardt, (1990, 1990), Mattheck et al., (1992) optimized a rectangular bar with a hole using an approach based on the concept of "living structures". They suggested adding material to regions of high stress and removing material from regions of low stress to achieve the optimal shape of the structure. Later various researchers proposed different methods for beam shape optimization. Kathiravan and Ganguli, (2007) worked on finding the optimum design of composite box beam under the constraints applied on strength and showed that the PSO-based method was able to find better solutions than traditional gradient-based optimization methods, as it was able to converge to the optimal solution faster. This is because PSO uses information from the entire population of solutions, rather than just the current best solution, to guide the search for the optimal solution. Jarraya et al., (2007) used the sequential quadratic programming (SQP) method for shape and thickness optimization of a beam, considering its linear behaviour. Nagy et al., (2009) proposed an isometric analysis for the shape optimization of the beam. They, in their approach, considered an alteration of both weights of the structure and the spatial location of the control point to get the optimized shape. They used a non-uniform B-spline (NURBS) based description for geometry. Pathak and Sehgal, (2009) gave a new zero-order method for the shape optimization of structures. They changed the shape of the design boundary satisfying the fully stressed design criteria and also used an artificial neural network to accelerate the convergence. Radaelli and Herder, (2016) presented the optimization in the shape of compliant beams for a given load-displacement response. Ozbasarn and Limaz, (2018) modified the Big-Bang-Big crunch algorithm to give an optimized shape for tapered I-beams having lateral-torsional buckling. They discussed the location of the inflection point and flange/web tapering to get the economical design and concluded that conventional design can be at most 35% worse than optimal design on the ground of economy consideration.

There has been an increased focus on the use of multi-objective optimization methods for beam shape optimization. Multi-objective optimization is an extension of traditional optimization methods that allows for the simultaneous optimization of multiple objectives, rather than just one. In the case of beam shape optimization, this can include objectives such as weight minimization, displacement constraint satisfaction, and stress constraint satisfaction. Wang et al., (2011) proposed a multi-objective optimization method for beam shape optimization using an improved non-dominated sorting genetic algorithm (NSGA-II). They demonstrated that their method was able to find a set of Pareto-optimal solutions that balanced the trade-offs between different objectives, providing design engineers with a range of options for different design scenarios.

In summary, the literature on beam shape optimization is vast and varied. Researchers have proposed a wide range of methods for solving the problem, including gradient-based optimization, genetic algorithms, PSO, EA, and multi-objective optimization. These methods differ in their underlying assumptions and algorithms, but they all aim to find the optimal shape of a beam that satisfies certain constraints and maximizes or minimizes a given objective function. It is important to note that most of these studies used numerical simulations and experiments to verify the proposed methodologies and results were presented for beams under specific loads and boundary conditions. These studies have proved the effectiveness of non-gradient methods in the field of shape optimization.

2.4 Literatures on optimization in sandwich beams

A steel-concrete-steel (SCS) sandwich beam is made up of two steel faceplates of the thickness (t_f) sandwiching a concrete core of thickness (t_c) generally abiding by the ratio of $t_f/t_c \ll 1$, connected together via mechanical connectors forming a compact unit to withstand externally applied loads. Solomon et al., (1976) proposed the concept of using a steel-concrete-steel (SCS) sandwich roadway slab in medium to long-span composite bridges to reduce the weight of the structure. This idea led to further research and development in the construction of SCS sandwich structures. One major area of focus in this research has been the optimization of sandwich structures to reduce weight. Huang and Alspaugh, (1974) worked on the weight minimization of sandwich beams under the constraints of maximum allowable deflection, shear stress, and bending stress. Gibson, (1984) proposed an analytical method for finding the core and face thickness and core density aimed towards minimization of the weight of foamed core with respect to a certain span length and stiffness. This analytical method was based on two observations: (1) for the foam core the shear modulus is proportional to the square of its density; (2) the shear and bending component sum up to give the total deflection of sandwich beams subjected to bending. Demsetz and Gibson, (1987) worked on minimizing the weight of sandwich plates subjected to stiffness constraints by developing a closed-form solution that worked on face and core thickness. Triantafillou and Gibson, (1987) used the criteria of simultaneous failure of the face and core of the sandwich beam or plate for finding the minimum weight design. Paydar and Park, (1990) also used the idea of minimum weight design. They worked on a varying-thickness sandwich beam which was symmetric about their axes and presented a small deflection theory for the determination of deformation and stresses in the same. Swanson and Kim, (2002) used strength constraints for geometric and core density optimization for sandwich beams under concentrated loads. Bergan et al., (2008) developed lightweight concrete mixes to be used as the core, testing the results through laboratory tests. Steeves, (2012) minimized the overall mass of the polymer foam core sandwich beam under a three-point bending load by optimizing the core density and geometry. The constraints used here were indentation strength and stiffness. Fan et al., (2018) worked on a multi-sandwich-panel composite structure to optimize its global stacking sequence using a genetic algorithm-based method. They verified its efficiency using a seven-sandwich-panel optimized to its strength and buckling considerations and deduced that by using non-identical core thickness in a multi-sandwich panel the total mass can be reduced. Al-Fatlawi et al., (2019) presented a study on the weight minimization of shipping and aeroplane containers using honeycomb core sandwich panels. They took weight as a design variable and constrained it on the basis of stiffness, skin wrinkling, core shear, and face sheet failures showing a weight reduction of 15% to 38% in different samples. Kondratiev and Gaidachuk, (2019) developed an optimization procedure for a sandwich-shelled composite structure with honeycomb filler taking into consideration the geometric and material parameters. They found that the use of irregularly shaped hexagonal honeycomb filler was more effective in the reduction of the overall weight of the system by around 2.1 to 12.3%. Sjølund et al., (2019) optimized the sandwich structure using Discrete Material and Thickness Optimization (DMTO). They optimized both the face and core plies simultaneously using the density design variable in accordance with the linear buckling and displacement constraints. Hanifehzadeh and Mousavi, (2019) presented an article predicting the structural behaviour of the SCS wall under a near-field blast load. They proposed an optimum ratio for the thickness of the rear to front plate to be 1.5 and that by using higher strength concrete (100 MPa) the backside deformation can be reduced by up to 25%. They performed all the simulations in the ABAQUS\EXPLICIT finite element package. Much work has also been done to find the best-suited mechanical shear connectors so as to have a better connection between the faceplates and core for enhanced performance of SCS structures. Sohel et al., (2012) proposed novel shear connectors, for example, cable shear and J-hook connectors, and studied their effectiveness in achieving the desired strength in SCS sandwich structures. Additionally for determination of the ultimate strength of SCS beams when used under different shear connectors they proposed an analytical method. Yan, (2015) presented a FEM analysis for the SCS sandwich beam having J-hook shear connectors and overlapped shear connectors and validated their results through experimental results for the same. Huang and Liew, (2016) designed SCS sandwich shells with multiple vaults to be used for the resistance of ice contact pressure. They used J-hook or headed shear studs as a mechanical connector and proposed a design equation for the prediction of resistance to shear in SCS curved sandwich panels. In the field of optimizing sandwich structures, there are two main criteria that have been focussed on: minimizing core density and reducing the thickness of the face and core. Most research and application have focused on constant face and core thickness, but as fabrication techniques improve, there is increasing demand for non-uniform cross-sections to achieve lighter-weight designs. To achieve this, it may be possible to reduce the amount of steel in the sandwich beam and replace it with concrete, which has a much lower weight density. However, this must be done in a way that does not compromise the structural integrity or functionality of the beam.

2.5 Literatures on optimization in pre-stressed beams

Pre-stressing of concrete has been a revolutionary technique towards counteracting the tensile weakness of concrete (Masterman, 1952). Pre-stressed concrete (PSC) structures have been used extensively worldwide in the field of construction for decades now (Dolan and Hamilton, 2019). As the research progressed and the necessity for optimization was felt in every area of life, the exigency for optimized design in PSC structures was also recognized and researched upon. Numerous optimization methodologies and formulations have been developed suggesting optimization of topology, dimension, shape, cable layout, pre-stressing force etc., for PSC structures.

The early optimization techniques were based on linear programming formulations where they found the optimal pre-stressing force and tendon co-ordinates for applied loading (Kirsch, 1972, 1973), minimum cost and weight for simply supported PSC beams (Erbatur et al., 1991). Later advancements towards non-linear formulations were adopted, incorporating methods like the Penalty Function Method coupled with Quasi-Newton (Saouma and Murad, 1984), functional constraints like flexural strength, material stress, and deflection (MacRae and Cohn, 1987), ε – constraint approach with minimum cost and minimum initial camber as design objectives (Lounis and Cohn, 1993), and combination of linear and non-linear formulation for tendon co-ordinate and concrete dimension respectively (Kirsch, 1997) in pursuance of optimized solutions for PSC structures. Generalized heuristic algorithms with constraints like stress, moment capacity, and camber were also propounded for the determination of concrete strength, location and the number of cables in order to minimize the constructional cost of PSC beams (Jones, 1985). Han et al., (1996) suggested a Discretized Continuum-type Optimality Criteria (DCOC) method using a parabolic profile, curbing the cost of construction of PSC T-beams. Single-objective (Barakat et al., 2003) and multi-objective optimization using fuzzy algorithms (Ohkubo et al., 1998), ε – constraints and reliability index (Barakat et al., 2004), hierarchically decomposed design optimization (Fang et al., 1994) for PSC structural components were also suggested. Progressively myriad algorithms by various researchers were suggested in the domain of optimization of PSC structures. Aydin and Ayvaz, (2009) used Genetic algorithms to achieve a 28% cost reduction in middle span bridge girders, Luo et al., (2013) used Drucker-Prager yield constraint for stress analysis in concrete and minimized the steel volume accordingly, Segura et al., (2017) minimized the cost and maximized the crack initiation time to obtain an optimum cross-section for PSC box girders, Mohammed et al., (2017) modelled PSC slabs considering contact algorithm and material non-linearity for optimization declaring strain energy as the objective function.

The positioning of cable profile in PSC structures plays a very pivotal role in the removal of tension, yet somehow it has received relatively less consideration towards incorporation in optimization procedures of PSC structures. In literatures, as per the load balancing concept (Lin and Burns, 1981) layout of cables has been suggested to follow the bending moment diagram. However, researchers gradually brainstormed to find the optimized layout of cables in accordance with the constraints applied to a given PSC structure (Eurviriyanukul and Askes, 2011; Semelawy et al., 2012). Amir and Shakour, (2018) proposed a simultaneous topology and shape optimization approach where they optimized the concrete topology and changed the shape of the tendon in PSC beams. Lately, uses of B-spline representation for boundaries and tendon profiles have gained the interest of researchers. Akhtar et al., (2008) used a B-spline profile to model the cables in PSC beams. Khan et al., (2010) adapted the B-spline profile to design one-way PSC slabs. Yoely et al., (2018) optimized the shape and topology of PSC beams expressing the boundaries as a B-spline curve, Zelickman and Amir, (2021) used second-order, three-dimensional B-spline to represent the cable profile for optimization of PSC slabs, later they added cost function (Zelickman and Amir, 2022) and achieved a 50% saving in cost of cables.

The majority of the research work accomplished towards achieving an optimized PSC structure can be categorized under two modus operandi (i) obtaining an optimum concrete section for a given cable profile and force (ii) finding an optimum cable profile and cable force for a given concrete dimension, but there are barely any literature suggesting optimization of both cable profile and concrete shape concurrently to get a final optimized state.

2.6 Conclusion from literature reviews

With the advancement of technology, the integration of analysis and optimization into the mechanical design process is becoming increasingly feasible. As computers become faster and more advanced structural analysis and optimization programs become widely available, it is becoming more realistic for designers to use these tools in their design process. Optimization plays a crucial role in using the results from analysis programs to improve a design systematically and efficiently. In the past, designers would use trial and error to improve a design, but optimization allows for a systematic approach to finding the best design. By using optimization algorithms, a designer can systematically search through the design space and find the best design with minimal effort. Additionally, integrating analysis and optimization into the design process allows for earlier detection of problems in the design. By performing analysis and optimization early in the design process, designers can identify and fix problems before they become too costly to fix. This also allows for better communication between designers and engineers as the analysis and optimization results can be used to validate and support design decisions. Furthermore, optimization algorithms can be used to not just improve the performance of a design but also to optimize for cost, weight, and manufacturing feasibility. This allows for the design of products that are not just high-performing but also cost-effective and easy to manufacture. By using these tools, designers can systematically improve their designs, identify and fix problems early in the design process, and optimize for multiple objectives such as performance, cost, weight and manufacturing feasibility. Zero-order/Non-gradient methods can be effectively used to achieve these targets.

These methods are particularly useful in situations where the objective function and/or constraints are not differentiable, or when the gradient information is not reliable or computationally expensive to obtain. The effectiveness of non-gradient optimization methods can be described by these four major points.

1. Non-gradient optimization methods are generally more robust than gradient-based methods. Because they do not rely on gradient information, they are less sensitive to the initial conditions and are less likely to get stuck in a local optimum. This makes them particularly useful for problems that have multiple local optima or for problems that are non-convex.
2. Non-gradient optimization methods are particularly well-suited for problems where the objective function and/or constraints are expensive to evaluate. These methods typically only require the objective function and/or constraint to be evaluated at a small number of points, making them more efficient when the cost of evaluating these functions is high. This makes them ideal for problems where

the objective function or constraints are based on computer simulations, or for problems that involve complex physical experiments.

3. Non-gradient optimization methods are also useful for problems where the gradients are not reliable. Gradients can be unreliable when the objective function or constraints have noise, or when they are not well-behaved. In such cases, non-gradient optimization methods can often still find good solutions, whereas gradient-based methods may fail.
4. Many Non-gradient optimization methods use a probabilistic approach which is a random search over the feasible design space. This allows for better exploration of the design space and can increase the chance of finding the global optimum. Additionally, many non-gradient optimization methods are designed to adaptively adjust the search strategy based on the information that is obtained from the evaluations, which can further increase the chances of finding the global optimum.

In conclusion, non-gradient optimization methods offer a number of benefits over gradient-based methods. They are generally more robust, well-suited for problems where the objective function and/or constraints are expensive to evaluate, and useful for problems where the gradients are not reliable. Furthermore, the randomness in many non-gradient optimization methods also allow for better exploration of the feasible design space increasing the chance of finding the global optimum. Non-gradient optimization methods are powerful optimization techniques that can be applied to a wide range of problems, and they are becoming increasingly popular in engineering, computer science and other fields.

2.7 Research gaps

Upon conducting a thorough review of the existing literature, certain areas of deficiency were identified in zero-order shape optimization that requires further research and attention in order to make the process more robust and acceptable. The current thesis aims to address some of the research gaps that were identified during our literature review. These gaps include:

1. Previous research on this topic has not incorporated the feature of automatic mesh refinement in zero-order methods of shape optimization, which is the process of adjusting the fineness of the mesh used in the simulation during the iterative solution process, after each iteration. This is an important aspect and the current study aims to address this gap.
2. Previous studies have failed to take into account the boundary smoothness after each iteration, which leads to the formation of sharp corners and subsequently results in an excessive concentration of stress at these locations. This is a crucial aspect that has been overlooked in prior research and the present study endeavours to address this significant deficiency by incorporating boundary smoothness as a vital component of the iterative solution process.
3. Prior investigations on the optimization of shape for beams constructed using composite materials (two or more), having initial imperfections have been found to be inadequate in terms of zero-order shape optimization. The present study aims to address these issues using the suggested approach based on the zero-order method.
4. Most of the research works on the optimization of Steel-concrete-steel (SCS) sandwich structures have been limited to the use of constant face and core thickness. This is due to the fact that traditional fabrication techniques were not

capable of producing sandwich elements with non-uniform cross-sections. However, with the advent of modern and advanced fabrication techniques, the need for lighter-weight designs has increased, and the use of sandwich elements with non-uniform cross-sections is likely to become more prevalent. The current study aims to address this gap by exploring the optimization of SCS structures with non-uniform cross-sections.

5. There are very limited works based on zero-order methods in shape and cable layout optimization. Most existing research only focuses on either finding the optimal concrete section for a given cable profile and force or determining the optimal cable profile and cable force for a given concrete dimension. There are no existing methods that suggest concurrent optimization of both the shape and layout of the cable. The current study aims to address this gap by exploring the possibility of concurrent optimization of shape and cable layout using zero-order methods.
6. Prior research on zero-order methods has primarily focused on single-criteria optimization, in which one objective such as mass, stress, deflection, or stiffness is selected as the primary goal. The current study aims to address this gap by exploring multi-criteria optimization using zero-order methods, where multiple objectives are taken into consideration simultaneously during the optimization process.

The above points as mentioned have been addressed in Chapters 3 to 7.