Appendix A

$$\lambda_1 = \frac{1}{\sqrt{2}}\sqrt{\frac{-(C_{12}^2 - C_{11}C_{22} + 2C_{12}C_{66} + \sqrt{(C_{12}^2 - C_{11}C_{22})(-C_{11}C_{22} + (C_{12} + 2C_{66})^2)})}{C_{22}C_{66}}}, \tag{A.1a}$$

$$\lambda_2 = -\frac{1}{\sqrt{2}}\sqrt{\frac{(C_{12}^2 - C_{11}C_{22} + 2C_{12}C_{66} + \sqrt{(C_{12}^2 - C_{11}C_{22})(-C_{11}C_{22} + (C_{12} + 2C_{66})^2)})}{C_{22}C_{66}}}, \tag{A.1b}$$

$$M_1(x,r) = \frac{\wedge_1}{\pi}\int_0^\infty \cos(sx)\sin(sr)ds, \tag{A.2a}$$

$$M_3(x,r) = \frac{1}{\pi}\int_0^\infty (m_1 e^{-s\lambda_1 h} + m_2 e^{-s\lambda_2 h})\sin(sr)\cos(sx)ds, \tag{A.2b}$$

$$M_4(x,r) = \frac{m_3}{\pi}\int_0^\infty (e^{-s\lambda_1 h} - e^{-s\lambda_2 h})\cos(sr)\cos(sx)ds, \tag{A.2c}$$

$$M_6(x,r) = \frac{-\wedge_2}{\pi}\int_0^\infty \cos(sr)\sin(sx)ds, \tag{A.2d}$$

$$M_7(x,r) = \frac{m_4}{\pi}\int_0^\infty (e^{-s\lambda_1 h} - e^{-s\lambda_2 h})\sin(sr)\sin(sx)ds, \tag{A.2e}$$

$$M_8(x,r) = \frac{1}{\pi}\int_0^\infty (-m_5 e^{-s\lambda_1 h} - m_6 e^{-s\lambda_2 h})\cos(sr)\sin(sx)ds, \tag{A.2f}$$

$$M_2(x,r) = M_5(x,r) = N_1(x,r) = 0, \tag{A.2g}$$

$$N_2(x,r) = \frac{1}{\pi}\int_0^\infty \frac{1}{s}(-n_1 e^{-s\lambda_1 h} - n_2 e^{-s\lambda_2 h} - \sigma_T e^{-skh})\cos(sx)\sin(sr)ds, \tag{A.3a}$$

$$N_3(x,r) = \frac{n_5}{\pi}\int_0^\infty \frac{1}{s}\sin(sr)\sin(sx)ds, \tag{A.3b}$$

$$N_4(x,r) = \frac{1}{\pi}\int_0^\infty \frac{1}{s}(n_3 e^{-s\lambda_1 h} + n_4 e^{-s\lambda_2 h} + \tau_T e^{-skh})\sin(sr)\sin(sx)ds, \tag{A.3c}$$

$$\sigma_j = C_{22}\lambda_j\gamma_j - C_{12}, \qquad \tau^{(j)} = C_{66}(\lambda_j + \gamma_j), j = 1,2, \tag{A.4a}$$

$$\sigma_T = C_{12}n_1 + C_{22}n_2 k - \beta_2, \qquad \tau_T = C_{66}(n_1 k - n_2), \tag{A.4b}$$

$$\wedge_1 = \frac{\sigma_2\tau_1 - \sigma_1\tau^{(2)}}{\tau^{(1)}\gamma_2 - \tau^{(2)}\gamma_1}, \qquad \wedge_2 = \frac{\sigma_1\tau_2 - \sigma_2\tau^{(1)}}{\sigma_1 - \sigma_2}, \tag{A.4c}$$

$$m_1 = -\frac{\sigma_1\tau_2}{\tau^{(1)}\gamma_2 - \tau^{(2)}\gamma_1}, \qquad m_2 = \frac{\sigma_2\tau}{\tau^{(1)}\gamma_2 - \tau^{(2)}\gamma_1}, \tag{A.4d}$$

$$m_3 = \frac{\sigma_1\sigma_2}{\sigma_1 - \sigma_2}, \qquad m_4 = \frac{\tau^{(1)}\tau^{(2)}}{\tau^{(1)}\gamma_2 - \tau^{(2)}\gamma_1}, \tag{A.4e}$$

$$m_5 = -\frac{\sigma_2\tau^{(1)}}{\sigma_1 - \sigma_2}, \qquad m_6 = \frac{\sigma_1\tau^{(2)}}{\sigma_1 - \sigma_2}, \tag{A.4f}$$

$$n_1 = -\frac{\sigma_1(\sigma_T + \sigma_2\eta_1}{\sigma_1 - \sigma_2}, \qquad n_2 = \frac{\sigma_2(\sigma_T + \sigma_1\eta_1}{\sigma_1 - \sigma_2}, \tag{A.4g}$$

$$n_3 = \frac{\tau^{(1)}(\sigma_T + \sigma_2\eta_1}{\sigma_1 - \sigma_2}, \qquad n_4 = -\frac{\tau^{(2)}(\sigma_T + \sigma_1\eta_1}{\sigma_1 - \sigma_2}, \tag{A.4h}$$

$$n_5 = \frac{\sigma_T(\tau^{(1)} - \tau^{(2)}) + n_1(\sigma_2\tau^{(1)} - \sigma_1\tau^{(2)})}{\sigma_1 - \sigma_2} + \tau_T. \tag{A.4i}$$

$$g_1(s) = \frac{(1 + \frac{\mu_1}{\mu_2})\ e^{sk^{(1)}h} + (1 - \frac{\mu_1}{\mu_2})\ e^{-sk^{(1)}h}}{(1 + \frac{\mu_1}{\mu_3})(1 + \frac{\mu_1}{\mu_2})\ e^{sk^{(1)}h} - (1 - \frac{\mu_1}{\mu_3})(1 - \frac{\mu_1}{\mu_2})\ e^{-sk^{(1)}h}}, \tag{A.5}$$

$$g_2(s) = -\frac{(1 + \frac{\mu_1}{\mu_3}) + (1 - \frac{\mu_1}{\mu_3})}{(1 + \frac{\mu_1}{\mu_3})(1 + \frac{\mu_1}{\mu_2})\ e^{sk^{(1)}h} - (1 - \frac{\mu_1}{\mu_3})(1 - \frac{\mu_1}{\mu_2})\ e^{-sk^{(1)}h}}, \tag{A.6}$$

$$g_3(s) = \frac{(1 + \frac{\mu_1}{\mu_2}) + (1 - \frac{\mu_1}{\mu_2})}{(1 + \frac{\mu_1}{\mu_3})(1 + \frac{\mu_1}{\mu_2})\ e^{sk^{(1)}h} - (1 - \frac{\mu_1}{\mu_3})(1 - \frac{\mu_1}{\mu_2})\ e^{-sk^{(1)}h}}, \tag{A.7}$$

$$g_4(s) = -\frac{(1+\frac{\mu_1}{\mu_3})\ e^{sk^{(1)}h} + (1-\frac{\mu_1}{\mu_3})\ e^{-sk^{(1)}h}}{(1+\frac{\mu_1}{\mu_3})(1+\frac{\mu_1}{\mu_2})\ e^{sk^{(1)}h} - (1-\frac{\mu_1}{\mu_3})(1-\frac{\mu_1}{\mu_2})\ e^{-sk^{(1)}h}}, \tag{A.8}$$

$$G_n^{(1)} = 2\sqrt{\pi}\frac{\Gamma(n+1/2)}{(n-1)!}, \tag{A.9}$$

$$G_n^{(2)} = 2\sqrt{\pi}\frac{\Gamma(2n-1/2)}{(2n-2)!}, \tag{A.10}$$

$$B_n(s) = \begin{cases} (-1)^{(n-1)/2}\cos(s\ (c+b)/2) & \text{if } n = 1,3,5,... \\ (-1)^{(n-2)/2}\sin(s\ (c+b)/2) & \text{if } n = 0,2,4,.... \end{cases} \tag{A.11}$$

$$\sum_{n=1}^{\infty} c_n\ k_{n1}(x) + \sum_{n=1}^{\infty} d_n\ l_{n1}(x) + \sum_{n=1}^{\infty} e_n\ m_{n1}(x) + \sum_{n=1}^{\infty} f_n\ n_{n1}(x)$$

$$+\sum_{n=1}^{\infty} a_n\ u_{n1}(x) + \sum_{n=1}^{\infty} b_n\ v_{n1}(x) = -p_1(x), \tag{A.12}$$

$$\sum_{n=1}^{\infty} c_n\ o_{n1}(x) + \sum_{n=1}^{\infty} d_n\ p_{n1}(x) + \sum_{n=1}^{\infty} e_n\ q_{n1}(x) + \sum_{n=1}^{\infty} f_n\ r_{n1}(x)$$

$$+\sum_{n=1}^{\infty} a_n\ w_{n1}(x) + \sum_{n=1}^{\infty} b_n\ z_{n1}(x) = 0, b < x < c, \tag{A.13}$$

$$\sum_{n=1}^{\infty} c_n\ k_{n2}(x) + \sum_{n=1}^{\infty} d_n\ l_{n2}(x) + \sum_{n=1}^{\infty} e_n\ m_{n2}(x) + \sum_{n=1}^{\infty} f_{n2}\ n_{n}(x)$$

$$+\sum_{n=1}^{\infty} a_n\ u_{n2}(x) + \sum_{n=1}^{\infty} b_n\ v_{n2}(x) = -p_2(x), \tag{A.14}$$

$$\sum_{n=1}^{\infty} c_n\ o_{n2}(x) + \sum_{n=1}^{\infty} d_n\ p_{n2}(x) + \sum_{n=1}^{\infty} e_n\ q_{n2}(x) + \sum_{n=1}^{\infty} f_n\ r_{n2}(x)$$

$$+\sum_{n=1}^{\infty} a_n\ w_{n1}(x) + \sum_{n=1}^{\infty} b_n\ z_{n1}(x) = 0, 0 < x < a. \tag{A.15}$$

where $k_{nj},\ l_{nj},\ m_{nj},\ n_{nj},\ o_{nj},\ p_{nj},\ q_{nj},\ r_{nj},\ u_{nj},\ v_{nj},\ w_{nj},\ z_{nj},\ j = 1,2$ are all known integral terms whose values are computed numerically.

$$k_{n1}(x) = -\sqrt{\frac{2}{\pi}}\int_0^{\infty}(Q_1(s)/s)J_n(l_o\ \ s) \times ((1/2 \times \cos(n\pi/2) \times \{\cos((x_o\ -x)s)$$

$$\begin{aligned}
&+\cos((x_o\ +x)s)\}+1/2\times\sin(n\pi/2)\times\{\sin((x_o\ +x)s)\\
&+\sin((x_o\ -x)s)\}))ds, \qquad (A.16)
\end{aligned}$$

$$\begin{aligned}
l_{n1}(x)=&\sqrt{\frac{2}{\pi}}\int_0^\infty(Q_2(s)/s-\beta_{2L}J_n(l_o\ s)\times(1/2(\cos(n\pi/2)\times\{\sin((x_o\ +x)s)\\
&+\sin((x_o\ -x)s)\}+\sin(n\pi/2)\times\{\cos((x_o\ +x)s)+\cos((x_o\ -x)s)\})ds\\
&-\sqrt{\frac{2}{\pi}}\frac{\beta_{2L}\sin\{n\pi/2-n\sin^{-1}((x_o\ -x)/l_o)\}}{2\{((l_o)^2-(x_o\ -x)^2)^{1/2}\}}\\
&+\sqrt{\frac{2}{\pi}}\frac{\beta_{2L}\times(l_o)^n}{\{((x_o\ +x)^2-l_o^2)^{1/2}\{(x_o\ +x)+((x_o\ +x)^2-l_o^2)^{1/2}\}^n\}}, \qquad (A.17)
\end{aligned}$$

$$m_{n1}(x)=2n\sqrt{\frac{2}{\pi}}\int_0^\infty Q_3(s)/s\times J_{2n}(as)\cos(sx)ds, \qquad (A.18)$$

$$n_{n1}(x)=(2n-1)\ \sqrt{\frac{2}{\pi}}\int_0^\infty Q_4(s)/s\times J_{2n-1}(as)\cos(sx)ds, \qquad (A.19)$$

$$\begin{aligned}
o_{n1}(x)=&\sqrt{\frac{2}{\pi}}\int_0^\infty(Q_7(s)/s-\beta_{7L})J_n(l_o\ s)\times(1/2(\cos(n\pi/2)\times\{\sin((x_o\ +x)s)\\
&-\sin((x_o\ -x)s)\}-\sin(n\pi/2)\times\{\cos((x_o\ +x)s)\\
&-\cos((x_o\ -x)s)\}))ds+\sqrt{\frac{2}{\pi}}\frac{\beta_{7L}\sin\{n\pi/2-n\sin^{-1}((x_o\ -x)/l_o)\}}{2\{((l_o)^2-(x_o\ -x)^2)^{1/2}\}}\\
&+\sqrt{\frac{2}{\pi}}\frac{\beta_{7L}\times(l_o)^n}{2\{((x_o\ +x)^2-l_o^2)^{1/2}\{(x_o\ +x)+((x_o\ +x)^2-l_o^2)^{1/2}\}^n\}}, \qquad (A.20)
\end{aligned}$$

$$\begin{aligned}
p_{n1}(x)=&\sqrt{\frac{2}{\pi}}\int_0^\infty Q_8(s)/s\times J_n(l_o\ s)\times(1/2(\cos(n\pi/2)\times\{\cos((x_o\ -x)s)\\
&-\cos((x_o\ +x)s)\}+\sin(n\pi/2)\times\{\sin((x_o\ -x)s)\\
&-\sin((x_o\ +x)s)\}))ds, \qquad (A.21)
\end{aligned}$$

$$q_{n1}(x)=2n\ \sqrt{\frac{2}{\pi}}\int_0^\infty Q_9(s)/s\times J_{2n}(as)\sin(sx)ds, \qquad (A.22)$$

$$r_{n1}(x)=(2n-1)\ \sqrt{\frac{2}{\pi}}\int_0^\infty Q_{10}(s)/s\times J_{2n-1}(as)\sin(sx)ds, \qquad (A.23)$$

$$\begin{aligned}
k_{n2}(x)=&\sqrt{\frac{2}{\pi}}\int_0^\infty(Q_{13}(s)/s)J_n(l_o\ s)\times((\cos(n\pi/2)\times\{\cos((x_o\ -x)s)\\
&+\cos((x_o\ +x)s)\}+\sin(n\pi/2)\times\{\sin((x_o\ +x)s)\\
&+\sin((x_o\ -x)s)\}))ds, \qquad (A.24)
\end{aligned}$$

$$\begin{aligned} l_{n2}(x) =& \sqrt{\frac{2}{\pi}} \int_0^\infty (Q_{14}(s)/s) J_n(l_o\ s) \times ((\cos(n\pi/2) \times \{\sin((x_o\ + x)s) \\ &+ \sin((x_o\ - x)s)\} - \sin(n\pi/2) \times \{\cos((x_o\ - x)s) \\ &+ \cos((x_o\ + x)s)\}))ds, \end{aligned} \tag{A.25}$$

$$m_{n2}(x) = 2n \sqrt{\frac{2}{\pi}} \int_0^\infty Q_{15}(s)/s \times J_{2n}(as) \cos(sx) ds, \tag{A.26}$$

$$\begin{aligned} n_{n2}(x) =& (2n-1) \sqrt{\frac{2}{\pi}} \int_0^\infty (Q_{16}(s)/s - \beta_{16L}) J_{2n-1}(as) \cos(sx) ds \\ &+ \beta_{16}^{L} \frac{\cos\big((2n-1)\sin^{-1}(x/a)\big)}{(a^2-x^2)^{1/2}}, \end{aligned} \tag{A.27}$$

$$\begin{aligned} o_{n2}(x) =& \sqrt{\frac{2}{\pi}} \int_0^\infty Q_{19}(s)/s \times J_n(l_o\ s) \times (1/2(\cos(n\pi/2) \times \{\sin((x_o\ + x)s) \\ &- \sin((x_o\ - x)s)\} + \sin(n\pi/2) \times \{\cos((x_o\ - x)s) \\ &- \cos((x_o\ + x)s)\}))ds, \end{aligned} \tag{A.28}$$

$$\begin{aligned} p_{n2}(x) =& \sqrt{\frac{2}{\pi}} \int_0^\infty Q_{20}(s)/s \times J_n(l_o\ s) \times (1/2(\cos(n\pi/2) \times \{\cos((x_o\ - x)s) \\ &- \cos((x_o\ + x)s)\} - \sin(n\pi/2) \times \{\sin((x_o\ + x)s) - \sin((x_o\ - x)s)\}))ds, \end{aligned} \tag{A.29}$$

$$\begin{aligned} q_{n2}(x) =& 2n \sqrt{\frac{2}{\pi}} \int_0^\infty (Q_{21}(s)/s - \beta_{21L}) J_{2n}(as) \sin(sx) ds \\ &+ \beta_{21L} \frac{\sin\big((2n)\sin^{-1}(x/a)\big)}{(a^2-x^2)^{1/2}} \end{aligned} \tag{A.30}$$

$$r_{n2}(x) = (2n-1) \sqrt{\frac{2}{\pi}} \int_0^\infty Q_{22}(s)/s \times J_{2n-1}(as) \sin(sx) ds, \tag{A.31}$$

$$u_{n1}(x) = \sqrt{\frac{2}{\pi}} \times (n) \int_0^\infty (Q_5(s)/s) \times J_n(sl_o) \times \sin(l_o s - n\pi/2) \times \cos(sx) ds, \tag{A.32}$$

$$v_{n1}(x) = \sqrt{\frac{2}{\pi}} \times (2n-1) \int_0^\infty (Q_6(s)/s) \times J_{2n-1}(as) \times \cos(sx) ds, \tag{A.33}$$

$$w_{n1}(x) = \sqrt{\frac{2}{\pi}} \times (n) \int_0^\infty (Q_{11}(s)/s) J_n(sl_o) \sin(l_o s - n\pi/2) \sin(sx) ds, \tag{A.34}$$

$$z_{n1}(x) = \sqrt{\frac{2}{\pi}} \times (2n-1) \int_0^\infty (Q_{12}(s)/s) \times J_{2n-1}(as) \times \sin(sx) ds, \tag{A.35}$$

$$u_{n2}(x) = \sqrt{\frac{2}{\pi}} \times (n) \int_0^\infty (Q_{17}(s)/s) \times J_n(sl_o) \times \sin(l_o s - n\pi/2) \cos(sx) ds, \tag{A.36}$$

$$v_{n2}(x) = \sqrt{\frac{2}{\pi}} \times (2n-1) \int_0^\infty (Q_{18}(s)/s) \times J_{2n-1}(as)\cos(sx)ds, \tag{A.37}$$

$$w_{n2}(x) = \sqrt{\frac{2}{\pi}} \times (n) \int_0^\infty (Q_{23}(s)/s) \times J_n(sl_o) \times \sin(l_o s - n\pi/2)\cos(sx)ds, \tag{A.38}$$

$$z_{n2}(x) = \sqrt{\frac{2}{\pi}} \times (2n-1) \int_0^\infty (Q_{24}(s)/s) \times J_{2n-1}(as)\cos(sx)ds. \tag{A.39}$$

where $Q_i(s)'$s are all known functions and $\beta_{iL} = Q_i(s)/s$ for very large values of s.

Bibliography

[1] Z. K. Eshkuvatov, N. M. A. N. Long, and M. Abdulkawi. Approximate solution of singular integral equations of the first kind with cauchy kernel. *Applied Mathematics Letters*, 22(5):651–657, 2009.

[2] R. N. Ghosh. Failure analysis: techniques and few case studies. Technical report, NML, 1997.

[3] A. Öchsner. *Continuum damage mechanics.* Springer, 2016.

[4] T. L. Anderson. *Fracture mechanics: fundamentals and applications.* CRC press, 2005.

[5] L.B. Freund. *Dynamic Fracture Mechanics.* Cambridge Monographs on Mechanics. Cambridge University Press, 1998.

[6] X.-C. Zhong and B. Wu. Thermoelastic analysis for an opening crack in an orthotropic material. *International Journal of Fracture*, 173(1):49–55, 2012.

[7] A. Tanwar, S. Das, E.-M. Craciun, and H. Altenbach. Interaction among interfacial offset cracks in composite materials under the anti-plane shear loading. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, 103(11):e202300081, 2023.

[8] B. Wu, D. Peng, and R. Jones. On the analysis of cracking under a combined quadratic thermal flux and a quadratic mechanical loading. *Applied Mathematical Modelling*, 68:182–197, 2019.

[9] H. J. Choi. Thermal stresses due to a uniform heat flow disturbed by a pair of offset parallel cracks in an infinite plane with orthotropy. *European Journal of Mechanics-A/Solids*, 63:1–13, 2017.

[10] T. Rich and D. Cartwright. Case studies in fracture mechanics. Technical report, Army Materials and Mechanics Research Center, 06 1977.

[11] T. L. Anderson. *Fracture mechanics: Fundamentals and Applications.* CRC press, 2017.

[12] A. A. Griffith. The phenomena of rupture and flow in solids. *Philosophical transactions of the Royal Society of London. Series A, containing papers of a mathematical or physical character*, 221(582-593):163–198, 1921.

[13] G. R. Irwin. Onset of fast crack propagation in high strength steel and aluminum alloys. Technical report, Naval Research Laboratory Washington, DC, USA, 1956.

[14] H. M. Westergaard. Bearing pressures and cracks: Bearing pressures through a slightly waved surface or through a nearly flat part of a cylinder, and related problems of cracks. *Journal of Applied Mechanics*, 2, 1939.

[15] M. L. Williams. On the stress distribution at the base of a stationary crack. *Journal of Applied Mechanics*, 1, 1957.

[16] A. A. Wells. Unstable crack propagation in metals: cleavage and fast fracture. In *Proceedings of the crack propagation symposium*, volume 1, page 26028. Cranfield, UK, 1961.

[17] J. R. Rice. A path independent integral and the approximate analysis of strain concentration by notches and cracks. *Journal of Applied Mechanics*, 1968.

[18] P. Kumar. *Elements of fracture mechanics*. McGraw-Hill Education LLC., 2009.

[19] S. Timoshenko. *History of strength of materials: with a brief account of the history of theory of elasticity and theory of structures*. Courier Corporation, 1983.

[20] E. Orowan. Fracture and strength of solids. *Reports on progress in physics*, 12(1):185, 1949.

[21] M. H Sadd. *Elasticity: theory, applications, and numerics*. Academic Press, 2009.

[22] T. C. T. Ting. *Anisotropic elasticity: theory and applications*, volume 45. Oxford university press, 1996.

[23] H. Deng, B. Yan, H. Su, X. Zhang, and X. Lv. An interaction integral method for calculating heat flux intensity factor with the xfem. *International Journal of Thermal Sciences*, 136:379–388, 2019.

[24] G. C. Sih. Heat Conduction in the Infinite Medium With Lines of Discontinuities. *Journal of Heat Transfer*, 87(2):293–298, 05 1965.

[25] C. K. Chao and R. C. Chang. Thermal interface crack problems in dissimilar anisotropic media. *Journal of Applied Physics*, 72(7):2598–2604, 10 1992.

[26] G. C. Sih, P. C. Paris, and G. R. Irwin. On cracks in rectilinearly anisotropic bodies. *International Journal of Fracture Mechanics*, 1:189–203, 1965.

[27] C. E. Inglis. Stresses in a plate due to the presence of cracks and sharp corners. *Spie Milestone series MS*, 137:3–17, 1997.

[28] H. Altenbach, J. Altenbach, and W. Kissing. Classification of composite materials. *Mechanics of Composite Structural Elements*, 2018.

[29] F. C. Moon and L. J. Broutman. Wave propagation and impact in composite materials. *Composite Materials Vol*, 7:259–332, 1975.

[30] H. Liebowitz. *Fracture: an advanced treatise. 1. Microscopic and macroscopic fundamentals*. Academic Press, 1968.

[31] G. W. Milton. *The theory of composites*. SIAM, 2022.

[32] J. R. Willis. A comparison of the fracture criteria of griffith and barenblatt. *Journal of the Mechanics and Physics of Solids*, 15(3):151–162, 1967.

[33] J. D. Eshelby. The elastic field of a crack extending non-uniformly under general anti-plane loading. *Journal of the Mechanics and Physics of Solids*, 17(3):177–199, 1969.

[34] B. V. Kostrov. Crack propagation at variable velocity. *Journal of Applied Mathematics and Mechanics*, 38(3):511–519, 1974.

[35] L. B. Freund and Y.J. Lee. Observations on high strain rate crack growth based on a strip yield model. *International Journal of Fracture*, 42:261–276, 1990.

[36] S. Itou. Thermal stresses around two parallel cracks in an infinite orthotropic plate under uniform heat flow. *Journal of Thermal Stresses*, 24(7):677–694, 2001.

[37] X-C Zhong, B. Wu, and K-S Zhang. Thermally conducting collinear cracks engulfed by thermomechanical field in a material with orthotropy. *Theoretical and Applied Fracture Mechanics*, 65:61–68, 2013.

[38] S. Thangjitham and H. J. Choi. Thermal stress singularities in an anisotropic slab containing a crack. *Mechanics of Materials*, 14(3):223–238, 1993.

[39] F. G. Tricomi. *Integral Equations*. Dover Books on Mathematics. Dover Publications, 2012.

[40] I. S. Gradshteyn and I. M. Ryzhik. *Table of integrals, series, and products*. Academic press, 2014.

[41] M. D. Raisinghania. *Integral equations and boundary value problems*. S. Chand Publishing, 2007.

[42] S. Itou. Thermal stress intensity factors of an infinite orthotropic layer with a crack. *International Journal of Fracture*, 103(3):279–291, 2000.

[43] P. M. Morse and H. Feshbach. Methods of theoretical physics. *American Journal of Physics*, 22(6):410–413, 1954.

[44] J. B. Lawrie and I. D. Abrahams. A brief historical perspective of the wiener–hopf technique. *Journal of Engineering Mathematics*, 59:351–358, 2007.

[45] A. Kisil, I. Abrahams, G. Mishuris, and S. Rogosin. The wiener–hopf technique, its generalizations and applications: constructive and approximate methods. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 477, 10 2021.

[46] S. Ponnusamy and H. Silverman. *Complex variables with applications*. Springer Science & Business Media, 2007.

[47] B. Noble. *The Wiener-Hopf Technique: Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations*. Pergamon Press, 1958.

[48] H. Altenbach. On the determination of transverse shear stiffnesses of orthotropic plates. *Zeitschrift für Angewandte Mathematik und Physik ZAMP*, 51(4):629–649, 2000.

[49] H. Altenbach. Mechanics of advanced materials for lightweight structures. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 225(11):2481– 2496, 2011.

[50] H. Altenbach, J. Altenbach, and W. Kissing. *Mechanics of composite structural elements*. Springer, 2nd edition, 2018.

[51] S. Das. Interaction between line cracks in an orthotropic layer. *International Journal of Mathematics and Mathematical Sciences*, 29(1):31–42, 2002.

[52] J. R. Rice et al. Mathematical analysis in the mechanics of fracture. *Fracture, an advanced treatise*, 2:191–311, 1968.

[53] M. W. Hyer and S. R. White. *Stress analysis of fiber-reinforced composite materials*. DEStech Publications, Inc, 2009.

[54] Y. M. Tsai. Orthotropic thermoelastic problem of uniform heat flow disturbed by a central crack. *Journal of Composite Materials*, 18(2):122–131, 1984.

[55] G. C. Sih. On the singular character of thermal stresses near a crack tip. *ASME Journal of Applied Mechanics*, 29(3):587–589, 1962.

[56] D. D. Ang and M. L. Williams. Combined stresses in an orthotropic plate having a finite crack. *ASME Journal of Applied Mechanics*, 28(3):372–378, 1961.

[57] A. N. Stroh. Dislocations and cracks in anisotropic elasticity. *Philosophical magazine*, 3(30):625–646, 1958.

[58] A. L. Florence and J. N. Goodier. The linear thermoelastic problem of uniform heat flow disturbed by a penny-shaped insulated crack. *International Journal of Engineering Science*, 1(4):533–540, 1963.

[59] A. L. Florence and J. N. Goodier. Thermal stresses due to disturbance of uniform heat flow by an insulated ovaloid hole. *ASME Journal of Applied Mechanics*, 27(4):635–639, 1960.

[60] S. Chang. Steady-state temperatures in an infinite medium split by a pair of coplanar cracks. *ASME Journal of Heat Transfer*, 110(2):283–289, 1988.

[61] C. Baoxing and Z. Xiangzhou. Orthotropic thermoelasticity problem of symmetrical heat flow disturbed by three coplanar cracks. *International Journal of Fracture*, 67(4):301–314, 1994.

[62] T. Zheng, Z. Zhu, B. Wang, and L. Zeng. Stress intensity factor for an infinite plane containing three collinear cracks under compression. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, 94(10):853–861, 2014.

[63] W. B. Peng and J. C. Sung. Interactions of two arbitrarily oriented cracks in a homogeneous anisotropic medium. *Applied Mathematical Modelling*, 27(9):701–715, 2003.

[64] C. Mauge and M. Kachanov. Anisotropic material with interacting arbitrarily oriented cracks. Stress intensity factors and crack - microcrack interactions. *International Journal of Fracture, Springer*, 65(2):115–139, 1994.

[65] R. T. Faal and S. J. Fariborz. Stress analysis of orthotropic planes weakened by cracks. *Applied Mathematical Modelling*, 31(6):1133–1148, 2007.

[66] W. K. Binienda and S. M. Arnold. Driving force analysis in an infinite anisotropic plate with multiple crack interactions. *International Journal of Fracture*, 71(3):213–245, 1995.

[67] H. Sekine. Thermoelastic interference between two neighbouring cracks (insulated cracks). *Transactions of the Japan Society of Mechanical Engineers*, 45:1058–1063, 1979.

[68] J. Aboudi, S.M. Arnold, and B.A. Bednarcyk. *Micromechanics of Composite Materials: A Generalized Multiscale Analysis Approach.* Butterworth-Heinemann, 2013.

[69] J. L. Nowinski. *Theory of thermoelasticity with applications*, volume 3. Springer, 1978.

[70] A. I. Lurie. *Theory of Elasticity.* Foundations of Engineering Mechanics. Springer, Berlin-Heidelberg, 2005.

[71] N. I. Muskhelishvili. *Singular integral equations: boundary problems of function theory and their application to mathematical physics.* Springer, Dordrecht, 2008.

[72] F. Erdogan. Mixed boundary value problems. *Mechanics Today*, 4:1–86, 2013.

[73] C. C. Chamis. Simplified composite micromechanics equations for hygral, thermal and mechanical properties. *Annual Conference of the Society of the Plastics Industry (SPI) Reinforced Plastics/Composites Institute*, 15:14–23, 1983.

[74] A. H. England. A crack between dissimilar media. *Journal of Applied Mechanics*, 32:400–402, 1965.

[75] F. Erdogan. Stress distribution in bonded dissimilar materials with cracks. *Journal of Applied Mechanics*, 32:403–410, 1965.

[76] J. R. Rice and G. C. Sih. Plane problems of cracks in dissimilar media. *Journal of Applied Mechanics*, 32:418–423, 1965.

[77] D. L. Clements. A crack between dissimilar anisotropic media. *International Journal of Engineering Science*, 9(2):257–265, 1971.

[78] J. L. Bassani and J. Qu. Finite crack on bimaterial and bicrystal interfaces. *Journal of the Mechanics and Physics of Solids*, 37(4):435–453, 1989.

[79] Z. Suo. Singularities, interfaces and cracks in dissimilar anisotropic media. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, 427(1873):331–358, 1990.

[80] T. C. T. Ting. Interface cracks in anisotropic bimaterials. *Journal of the Mechanics and Physics of Solids*, 38(4):505–513, 1990.

[81] H. Gao, M. Abbudi, and D. M. Barnett. Interfacial crack-tip field in anisotropic elastic solids. *Journal of the Mechanics and Physics of Solids*, 40(2):393–416, 1992.

[82] X. Wang and Y. Shen. Exact solution for mixed boundary value problems at anisotropic piezoelectric bimaterial interface and unification of various interface defects. *International Journal of Solids and Structures*, 39(6):1591–1619, 2002.

[83] J. R. Willis. Fracture mechanics of interfacial cracks. *Journal of the Mechanics and Physics of Solids*, 19(6):353–368, 1971.

[84] X-F. Wu, Y. A. Dzenis, and W-S. Zou. Interfacial edge crack between two bonded dissimilar orthotropic strips under antiplane point loading. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik: Applied Mathematics and Mechanics*, 83(6):419–422, 2003.

[85] X. Wang and E. Pan. Antiplane shear deformations of an anisotropic elliptical inhomogeneity with imperfect or viscous interface. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik: Applied Mathematics and Mechanics*, 88(2):142–150, 2008.

[86] M. Kachanov. A simple technique of stress analysis in elastic solids with many cracks. *International Journal of Fracture*, 28(1):R11–R19, 1985.

[87] W. Binienda, A. S. D. Wang, and F. Delale. Analysis of bent crack in unidirectional fibre reinforced composites. *International Journal of Fracture*, 47(1):1–24, 1991.

[88] T. Sadowski, L. Marsavina, E-M. Craciun, and M. Kneć. Modelling and experimental study of parallel cracks propagation in an orthotropic elastic material. *Computational Materials Science*, 52(1):231–235, 2012.

[89] X-F. Li and Kang Y. Lee. Closed-form solution for an orthotropic elastic strip with a crack perpendicular to the edges under arbitrary anti-plane shear. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik: Applied Mathematics and Mechanics*, 89(5):370–382, 2009.

[90] A. C. Eringen. Continuum mechanics at the atomic scale. Technical report, Princton University, Department of Civil Engineering, 1977.

[91] A. C. Eringen. Line crack subject to shear. *International Journal of Fracture*, 14(4):367–379, 1978.

[92] A. C. Eringen. Line crack subject to antiplane shear. *Engineering Fracture Mechanics*, 12(2):211–219, 1979.

[93] Z. Zhou, J. Han, and S. Du. Investigation of a griffith crack subject to anti-plane shear by using the non-local theory. *International Journal of Solids and Structures*, 36(26):3891–3901, 1999.

[94] W. G. Knauss. An observation of crack propagation in anti-plane shear. *International Journal of Fracture Mechanics*, 6(2):183–187, 1970.

[95] M. Zappalorto and M. Salviato. Antiplane shear stresses in orthotropic plates with lateral blunt notches. *European Journal of Mechanics-A/Solids*, 77:103815, 2019.

[96] S. Karan, P. Mandal, S. Basu, and S. C. Mandal. Interaction of shear waves with semi-infinite moving crack inside of a orthotropic media. *Waves in Random and Complex Media*, pages 1–17, 2021.

[97] E-M. Craciun, T. Sadowski, and A. Răbâea. Stress concentration in an anisotropic body with three equal collinear cracks in mode ii of fracture. i. analytical study. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, 94(9):721–729, 2014.

[98] S. Das. Interaction of moving interface collinear griffith cracks under antiplane shear. *International Journal of Solids and Structures*, 43(25-26):7880–7890, 2006.

[99] N. Gorbushin, V. A. Eremeyev, and G. Mishuris. On stress singularity near the tip of a crack with surface stresses. *International Journal of Engineering Science*, 146:103183, 2020.

[100] H. J. Choi. Thermal stresses due to a uniform heat flow disturbed by a pair of offset parallel cracks in an infinite plane with orthotropy. *European Journal of Mechanics-A/Solids*, 63:1–13, 2017.

[101] A. Tanwar, R. Singh, S. Das, and H. Altenbach. Interaction among offset parallel cracks in an orthotropic plane under thermo-mechanical loading. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, page e202100593, 2022.

[102] Z-G. Zhou, Y. Chen, and B. Wang. The behavior of two parallel interface cracks in magneto–electro–elastic materials under an anti-plane shear stress loading. *Composite Structures*, 77(1):97–103, 2007.

[103] S. Das and B. Patra. Moving griffith crack at the interface of two dissimilar orthotropic half planes. *Engineering Fracture Mechanics*, 54(4):523–531, 1996.

[104] C. O. Horgan. Anti-plane shear deformations in linear and nonlinear solid mechanics. *SIAM review*, 37(1):53–81, 1995.

[105] A. Erdelyi, W. Magnus, F. Oberhettinger, and F.G. Tricomi. *Tables of Integral Transforms: Vol.: 2*. McGraw-Hill Book Company, Incorporated, 1954.

[106] J. S. Lee, S. M. Kwon, K. Y. Lee, and J. H. Kwon. Anti-plane interfacial yoffe-crack between a piezoelectric and two orthotropic layers. *European Journal of Mechanics-A/Solids*, 21(3):483–492, 2002.

[107] D. L. Clements. A crack between dissimilar anisotropic media. *International Journal of Engineering Science*, 9(2):257–265, 1971.

[108] Y. Shindo, H. Nozaki, and H. Higaki. Impact response of a finite crack in an orthotropic strip. *Acta Mechanica*, 62(1-4):87–104, 1986.

[109] C.-Q. Ru and P. Schiavone. On the elliptic inclusion in anti-plane shear. *Mathematics and Mechanics of Solids*, 1(3):327–333, 1996.

[110] J. K. Knowles. The finite anti-plane shear field near the tip of a crack for a class of incompressible elastic solids. *International Journal of Fracture*, 13(5):611–639, 1977.

[111] Z. C. Ou and X. Wu. On the crack-tip stress singularity of interfacial cracks in transversely isotropic piezoelectric bimaterials. *International Journal of Solids and Structures*, 40(26):7499–7511, 2003.

[112] G. Mikhasev, Barış Erbaş, and V. A. Eremeyev. Anti-plane shear waves in an elastic strip rigidly attached to an elastic half-space. *International Journal of Engineering Science*, 184:103809, 2023.

[113] S. Karan, S. K. Panja, S. Basu, and S. C. Mandal. Edge crack subject to anti-plane shear wave in an orthotropic strip. *Journal of Elasticity*, pages 1–15, 2023.

[114] W. G. Knauss. Stresses in an infinite strip containing a semi-infinite crack. *Journal of Applied Mechanics*, 33(2):356–362, 1966.

[115] J. R. Rice. Stresses in an infinite strip containing a semi-infinite crack. *Trans. ASME, Ser. E, J. Appl. Mech.*, 34:248–250, 1967.

[116] N. Hasebe. Stress analysis for an orthotropic elastic half plane with an oblique edge crack and stress intensity factors. *Acta Mechanica*, 232:967–982, 2021.

[117] N. Wiener. Uber eine klasse singularer integralgleichungen. *Sitz. Ber. Preuss. Akad. Wiss., Phys.-Math.*, 1:696–706, 1931.

[118] J. B. Keller. Progress and prospects in the theory of linear wave propagation. *SIAM Review*, 21(2):229–245, 1979.

[119] A. V. Kisil, I. D. Abrahams, G. Mishuris, and S. V. Rogosin. The wiener–hopf technique, its generalizations and applications: constructive and approximate methods. *Proceedings of the Royal Society A*, 477(2254):20210533, 2021.

[120] E. H. Yoffe. Lxxv. the moving griffith crack. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 42(330):739–750, 1951.

[121] J. R. Rice. Stresses Due to a Sharp Notch in a Work-Hardening Elastic-Plastic Material Loaded by Longitudinal Shear. *Journal of Applied Mechanics*, 34(2):287–298, 1967.

[122] F. Nilsson. Dynamic stress-intensity factors for finite strip problems. *International Journal of Fracture Mechanics*, 8:403–411, 1972.

[123] C. Atkinson and C. H. Popelar. Antiplane dynamic crack propagation in a viscoelastic strip. *Journal of the Mechanics and Physics of Solids*, 27(5-6):431–439, 1979.

[124] J. Chaoufi, D. Gamby, and D. Benicchio. Asymptotic stress field in a cracked orthotropic strip. *Composite structures*, 32(1-4):467–475, 1995.

[125] A. K. Singh and A. K. Singh. Dynamic stress concentration of a smooth moving punch influenced by a shear wave in an initially stressed dry sandy layer. *Acta Mechanica*, 233(5):1757–1768, 2022.

[126] P. Basak and S. C. Mandal. Semi-infinite moving crack in an orthotropic strip. *International Journal of Solids and Structures*, 128:221–230, 2017.

[127] S. Naskar and S. C. Mandal. Moving semi-infinite crack between dissimilarorthotropic strips. *Waves in Random and Complex Media*, pages 1–16, 2022.

[128] S. Itou. Stress intensity factors around a moving griffith crack in an infinite elastic layer between two elastic half-planes. *International Journal of the Society of Materials Engineering for Resources*, 9(1-2):32–38, 2001.

[129] S. Itou. Strain energy release rate for an interface crack in linearized couple-stress theory. *Engineering fracture mechanics*, 40(2):421–432, 1991.

[130] X. Wang, E. Pan, and W. J. Feng. Anti-plane Green's functions and cracks for piezoelectric material with couple stress and electric field gradient effects. *European Journal of Mechanics-A/Solids*, 27(3):478–486, 2008.

[131] B. Wu, D. Peng, and R. Jones. On thermoelastic analysis of two collinear cracks subject to combined quadratic thermo-mechanical load. *Applied Mathematics and Computation*, 421:126905, 2022.

[132] Y. Zhou, X. Li, and D. Yu. A partially insulated interface crack between a graded orthotropic coating and a homogeneous orthotropic substrate under heat flux supply. *International Journal of Solids and Structures*, 47(6):768–778, 2010.

[133] J. Wang, M. Dai, and C.-F. Gao. The effect of interfacial thermal resistance on interface crack subjected to remote heat flux. *Zeitschrift für angewandte Mathematik und Physik*, 71:1–21, 2020.

[134] C. K. Chao and R. C. Chang. Thermal interface crack problems in dissimilar anisotropic media. *Journal of Applied Physics*, 72(7):2598–2604, 1992.

[135] M. Rasouli and M. Jafari. Thermal stress analysis of infinite anisotropic plate with elliptical hole under uniform heat flux. *Journal of Thermal Stresses*, 39(11):1341–1355, 2016.

[136] M. Jafari and M. Jafari. Effect of uniform heat flux on stress distribution around a triangular hole in anisotropic infinite plates. *Journal of Thermal Stresses*, 41(6):726–747, 2018.

[137] E. J. Brown and F. Erdogan. Thermal stresses in bonded materials containing cuts on the interface. *International Journal of Engineering Science*, 6(9):517–529, 1968.

[138] A.-Y. Kuo. Interface crack between two dissimilar half spaces subjected to a uniform heat flow at infinity—open crack. *Journal of Applied Mechanics*, 57(2):359–364, 1990.

[139] K. Y. Lee and S.-J. Park. Thermal stress intensity factors for partially insulated interface crack under uniform heat flow. *Engineering Fracture Mechanics*, 50(4):475–482, 1995.

[140] C. Wen-Hwa and C. Chi-Lone. Heat conduction analysis of a plate with multiple insulated cracks by the finite element alternating method. *International journal of solids and structures*, 31(10):1343–1355, 1994.

[141] W.-H. Chen and Y.-M. Tu. Thermal analysis for two-dimensional fracture problems using the boundary element alternating method. *Computers & structures*, 50(3):401–408, 1994.

[142] G. C. Sih. Mechanics of fracture initiation and propagation: surface and volume energy density applied as failure criterion. *Journal of Applied Mechanics*, 11, 2012.

[143] S. Das and B. Patra. Stress intensity factors for moving interfacial crack between bonded dissimilar fixed orthotropic layers. *Computers & Structures*, 69(4):459–472, 1998.

[144] S. Das. Interaction of moving interface collinear Griffith cracks under antiplane shear. *International Journal of Solids and Structures*, 43(25):7880–7890, 2006.

[145] S. Das and A. Tanwar. Study of an interfacial semi-infinite crack in a composite structure. *Acta Mechanica*, pages 1–17, 2024.

[146] W. F. Yau. Axisymmetric slipless indentation of an infinite, elastic cylinder. *SIAM Journal on Applied Mathematics*, 15(1):219–227, 1967.

[147] S. Itou. Transient dynamic stress intensity factors around three stacked parallel cracks in an infinite medium during passage of an impact normal stress. *International Journal of Solids and Structures*, 78:199–204, 2016.

[148] S. Itou. Stress intensity factors for three cracks at the interfaces of a graded layer bonding two different materials. *Applied Mathematical Modelling*, 37(4):2516–2530, 2013.

[149] Y. Song, Y. Lu, and H. Hu. Modified multiplying-factor integration method for solving exponential function dual integrals in crack problems. *Acta Mechanica Sinica*, 38(6):421287, 2022.

[150] N. Hasebe, K. Tamai, and T. Nakamura. Analysis of kinked crack under uniform heat flow. *Journal of engineering mechanics*, 112(1):31–42, 1986.

[151] D. Huachao, Y. Bo, S. Honghong, Z. Xiaomin, and L. Xin. Study on transient heat flux intensity factor with interaction integral. *International Journal of Thermal Sciences*, 146:106014, 2019.

[152] W. Thomson. 4. on a mechanical theory of thermo-electric currents. *Proceedings of the Royal society of Edinburgh*, 3:91–98, 1857.

[153] M. A. Biot and D. G. Willis. The elastic coefficients of the theory of consolidation. *American Society of Mechanical Engineers*, 1957.

[154] P. Chadwick and B. Powdrill. Singular surfaces in linear thermoelasticity. *International Journal of Engineering Science*, 3(6):561–595, 1965.

[155] D. S. Chandrasekharaiah. Thermoelasticity with Second Sound: A Review. *Applied Mechanics Reviews*, 39(3):355–376, 1986.

[156] H. W. Lord and Y. Shulman. A generalized dynamical theory of thermoelasticity. *Journal of the Mechanics and Physics of Solids*, 15(5):299–309, 1967.

[157] M. B. Nazari and M. M. Rokhi. Evaluation of sifs for cracks under thermal impact based on green-naghdi theory. *Theoretical and Applied Fracture Mechanics*, 107:102557, 2020.

[158] D. Y. Tzou. A Unified Field Approach for Heat Conduction From Macro- to Micro-Scales. *Journal of Heat Transfer*, 117(1):8–16, 1995.

List of Publications

- **A. Tanwar**, R. Singh, S. Das, H. Altenbach: *Interaction among offset parallel cracks in an orthotropic plane under thermo-mechanical loading.* ZAMM - Zeitschrift für Angewandte Mathematik und Mechanik. 102(7), e202100593, (2022), 10.1002/zamm.202100593.

- **A. Tanwar**, S. Das, E-M Craciun, H Altenbach: *Interaction among interfacial offset cracks in composite materials under the anti-plane shear loading*, ZAMM -Zeitschrift für Angewandte Mathematik und Mechanik. 103(11), e202300081, (2023), 10.1002/zamm.202300081.

- S. Das, **A. Tanwar**: *Study of an interfacial semi-infinite crack in a composite structure*, Acta Mechanica. 235, 4961–4977, (2024). 10.1007/s00707-024-03980-5

- **A. Tanwar**, S. Das, *Partially insulated cracks in orthotropic materials under steady state thermo-mechanical loadings.* Journal of Thermal Stresses, 47(12), 1566–1594, (2024). 10.1080/01495739.2024.2421795

- S.S. Das, **A. Tanwar**, S. Das, E-M Craciun, *Wiener–Hopf method to solve the anti-plane problem of moving semi-infinite crack in orthotropic composite materials*, Mathematics and Mechanics of Solids. 29(7):1311-1324, (2024). 10.1177/10812865231224348.

- **A. Tanwar**, S. S. Das, S Das, *Investigation of thermally conducting collinear cracks in an orthotropic composite medium under uniform heat flow*, **Communicated**

List of Figures

1.1 The Liberty ship S.S. Schenectady, which, in 1943, failed before leaving the shipyard. The incidents drew major attention to the study of cracks. Source: WordPress . . . 2

1.2 The Great Boston Molasses Disaster. Twenty-one people were killed, and more than 150 buildings were destroyed as the result of 2.3 million gallons of molasses flooding North Boston. Investigations indicated the probable cause to be brittle fractures. . . . 3

1.3 Diagram showing life prediction procedure and failure criteria (Fracture Mechanics Approach) [2]. . . . 5

1.4 Diagram to depict the approach taken to tackle a problem in LEFM [3]. . . . 7

1.5 Stress-strain curve [3]. . . . 9

1.6 Stress components distribution . . . 10

1.7 Coordinate axis diagram for a crack of length $2a$ [4]. . . . 13

1.8 Different modes of fracture based on the stress component acting on the crack surface [5]. . . . 15

1.9 Chebyshev Polynomials of the first kind for different values of n. . . . 25

2.1 An infinite orthotropic plane with uniform heat flow of magnitude q_o along the y-direction. . . . 47

2.2 Variation of normalized mode I SIF at the crack tip a for different values of $(e-b)/(e+b)$. . . . 62

2.3 Variations of (a) $K_{I(e-b)}$ and (b) $K_{I(e+b)}$ for different values of $a/(e+b)$. 63

2.4 Plot of normalized mode I SIF at the crack tip a for different values of a/h. . . . 63

2.5 Plots of (a) $K_{I(e-b)}$ and (b) $K_{I(e+b)}$ for different values of a/h. . . . 64

3.1 Geometry of the problem with three cracks at the interfaces of orthotropic materials under anti-plane shear traction. . . . 70

3.2 The plots of the normalised SIFs K_a^* vs. crack length ratio d/a for $h=0,\ h=1,\ h=2$ for aluminium, graphite epoxy, epoxy as media 1, 2 and 3, respectively for Case 1. . . . 80

3.3 The plots of the normalised SIFs K_b^* vs. crack length ratio d/a for $h = 0,\ h = 1,\ h = 2$ for aluminium, graphite epoxy, epoxy as media 1, 2 and 3, respectively for Case 1. 80
3.4 The plots of the normalised SIFs K_c^* vs. crack length ratio d/a for $h = 0,\ h = 1,\ h = 2$ for aluminium, graphite epoxy, epoxy as media 1, 2 and 3, respectively for Case 1. 81
3.5 The plots of the normalised SIFs K_a^* vs. crack length ratio d/a for $h = 0,\ h = 1,\ h = 2$ for aluminium, graphite epoxy, graphite epoxy as media 1, 2 and 3, respectively for Case 2. 81
3.6 The plots of the normalised SIFs K_b^* vs. crack length ratio d/a for $h = 0,\ h = 1,\ h = 2$ for aluminium, graphite epoxy, graphite epoxy as media 1, 2 and 3, respectively for Case 2. 81
3.7 The plots of the normalised SIFs K_c^* vs. crack length ratio d/a for $h = 0,\ h = 1,\ h = 2$ for aluminium, graphite epoxy, graphite epoxy as media 1, 2 and 3, respectively for Case 2. 82
3.8 The plots of the normalised SIFs K_a^* vs. crack length ratio d/a for $h = 0, h = 1, h = 2$ for aluminium, epoxy, epoxy as media 1, 2 and 3, respectively for Case 3. 82
3.9 The plots of the normalised SIFs K_b^* vs. crack length ratio d/a for $h = 0, h = 1, h = 2$ for aluminium, epoxy, epoxy as media 1, 2 and 3, respectively for Case 3. 82
3.10 The plots of the normalised SIFs K_c^* vs. crack length ratio d/a for $h = 0, h = 1, h = 2$ for aluminium, epoxy, epoxy as media 1, 2 and 3, respectively for Case 3. 83

4.1 Geometry of the problem. 90
4.2 Plot of stress intensity factor vs. crack velocity for different values of h. 106
4.3 Plot of normalised stress intensity factor vs. crack velocity for different h_1/h ratios for the considered model with dissimilar strips. . . . 107
4.4 Plot of normalised stress intensity factor vs. strip width ratios for different crack velocities for the considered model with dissimilar strips. 107
4.5 Plot of normalised stress intensity factor vs. crack velocity vs. h_1/h ratios for the considered model with dissimilar strips. 108
4.6 Plot of normalised stress intensity factor vs. crack velocity for different h_1/h ratios for the considered model with two dissimilar strips sandwiched between similar strips. 108
4.7 Plot of normalised stress intensity factor vs. strip width ratios for different crack velocities for the considered model with two dissimilar strips sandwiched between similar strips. 109
4.8 Plot of normalised stress intensity factor vs. crack velocity vs. h_1/h ratios for the considered model with two dissimilar strips sandwiched between similar strips. 109

4.9 Plot of normalised stress intensity factor vs. crack velocity for different h_1/h ratios when strip 3 is the same as strip 1 and strip 4 is the same as strip 2. 110
4.10 Plot of normalised stress intensity factor vs. strip width ratios for different crack velocities when strip 3 is the same as strip 1 and strip 4 is the same as strip 2. 110
4.11 Plot of normalised stress intensity factor vs. crack velocity vs. h_1/h ratios when strip 3 is the same as strip 1 and strip 4 is the same as strip 2. 111
4.12 Plot of normalised stress intensity factor vs. crack velocity for different C_{44} ratios for two bonded strips. 111
4.13 Plot of normalised stress intensity factor vs. h_1/h for different C_{44} ratios for two bonded strips. 112
4.14 Plots of absolute crack energy for different h_1/h and $V/C_s^{(1)}$ (a)Abs$(W/\sigma_0^2 h)$ vs. crack length for various h_1/h. 112
4.15 Plots of absolute crack energy for different h_1/h and $V/C_s^{(1)}$ Abs$(W/\sigma_0^2 h)$ vs. crack length for various $V/C_s^{(1)}$. 113
4.16 Plots of absolute crack energy for different h_1/h and $V/C_s^{(1)}$. 113
4.17 Plots of the effects of $C_{55}^{(1)}/C_{55}^{(2)}$ on the stress magnification factor for the considered model. 113

5.1 Geometry of the partially insulated crack model under uniform heat flux and mechanical loading in orthotropic medium. 120
5.2 Geometry of the considered model for in-plane shear. 134
5.3 Plots of q_{oc}/q_o vs x/a when only central crack is considered in orthotropic plane. 145
5.4 Comparative study between thermal medium model [6] and thermal conducting model for present study by fixing $(c-b)/2a = 0.0,\ h/2a = 0.5$. 145
5.5 Plots of dimensionless heat flux q_{oc}/q_o for central crack vs thermal conductivity h_c by fixing $(c-b)/2a = 0.0,\ h/2a = 0.5$. 145
5.6 Plots of Dimensionless heat flux q_{oc}/q_o for central crack vs non-dimensional thermal conductivity R_c by fixing $(c-b)/2a = 1.0,\ h/2a = 0.5$ . . . 146
5.7 3-D plot for heat flux vs dimensional thermal conductivity R_c vs x/a by fixing $(c-b)/2a = 1.0,\ h/2a = 0.5$ 146
5.8 Plots of Dimensionless heat flux q_{oc}/q_o for upper crack vs non-dimensional thermal conductivity R_c by fixing $(c-b)/2a = 1.0,\ h/2a = 0.5$ 146
5.9 3-D plot vs dimensional thermal conductivity R_c vs x/a by fixing $(c-b)/2a = 1.0,\ h/2a = 0.5$. 147
5.10 Plots of Dimensionless heat flux q_{oc}/q_o for upper crack vs $(c+b-2x)/(c-b)$ for different values of non-dimensional thermal conductivity R_c. 147

5.11 The plot of Dimensionless heat flux q_{oc}/q_o for upper crack vs x/a for different values of non-dimensional thermal conductivity R_c. 147

5.12 Plots of normalised temperature difference for different values of R_c for upper crack $b < x < c$. 148

5.13 Plots of normalised temperature difference for different values of R_c for $0 < x < a$. 148

5.14 Plots of Normalised Heat flux intensity factor at crack tip $x = a$ for different crack length ratios $(c - b)/2a$. 148

5.15 Plots of Normalised Heat flux intensity factor at crack tip $x = a$ for different strip width ratios $h/2a$. 149

5.16 Plots of Normalised Heat flux intensity factor at crack tip $x = b$ for different crack length ratios $(c - b)/2a$. 149

5.17 Plots of Normalised Heat flux intensity factor at crack tip $x = b$ for different strip width ratios $h/2a$. 149

5.18 Plots of Normalised Heat flux intensity factor at crack tip $x = c$ for different crack length ratios $(c - b)/2a$. 150

5.19 Plots of Normalised Heat flux intensity factor at crack tip $x = c$ for different strip width ratios $h/2a$. 150

5.20 Plots of Normalised Heat flux intensity factor at crack tip $x = a$ for different thermal conductivity coefficient ratios for different materials. 150

5.21 Plots of normalised heat flux intensity factor for different thermal conductivity coefficient ratios for different materials for $x = b$. 151

5.22 Plots of normalised heat flux intensity factor for different thermal conductivity coefficient ratios for different materials for $x = c$. 151

5.23 Plots of normalised stress intensity factor of mode-I at crack tip $x = a$ for different $h/2a$ when $h_c = 0.0$. 151

5.24 Plots of normalised stress intensity factor of mode-I at crack tip $x = a$ for different $h/2a$ when $h_c = 0.5$. 152

5.25 Plots of normalised stress intensity factor of mode-I at crack tip $x = b$ for different $h/2a$ when $h_c = 0.0$. 152

5.26 Plots of normalised stress intensity factor of mode-I at crack tip $x = b$ for different $h/2a$ when $h_c = 0.5$. 152

5.27 Plots of normalised stress intensity factor of mode-I at crack tip $x = c$ for different $h/2a$ when $h_c = 0.0$. 153

5.28 Plots of normalised stress intensity factor of mode-I at crack tip $x = c$ for different $h/2a$ when $h_c = 0.5$. 153

5.29 Plots of normalised stress intensity factor of mode-II at crack tip $x = a$ for different $h/2a$ when $h_c = 0.0$. 153

5.30 Plots of normalised stress intensity factor of mode-II at crack tip $x = a$ for different $h/2a$ when $h_c = 0.5$. 154

5.31 Plots of normalised stress intensity factor of mode-II at crack tip $x = b$ for different $h/2a$ when $h_c = 0.0$. 154

5.32 Plots of normalised stress intensity factor of mode-II at crack tip $x = b$ for different $h/2a$ when $h_c = 0.5$. 154
5.33 Plots of normalised stress intensity factor of mode-II at crack tip $x = c$ for different $h/2a$ when $h_c = 0.0$. 155
5.34 Plots of normalised stress intensity factor of mode-II at crack tip $x = c$ for different $h/2a$ when $h_c = 0.5$. 155
5.35 Plots of normalised opening displacement of crack at crack tip $x = a$ for different $h/2a$ when $h_c = 0.0$. 155
5.36 Plots of normalised opening displacement of crack at crack tip $x = a$ for different $h/2a$ when $h_c = 0.5$. 156
5.37 Plots of normalised opening displacement of crack at crack tip $x = b$ and $x = c$ for different $h/2a$ when $h_c = 0.0$ 156
5.38 Plots of normalised opening displacement of crack at crack tip $x = b$ and $x = c$ for different $h/2a$ when $h_c = 0.5$. 156
5.39 Plots of normalised sliding displacement of crack for different $h/2a$ tip $x = a$ 157
5.40 Plots of normalised sliding displacement of crack for different $h/2a$ tip $x = b$ and $x = c$ 157

List of Tables

2.1 Values of material constants for the considered orthotropic material. . 62

3.1 Table of values of material parameters used in the problem. 77

4.1 Material properties for the considered orthotropic materials. 106

5.1 Properties for the considered materials [7, 8, 9]. 143
5.2 Error analysis of values of approximate series (LHS) and exact values of RHS in equations (5.51) for $(c-b)/2a = l_o/a = 0.5$, $h/2a = 0.5$. . . 143
5.3 Error analysis of values of approximate series (LHS) and exact values of RHS in equations (5.52) for $(c-b)/2a = l_o/a = 0.5$, $h/2a = 0.5$. . . 144
5.4 Error analysis for different values of truncated terms n for (5.51) when $(c-b)/2a = l_o/a = 0.5$, $h/2a = 0.5$. 144
5.5 Different cases based on thermal conductivity coefficient. 144

Symbols and Abbreviations

a	half-crack length
FM	Fracture Mechanics
SIF	Stress intensity factor
NSIF	Normalised SIF
K_a	stress intensity factor at tip $x = a$.
$K_{I/II/III}$	stress intensity factors of mode-I/II/III, respectively
LEFM	Linear elastic fracture mechanics
σ	stress tensor
σ_{ii}	normal stress component in $i = x,\ y,\ z$ plane
τ_{ij}	shear stress component in i plane and j
N	S.I. unit of force, Newton
M	notation for S.I. unit meter
ϵ	strain tensor with components in the form ϵ_{ij}
(r, θ)	polar coordinate system denoting distance from the crack tip and angle from crack surface, respectively
π	mathematical constant symbol Pi
q_i	heat flux in direction i
k_i	thermal conductivity coefficients in direction i
q_o	constant heat flux flowing through the plane
$(u, v) = (u(x, y), v(x, y))$	displacements in $x,\ y$ directions, respectively
$_{,i}$	subscript denoting partial derivative with respect to variable i
$_{,ij}$	subscript denoting second order partial derivative with respect to variables $i,\ j$
ρ	material density
ι	imaginary unit number, $\iota = \sqrt{-1}$

$T_n(x)$	Chebyshev polynomial of the first kind of order n
$U_n(x)$	Chebyshev polynomial of the second kind of order n
P_n	Jacobi polynomial of the first kind of order n
$\binom{n}{m}$	binomial coefficient with formula $\frac{n!}{m!(n-m)!}$
$J_n(x)$	Bessel function
$K(x,y)$	kernel function
δ_{nm}	Kronecker delta function
$w(x)$	weight function
∇	gradient vector
Δ	delta
$\cdot$	dot product
$T(x,y)$	temperature field
t	time
$\mathbb{R}$	set of real numbers

Note: All the mathematical symbols have usual meanings unless stated otherwise