

Some Problems on Stability Analysis and Synchronization of Neural Networks



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by

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Chapter 6

Conclusion and Future Works

This thesis is dedicated to the problems of synchronization between neural networks under the various parameters. The focus is given to the mathematical analysis which is needed when synchronization problems are studied. The major issue that comes in synchronization of neural networks is stability analysis of nonlinear differential equations. Thus, the stability analysis of error systems has been investigated in effective ways and derived less conservative results than the existing one. The primary focus has been given to study synchronization of general neural network models as well as complex valued neural network models. In the first chapter we elaborately introduced artificial neural networks from the origin of motivation to their modeling in mathematical equations. The basic definitions and properties of DDE have been given. The physical interpretations are given on Hopfield, Complex-valued and Recurrent neural networks. These models are extended for bounded and unbounded time-varying delays, and uncertainties. The first chapter was finished by introducing matrix measure theory that has been applied in most of the chapters of the thesis to investigate stability analysis.

In the second chapter, the global exponential synchronization criteria of the complex-valued recurrent neural networks(CVRNNs) in the presence of uncertain parameters with time-varying bounded and unbounded delay terms have been investigated. Based on Halanay inequality and matrix measure approach, the global exponential

synchronization is studied for two cases. The first case is the synchronization of CVRNNs in the presence of uncertain parameters with time-varying bounded and unbounded delay terms and second one is the concerned synchronization in the absence of uncertain terms with same bounded and unbounded time-varying delay terms. The synchronization of the addressed CVRNNs is achieved with the help of Lyapunov functional, and several sufficient criteria and theorems.

In the third chapter, the global quasi-synchronization of CVRNNs is investigated. When scaling factor consists of unit constant then the synchronization scheme becomes global quasi synchronization. Since time-delays in neural network may affect their dynamics, so the CVRNNs models are considered with time-varying delays. It is shown that global quasi-synchronization of CVRNNs can not be completed, i.e., the trajectory of error system fluctuate in a small compact domain of radius equal to synchronization error bound. Thus, we call this type of synchronization a global quasi-synchronization.

In the fourth chapter, the focus is given on the quasi-projective synchronization for a class of non-identical memristor-based CVRNNs with time-varying delays and parameters' mismatched. Here, the quasi-projective synchronization is presented in two ways based on the Halanay inequality and some useful techniques. Some novel sufficient conditions are firstly derived for finding the quasi-projective synchronization criteria of non-identical memristor-based CVRNNs with time-varying delays and parameters' mismatched by using the matrix measure method. And then, by using the Lyapunov-Krasovskii functional, another sufficient stabilization condition for quasi-projective synchronization of the concerned model is presented.

In the fifth chapter, the quasi-projective synchronization of inertial complex-valued

recurrent neural networks (ICVRNNs) with mixed time-varying delay and mismatched parameters is investigated. By using an appropriate variable transformation, the order of the ICVRNN differential system is reduced, and then, by applying the real decomposition method, it is separated into real and imaginary components. The matrix measure approach with the nonlinear Lipschitz activation functions is employed in the ICVRNN model. Through the proper description of the matrix measure approach, some sufficient conditions have been derived for the quasi-projective synchronization criteria of the considered model through designing a suitable controller. Here, some significant results have been provided for the ICVRNNs with mismatched parameters and mixed time-varying delay.

In my future research work, the effects of time impulses on stability and synchronization problems of delayed complex valued neural networks will be studied. An impulsive phenomenon exists universally in a wide variety of evolutionary processes where the state is changed abruptly at certain moments of time, involving such fields as chemical technology, population dynamics, physics and economics [123]. It has also been shown that an impulsive phenomenon exists likewise in neural networks [124]. For instance, during the implementation of electronic networks, when a stimulus from the body or the external environment is received by receptors, the electrical impulses will be conveyed to the neural networks and an impulsive phenomenon which is called impulsive perturbations arises naturally [125]. The impulsive perturbation of neural networks can affect the dynamical behaviors of the neural networks, same as time delay effect [126]. Therefore, it is necessary to consider both impulsive effect and delay in the study of the stability of neural networks [127]. Thus, it will be tried to investigate the following possible problems in my future research work:

1. Extending all the results [128, 129] to the time impulsive effects.

2. The obtained results [130] for higher dimensional neural networks can be extended to the quaternion-valued and octanion-valued neural networks.
3. The focus will be given in developing the mathematical techniques to avoid complexity in calculation. Also the effort will be given to apply some more effective methods like Linear Matrix Inequality(LMI).
