
Abstract

The approximation theory of linear positive operators has become a significant area of research over the past few decades. Approximation is a key concept in mathematical analysis, representing a qualitative characteristic of linear positive operators, which plays a crucial role in this field. The systematic exploration of approximation theory began in the early 1920s. But, the study of Positive linear operators became crucial after the "Bohman-Korivkin theorem." Some of the studies about the positive linear operators can be seen in [1, 2, 3, 4, 5, 7, 9, 10, 11]

This thesis focuses on the approximation behavior of various positive linear operators along with their generalized forms and integral modifications and the properties related to approximation, rates of convergence, error estimation, and graphical comparisons. Also, examining the convergence properties of complex type operators, which extend real-valued operators into the complex plane is another goal. Solving Integral Equations using positive linear operators has been another important area of research, so contributing in such direction is also a motive of the research. The systematic delineation of the thesis is as follows: Chapter 1 is about the introduction and a brief literature survey of the topic, basic notation, and definitions concerning approximation theory, which will be used in the subsequent chapters.

Chapter 2 is dedicated to the study of two Bernstein-type operators. The first one is about a new modification of the Bernstein-Kantorovich operators, which approximate integrable functions. A Korovkin-type theorem and a weighted approximation property for the newly defined operators are also studied. Further, their rate of convergence using the modulus of continuity and Ditzian-Totik moduli of smoothness is highlighted. To determine their exact order of approximation, a Voronovskaja-type theorem, as well as a Grüss Voronovskaja-type result have been studied. In the end, their convergence is confirmed through graphical examples using two test functions. The second one is on the study of a blending-type generalization of the λ -Bernstein operators. The important approximation properties, as well as the rate of convergence, are studied. The direct error estimation for a special class of functions, i.e. for the set of functions having derivative of bounded variation, is given.

Chapter 3 deals with another integral-type generalization. A study on the approximation properties of a Stancu-type generalization of a modified Szász operator is conducted. Its approximation properties using the 1st and 2nd-order modulus of

smoothness are analyzed. An upper estimation of the error bound for the Lipschitz-type class of functions is provided. In the end, the importance of the generalization is shown using some graphical examples.

In Chapter 4, we have extended the study of the Korovkin-type theorem to general probability measure space for a new class of Bernstein-Kantorovich operators. In [12], D. Popa proved an Intermediate Voronovskaja-type theorem for Lotosky-Kantorovich operators. Following that, here, an intermediate Voronovskaja-type theorem is studied for the newly defined operators. Then, the convergence properties of these operators using the usual modulus of continuity are established. An upper estimation for the Lipschitz-type space is given, and a study is conducted for the error-bound estimation for a special class of functions, namely, the functions with derivatives of bounded variations.

In 1936, Bernstein himself started studying complex Bernstein operators [6] for analytic functions and showed their convergence. However, S. Gal [8] studied the quantitative upper estimation of the error in the convergence. In this thesis, Chapter 5 is devoted to a complex-type generalization. The complex variant of the α -Bernstein-Durrmeyer operators is introduced and their convergence properties have been studied. Also, the quantitative upper estimation and Voronovskaja type result to validate the natural extension of the α -Bernstein-Durrmeyer operators from the real domain to compact discs in the complex argand plane are obtained. Along with the analytic functions, a note on the order of convergence of their higher-order derivatives is also highlighted.

Another important aspect is the application of positive linear operators in other fields of Mathematics. In Chapter 6, an application is shown to solve the Volterra Integral Equations of both the 1st and 2nd kind. The study is done using the λ -Bernstein operators, which have more modeling flexibility than the traditional Bernstein operators due to the shape parameter λ . The actual process of approximation is provided in the form of algorithms and the error of approximation is established to confirm the convergence. Also, we have done some experiments with some test problems as well as practical problems with the motive of getting less error for at least some values of λ , and at the end, we pointed out some examples where we got less root mean square error by using the λ -Bernstein operators than the usual Bernstein operators for at least some values of λ .

This thesis can be seen as a valuable addition to the understanding of Approximation concepts using positive linear operators which is a highly regarded research field. The thesis will attract a wide range of readers, including those interested in functional analysis, operator theory, approximation theory and numerical analysis.

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