3 Bianchi Type-II Cosmological Models with String and Bulk Viscous Fluid in Lyra Geometry

3.1 Introduction

The present day universe is satisfactorily described by spatially homogeneous and isotropic Friedmann-Robertson-Walker space-time. But at smaller scales, the universe is neither homogeneous and isotropic nor do we expect the universe possess to these properties in it's early stages of evolution. Spatially homogeneous and anisotropic cosmological models have been widely studied in general relativity in different contexts in search of a realistic models of the universe in its early stages. Bianchi type II space-times have richer structure both geometrically and physically for describing the early stages of evolution of the universe. Asseo and Sol (1987) emphasized the importance of Bianchi type-II space-times which play a fundamental role in constructing cosmological models suitable for the description of the early

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stages of evolution of the universe.

The adequacy of isotropic perfect fluid cosmological models describing the present state of the universe is no basis to expect that they are equally suitable for describing the early stages of evolution of the universe. It is held that at the early stages of evolution of the universe when radiation in the form of photons as well as neutrino decoupled, the matter behaved like a viscous fluid. Since viscosity counteracts the gravitational collapse, a different picture of the initial stage of the universe may appear due to dissipative processes caused by viscosity. Misner (1967,1968) has studied the effect of viscosity on the evolution of the universe and suggested that the strong dissipation due to neutrino viscosity may be considerably reduce the anisotropy of the black body radiation. Murphy (1973) has obtained an exact zero curvature FRW cosmological model with bulk viscosity alone which exhibits an interesting feature that the big-bang singularity appears in the infinite past. Belinski and Khalatnikov (1975), while investigating influence of viscosity, have found that near the initial singularity the gravitational field creates matter. Padmanabhan and Chitre (1987) have shown that the bulk viscosity leads to an inflationary-like solution. It has also shown that the bulk viscosity acts like a negative energy field in an expanding universe (Johari and Sudarshan (1979)).

String cosmology has been a subject of considerable interest since long. Cosmic strings are topologically stable in the early universe (Kibble (1976)). That arise during the phase transition after big-bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theory (Zel'dovich (1980)), Kibble (1980), Everelt (1981)). It is held that cosmic strings give rise to density perturbations which lead to the formation of galaxies. The cosmic strings have stress-energy and couple to gravitational field. Letelier (1979, 1983), Stachel (1980) initiated the general relativistic treatment of strings and formulated the energy-momentum tensor of classical massive string and presented some cosmological solutions for massive strings in Bianchi type-I and Kantowski-Sachs space-times. Since then many researchers have obtained strings cosmological models with different Bianchi symmetries.

Lyra (1951) proposed a modification in Riemannian geometry by introducing a guage-function into the structureless manifold as a result a displacement field arises naturally. Sen (1957) has shown that the static model with finite density in Lyra's geometry is similar to the static Einstein model. Halford (1970) has developed a cosmological theory in Lyra's geometry which give rise to nonstatic perfect fluid universe models. The Lyra's geometry is very close to the spirit of Einstein's principle of geometrization since both scalar and tensor field have more or less a geometrical significance. Rao et al. (2012) have presented Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in Lyra geometry. A lot of works has been carried out by many authors in different physical context within the framework of Lyra's geometry.

It is worth to mention some recent works which have some relevance to the present work. Pradhan and Pandey (2003) have studied cosmological models based on Lyra's geometry with a constant displacement field vector. However, the restriction of the constant displacement field vector is a coincidence and there is no a priori reason for it. Kumar (2011) studied a spatially homogeneous and anisotropic

Bianchi type-II model representing massive strings. Tyagi and Sharma (2011) have obtained a lhasocally rotationally symmetric Bianchi type-II magnetized string cosmological model with bulk viscous fluid in general relativity. Chawala et al. (2013) have presented an anisotropic Bianchi type-I cosmological model in string cosmology with variable deceleration parameter. Agrawal et al.(2012) have investigated perfect fluid Bianchi type-II string cosmological models in normal guage for Lyra's manifold with constant deceleration parameter.

In this chapter, we obtain a one parameter family of spatially homogeneous Bianchi type-II string cosmological models in the presence of a bulk viscous fluid within the framework of Lyra's geometry with time-dependent displacement vector. The chapter is organized as follows: In Sec.(3.2), we discuss the metric and the field equations. We obtain the solution of the field equations with the assumption that the component σ_1^1 of the shear tensor σ_i^j is proportional to the expansion scalar θ and bulk viscosity coefficient is proportional to the power function of energy density in Sec.(3.3). In Sec.(3.4), we discuss the physical and kinematical behavior of the cosmological model which are suitable for describing the early stages of evolution of physical universe in agreement with recent Supernovae observations. Some concluding remarks are given Sec.(3.5).

3.2 The Metric and Field Equations

We consider the totally anisotropic and spatially homogeneous Bianchi type-II space-time in the form

$$ds^2 = dt^2 - A^2(dx - zdy)^2 - B^2dy^2 - C^2dz^2 \tag{3.1}$$

where the metric potentials are functions of time t alone.

The energy-momentum tensor for a cloud of massive strings with a bulk viscous fluid is given by

$$T_i^j = (\rho + \overline{p})u_i u^j - \overline{p} g_i^j + \lambda x_i x^j \tag{3.2}$$

where ρ is the proper energy density for a cloud of strings with particles attached to them. λ is the string tension density, $\overline{p}$ is the effective pressure, u^i is the four velocity vector of the particles, x^i is a unit vector representing the direction of string so that $x^1 \neq 0, x^2 = x^3 = x^4 = 0$. The vectors u^i and x_i satisfy the conditions

$$u^i u_i = -x^i x_i = 1, \quad u^i x_i = 0. \tag{3.3}$$

Choosing x^i parallel to $\frac{\partial}{\partial x}$,we have

$$x^i = (A^{-1}, 0, 0, 0). \tag{3.4}$$

The effective pressure $\overline{p}$ and isotropic pressure p are related by

$$\overline{p} = p - \xi\theta. \tag{3.5}$$

If the particle density of the configuration is denoted by ρ_p, then

$$\rho = \rho_p + \lambda. \tag{3.6}$$

The field equations in gravitational units $c = 1, 8\pi G = 1$, in normal guage for Lyra's manifold are:

$$R_i^j - \frac{1}{2}Rg_i^j + \frac{3}{2}\phi_i\phi^i - \frac{3}{4}g_i^j\phi_k\phi^k = -T_i^j \tag{3.7}$$

where ϕ_i is the displacement vector defined as

$$\phi_i = (0, 0, 0, \beta(t)) \tag{3.8}$$

and other symbols have their usual meanings as in Riemannian geometry (Sen (1957)).

In a comoving coordinate system, the field equations (3.7) together with equation (3.2) for the line-element (3.1) lead to the following system of equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{3}{4}\frac{A^2}{B^2C^2}+\frac{3}{4}\beta^2=-\overline{p}+\lambda, \tag{3.9}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}-\frac{1}{4}\frac{A^2}{B^2C^2}+\frac{3}{4}\beta^2=-\overline{p}, \tag{3.10}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{4}\frac{A^2}{B^2C^2}+\frac{3}{4}\beta^2=-\overline{p}, \tag{3.11}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{1}{4}\frac{A^2}{B^2C^2}+\frac{3}{4}\beta^2=-\rho. \tag{3.12}$$

The energy conservation equation $T^{ij}_{;j}$ leads to the following equation

$$\dot{\rho}+(\rho+\overline{p})\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\lambda\frac{\dot{A}}{A}=0 \tag{3.13}$$

which is a consequence of the field equations (3.9)-(3.12). The conservation of R.H.S. of equation (3.7) provides

$$(R^j_i-\frac{1}{2}g^j_iR)_{;j}+\frac{3}{2}(\phi_i\phi^j)_{;j}-\frac{3}{4}(g^j_i\phi_k\phi^k)_{;j}=0 \tag{3.14}$$

which, after straightforward calculation, leads to

$$\dot{\beta}+\beta\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{3.15}$$

Thus, Eq.(3.13) combined with Eq.(3.15) is the resulting equation when energy conservation equation is satisfied in the given system. It deserves to mention that the conservation equation in the Lyra's manifold is not satisfied as in general relativity. In fact, the conservation equation in Lyra's manifold is satisfied only on giving some special condition on the displacement vector β as shown above.

For the metric (3.1) the dynamical parameters viz. spatial volume V, expansion scalar θ, shear scalar σ and Hubble parameter H for Bianchi type-II metric are given by

$$V = ABC = a^3, \tag{3.16}$$

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{3.17}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2, \tag{3.18}$$

$$\theta = 3H. \tag{3.19}$$

3.3 Solutions of Field Equations

Equations (3.9)-(3.12) and (3.15) are five equations in seven unknown A, B, C, ρ, $\overline{p}$, λ and β. We need two extra constraints to obtain explicit solutions of the system of equations. We first assume that the component σ_1^1 of the shear tensor σ_i^j is proportional to the expansion scalar θ, which leads to the following relation between the metric potentials

$$A = (BC)^m \tag{3.20}$$

where m is a positive constant. The motive behind assuming this condition is explained with reference to Thorne (1967). The observations of the velocity red-shift relation for extragalactic source suggest that Hubble expansion of the universe is isotropic today within$\approx$ 30%.

Now, we assume that the deceleration parameter q has the constant value given by

$$q = -1 + \frac{1}{n} \tag{3.21}$$

where n is a parameter. It is obvious that the deceleration parameter is negative for $n < 0$ and $n > 1$ and is positive for $0 < n < 1$. Substituting Eq.(3.21) in Eq.(1.34) and integrating the resulting equation, we obtain the average scale factor a of the form

$$a = (k_1 t + k_2)^n \tag{3.22}$$

where k_1 and k_2 are arbitrary constants.

Combining Eqs.(3.16), (3.20) and (3.22), we find that

$$A = (k_1 t + k_2)^{\frac{3nm}{m+1}}. \tag{3.23}$$

From Eq.(3.20) and Eq.(3.23), we obtain

$$BC = (k_1 t + k_2)^{\frac{3n}{m+1}}. \tag{3.24}$$

Subtracting Eq.(3.10) from Eq.(3.11), we get

$$\frac{\ddot{B}}{B} - \frac{\dot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \tag{3.25}$$

Equation (3.25), on integration, yields

$$\frac{B}{C} = c_1 \exp\left\{c_2 \int \frac{dt}{a^3}\right\} \tag{3.26}$$

where c_1 and c_2 are integration constants.

Inserting the value of a from Eq.(3.22) in Eq.(3.26) and integrating, we obtain

$$\frac{B}{C} = c_1 \exp\left\{\frac{c_2 (k_1 t + k_2)^{1-3n}}{k_1(1-3n)}\right\}, n \neq \frac{1}{3}. \tag{3.27}$$

From Eq.(3.24) and Eq.(3.27), the metric potentials B and C are obtained as

$$B = \sqrt{c_1}(k_1 t + k_2)^{\frac{3n}{2(m+1)}} \exp\left\{\frac{c_2 (k_1 t + k_2)^{1-3n}}{2k_1(1-3n)}\right\}, \tag{3.28}$$

$$C = \frac{1}{\sqrt{c_1}}(k_1 t + k_2)^{\frac{3n}{2(m+1)}} \exp\left\{-\frac{c_2(k_1 t + k_2)^{1-3n}}{2k_1(1-3n)}\right\}. \tag{3.29}$$

For these solutions, the geometry of universe is described by the line-element

$$ds^2 = dt^2 - (k_1 t + k_2)^{\frac{6mn}{m+1}}(dx - zdy)^2 - c_1(k_1 t + k_2)^{\frac{3n}{m+1}} \exp\left\{\frac{c_2(k_1 t + k_2)^{1-3n}}{k_1(1-3n)}\right\} dy^2$$
$$-\frac{1}{c_1}(k_1 t + k_2)^{\frac{3n}{m+1}} \exp\left\{-\frac{c_2(k_1 t + k_2)^{1-3n}}{k_1(1-3n)} dy^2\right\} dz^2. \tag{3.30}$$

The model (3.30) represents an anisotropic Bianchi type II cosmological universe filled with a bulk viscous fluid in the framework of Lyra geometry.

3.4 Physical and Kinematical Behaviors of the Model

Now using Eqs.(3.23), (3.28) and (3.29) in Eq.(3.15), we get

$$\frac{\dot{\beta}}{\beta} = \frac{-3nk_1}{k_1 t + k_2} \tag{3.31}$$

which on integration gives

$$\beta = \frac{M}{(k_1 t + k_2)^{3n}} \tag{3.32}$$

where M is integration constant.

The expression for effective pressure ($\overline{p}$) for the model (3.30) is given as follows:

$$\overline{p} = -\left[\frac{nk_1^2(36m^2 n - 12m^2 - 18m + 18mn + 9n - 6)}{4(m+1)^2(k_1 t + k_2)^2} + \frac{3M^2 + c_2^2}{4(k_1 t + k_2)^{6n}} + \frac{1}{4}(k_1 t + k_2)^{\frac{6n(m-1)}{(m+1)}}\right]. \tag{3.33}$$

The expression for proper energy density(ρ) of the model (3.30) is obtained as

$$\rho = \frac{9n^2 k_1^2(4m+1)}{4(m+1)^2(k_1 t + k_2)^2} + \frac{3M^2 - c_2^2}{4(k_1 t + k_2)^{6n}} - \frac{1}{4}(k_1 t + k_2)^{\frac{6n(m-1)}{(m+1)}}. \tag{3.34}$$

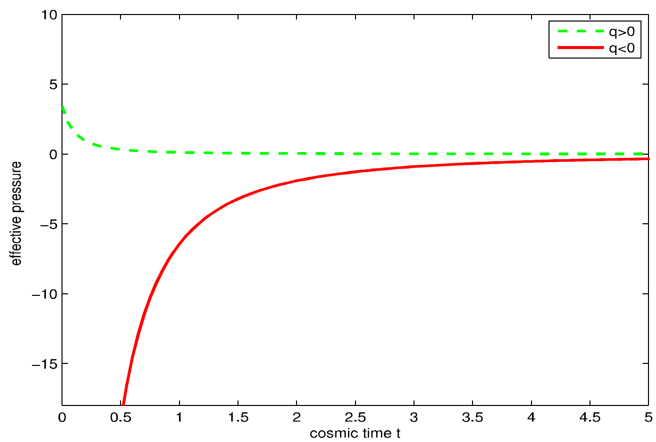

Figure 3.1: The plot of effective pressure $\overline{p}$ vs. cosmic time t

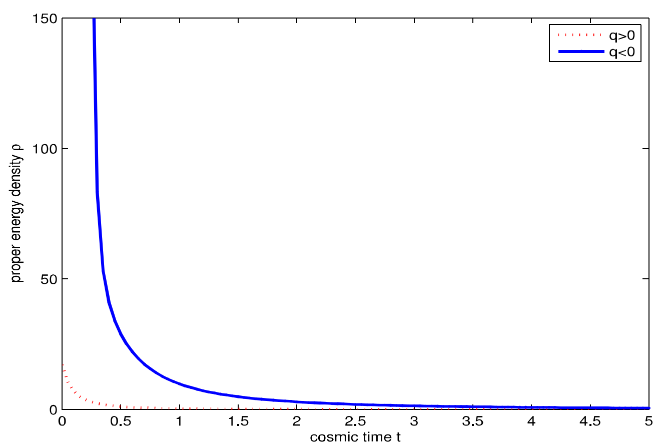

Figure 3.2: The plot of proper density ρ vs. cosmic time t

The expression for the string tension (λ) for the model (3.30) is obtained as,

$$\lambda = \frac{6nk_1^2(2m^2 - 3mn + m + 3n - 6m^2n - 1)}{4(m+1)^2(k_1t + k_2)^2} - (k_1t + k_2)^{\frac{6n(m-1)}{(m+1)}}. \tag{3.35}$$

The expression for the particle density (ρ_p) is given as follows:

$$\rho_p = \frac{nk_1^2(54mn - 9n - 12m^2 - 6m + 36m^2n + 6)}{4(m+1)^2(k_1t + k_2)^2} + \frac{3M^2 - c_2^2}{4(k_1t + k_2)^{6n}} + \frac{3}{4}(k_1t + k_2)^{\frac{6n(m-1)}{(m+1)}}. \tag{3.36}$$

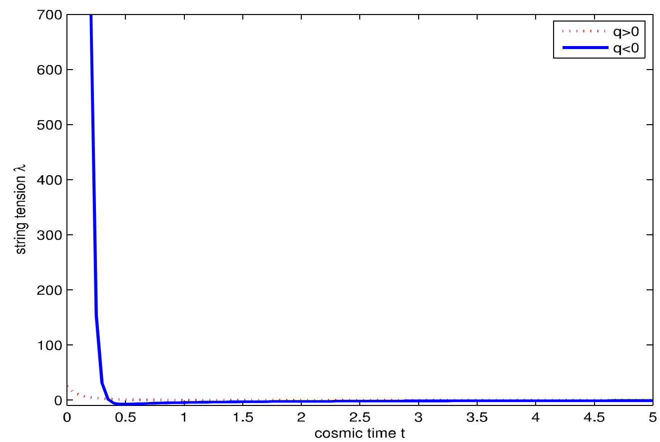

Figure 3.3: The plot of string tension λ vs. cosmic time t

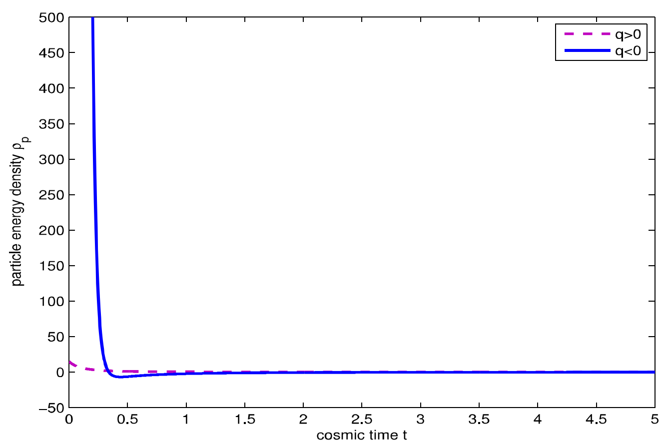

Figure 3.4: The plot of particle energy density ρ_p vs. cosmic time t

For $n > 1$, the model is accelerating whereas for $n < 1$ the model is decelerating. Here we have compared two modes of universe through graphical analysis of various parameters . We have chosen $n = 0.4$, i,e $q > 0$ to describe decelerating phase while the accelerating phase has been accounted by choosing $n = 2.4$ i.e. $q < 0$, the other constants are chosen as $m = 0.5$, $c_2 = 0.2$, $M = .009$, $k_1 = 1.5$ and $k_2 = 0.3$. Fig.(3.1) depicts the variation of effective pressure versus cosmic time in the two modes of the evolution of the universe. We observe that the pressure is positive in the decelerating universe which decreases with the evolution of the universe. But in the accelerating phase, negative pressure dominates the universe, as expected. In both cases, the pressure becomes negligible at late time. In fig.(3.2) the proper energy density has been graphed versus time. It is evident that the proper energy density remains positive in both modes of evolution. However, it decreases sharply with time in decelerating phase more than accelerating phase. Fig.(3.3) and fig.(3.4) shows nature of string tension and particle energy density versus cosmic time t in decelerating and accelerating modes respectively. It is evident that λ and ρ_p are positive in both modes of evolution. Both are decreasing function of time and for sufficiently large times, λ and ρ_p tend to zero. Therefore, strings disappear from the universe at late time which is consistent with present day observation.

In most investigations involving bulk viscosity, $\xi(t)$ is assumed to be a simple power function of energy density (Santos et al.(1985), Maartens (1995), Zimdhal (1996), Pavon et al.(1991)). Here we assume

$$\xi(t) = \xi_0 \rho^{\eta} \tag{3.37}$$

where ξ_0 and η are real constant. For low density η may be equal to unity as used in Murphy (1973) and corresponds to radiating fluid. Further, $0 \leq \eta \leq \frac{1}{2}$ is a more suitable assumption to obtain realistic cosmological models near big bang singularity (Belinski and Kalatnikov (1975)). Therefore, for simplicity and to obtain realistic models of physical importance, we adopt the following three cases as $\eta = 0, \frac{1}{2}, 1$. From Eqs. (3.5), (3.17), (3.33), (3.34) and (3.37) we get

$$p = -\left[\frac{nk_1^2(36m^2n - 12m^2 - 18m + 18mn + 9n - 6)}{4(m+1)^2(k_1t+k_2)^2} + \frac{3M^2 + c_2^2}{4(k_1t+k_2)^{6n}} + \frac{1}{4}(k_1t+k_2)^{\frac{6n(m-1)}{m+1}}\right]$$
$$+\frac{3nk_1\xi_0}{(k_1t+k_2)}\left[\frac{9n^2k_1^2(4m+1)}{4(m+1)^2(k_1t+k_2)^2} + \frac{3M^2 - c_2^2}{4(k_1t+k_2)^{6n}} - \frac{1}{4}(k_1t+k_2)^{\frac{6n(m-1)}{m+1}}\right]^{\eta}. \tag{3.38}$$

When $\eta = 0$, Eq.(3.37) reduces to $\xi = \xi_0$ =constant. Hence in this case Eq.(3.38) becomes

$$p = -\Big[\frac{nk_1^2(36m^2n - 12m^2 - 18m + 18mn + 9n - 6)}{4(m+1)^2(k_1t+k_2)^2} + \frac{3M^2 + c_2^2}{4(k_1t+k_2)^{6n}} + \frac{1}{4}(k_1t+k_2)^{\frac{6n(m-1)}{m+1}}\Big] + \frac{3nk_1\xi_0}{(k_1t+k_2)}. \quad (3.39)$$

When $\eta = \frac{1}{2}$, Eq.(3.37) reduces to $\xi = \xi_0\rho^{\frac{1}{2}}$. Hence in this case, Eq.(3.38) with use of equation (3.34) leads to

$$p = -\Big[\frac{nk_1^2(36m^2n - 12m^2 - 18m + 18mn + 9n - 6)}{4(m+1)^2(k_1t+k_2)^2} + \frac{3M^2 + c_2^2}{4(k_1t+k_2)^{6n}} + \frac{1}{4}(k_1t+k_2)^{\frac{6n(m-1)}{m+1}}\Big] + \frac{3\xi_0 nk_1}{(k_1t+k_2)}\Big[\frac{9n^2k_1^2(4m+1)}{4(m+1)^2(k_1t+k_2)^2} + \frac{3M^2 - c_2^2}{4(k_1t+k_2)^{6n}} - \frac{1}{4}(k_1t+k_2)^{\frac{6n(m-1)}{m+1}}\Big]^{\frac{1}{2}}. \quad (3.40)$$

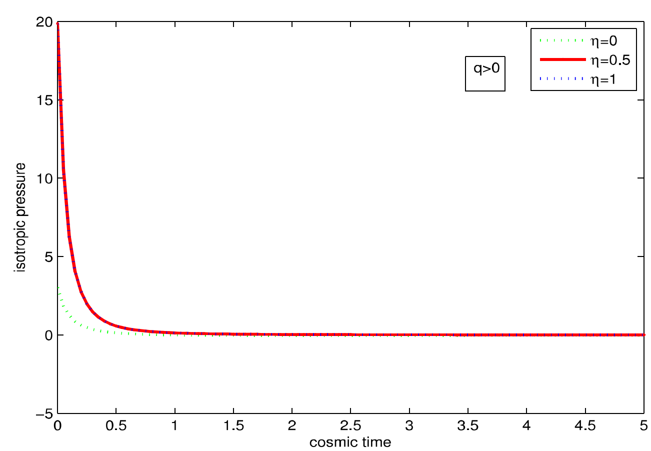

Figure 3.5: The plot of isotropic pressure p vs. cosmic time t for $q > 0$

When $\eta = 1$, Eq.(3.37) reduces to $\xi = \xi_0\rho$. Hence in this case, Eq.(3.38) with use of Eq.(3.34), leads to

$$p = -\Big[\frac{nk_1^2(36m^2n - 12m^2 - 18m + 18mn + 9n - 6)}{4(m+1)^2(k_1t+k_2)^2} + \frac{3M^2 + c_2^2}{4(k_1t+k_2)^{6n}} + \frac{1}{4}(k_1t+k_2)^{\frac{6n(m-1)}{m+1}}\Big] + \frac{3\xi_0 nk_1}{(k_1t+k_2)}\Big[\frac{9n^2k_1^2(4m+1)}{4(m+1)^2(k_1t+k_2)^2} + \frac{3M^2 - c_2^2}{4(k_1t+k_2)^{6n}} - \frac{1}{4}(k_1t+k_2)^{\frac{6n(m-1)}{m+1}}\Big]. \quad (3.41)$$

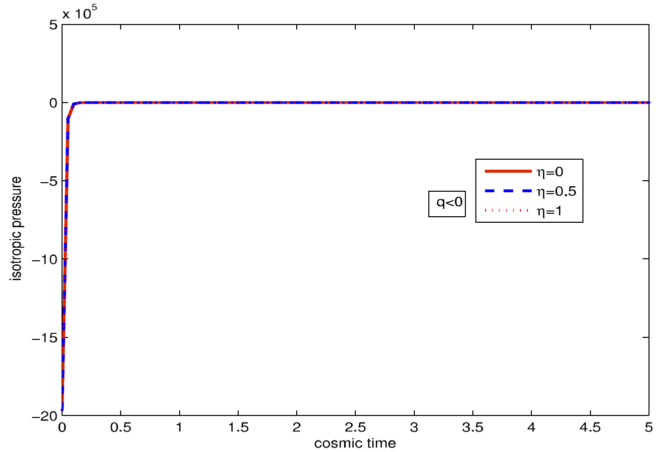

Figure 3.6: The plot of isotropic pressure p vs. cosmic time t for $q < 0$

Fig.(3.5) shows the positive and decreasing behavior of isotropic pressure for different values of η in decelerating phase whereas fig.(3.6) depicts negative and increasing behavior of isotropic pressure in accelerating mode of universe.

The kinematical parameters of the model (3.30) have following expressions:

$$V = (k_1 t + k_2)^{3n}, \tag{3.42}$$

$$\theta = \frac{3nk_1}{k_1 t + k_2}, \tag{3.43}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{36m^2n^2k_1^2 + 18n^2k_1^2 - 6n^2k_1^2(m+1)^2}{4(m+1)^2(k_1 t + k_2)^2} + \frac{2c_2^2}{4(k_1 t + k_2)^{6n}}\right], \tag{3.44}$$

$$H = \frac{nk_1}{(k_1 t + k_2)}, \tag{3.45}$$

$$A_m = \frac{(2m-1)^2}{2(m+1)^2} + \frac{1}{6}\frac{c_2^2}{\{n^2k_1^2(k_1 t + k_2)^{6n-2}\}}. \tag{3.46}$$

It is evident that the energy condition $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied under the appropriate choice of constants. The parameters p, ρ, ρ_p and λ start off with extremely large values, which continue to decrease with the expansion of the universe provided $m < 1$. In particular, the large values of ρ_p and λ in the beginning suggest that the string dominates the early universe. For sufficiently large times, the ρ_p and λ become negligible. Therefore, the strings disappear from the universe for large times.

We observe that the spatial volume V is zero at $t = -\frac{k_2}{k_1}$. At this epoch the energy density, the pressure and shear expression, Hubble parameter all have infinite values. Therefore, the model has point type singularity at $t = -\frac{k_2}{k_1}$. These physical and kinematical parameters are decreasing functions of time which ultimately tend to zero as $t \to \infty$. The anisotropic parameter increases with time for $n < \frac{1}{3}$ and decrease for $n > \frac{1}{3}$. Thus, it depends upon n. Since $\frac{\sigma}{\theta}$ does not tend to zero at $t \to \infty$, the anisotropy in the model is maintained throughout the expansion. For $n > 1$ the model is accelerating in the presence of dark energy which is consistent with present day observation whereas for $0 < n < 1$, DP and pressure are positive so the model is decelerating.

3.5 Conclusions

In summary, we have considered the field equations for a viscous fluid together with massive string within the framework of Lyra's geometry for a spatially homogeneous and Bianchi type II space time. The presence of bulk viscosity is to bring a change in the perfect fluid model. Bulk viscosity is expected to play an important role in the early evolution of the universe. We have obtained exact solutions of the field equations by assuming that the space-time admits a constant deceleration parameter. For $n > 1$, the cosmological model represents an accelerated expanding universe having initial big-bang singularity at $t = -\frac{k_2}{k_1}$ and for $0 < n < 1$, we have obtained a family of decelerating models of the universe. It is worthwhile to mention the work of Vishwakarma (2003), where he has shown that the decelerating models are also consistent with recent CMB observations model by WMAP, as well as the high red-shift supernovae Ia data including 1997 iff at $z = 1.755$. The role of viscosity coefficient and strings are also discussed. It has been found that the displacement vector β has a large value at the beginning of universe and reduces fast during the evolution of the universe, so it behaves like cosmological term Λ in the normal guage treatment.