Chapter 6

Conclusion and future work

This chapter summarizes the main conclusions drawn from the thesis and suggests future research directions. Chapter 1 introduces the thesis, discussing topics like the diffusion process and the reaction-advection-diffusion equation, as well as the historical background of fractional calculus theory and related definitions. It covers basic concepts of fractional order reaction-advection-diffusion equations, includes a literature review, and introduces Lucas and Vieta-Fibonacci polynomials, which have been used in the methods discussed.

In Chapter 2, a method is introduced to obtain the approximate solutions for a two-dimensional nonlinear time-fractional diffusion equation using Lucas polynomials and their operational matrices. The efficiency of the proposed scheme is demonstrated through maximum absolute error on existing problems, with accuracy improving by increasing polynomial terms. The applicability to other types of multi-term two-dimensional fractional order PDEs is also suggested.

In chapter 3, a novel operational matrix method has been developed with the help of shifted Vieta-Fibonacci polynomials. Two operational matrices for fractional and integer order derivatives have been derived. Validation of the efficiency of the method is demonstrated through its application to existing problems, indicating excellent accuracy. In the near future, the proposed method can also be applied to find numerical solutions of two-dimensional and three-dimensional time and space fractional-order reaction-advection-diffusion equations.

chapter 4 introduces the Vieta-Fibonacci wavelet and collocation method for solving the three-component time-fractional order Brusselator reaction-diffusion system. It discusses the uniqueness, existence of solution, and Ulam-Hyers stability of the aforementioned model along with comprehensive convergence analysis to highlight the method's effectiveness. The method proposed in this chapter can also be used to approximate some other types of fractional partial differential equations.

Chapter 5 presents the C-F approximation of shifted Legendre polynomials using the defined C-F derivative. The spectral collocation method is applied to the model, reducing it to a system of fractional PDEs, which are solved by a finite difference scheme to obtain a numerical solution. Numerical examples demonstrate the method's accuracy by finding the absolute errors and convergence rates, indicating the effectiveness of the method.

6.1 Future work

Fractional-order partial differential equations are used to model various real-life problems. In particular, reaction-advection-diffusion equations are extensively utilized to describe complex processes involving the transport of substances through a medium, such as pollutants in groundwater or chemical reactions in biological systems. However, it is a tedious task to find the exact solution of such type of fractional order model, especially in nonlinear cases. So, this thesis attempts to develop numerical methods to obtain the numerical approximation for some fractional-order mathematical models. In upcoming research, it is expected to develop new numerical tools which will be adequate to solve the two/three dimensional nonlinear problems with some other fractional operators.

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