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Appendix A

1. Einstein summation convention

If an index occurs twice in any one term, summation is taken from 1 to 3. For instance,

$$\begin{aligned}
 C_{ijkl}\varepsilon_{kl} &= C_{ij1l}\varepsilon_{1l} + C_{ij2l}\varepsilon_{2l} + C_{ij3l}\varepsilon_{3l} \\
 &= (C_{ij11}\varepsilon_{11} + C_{ij12}\varepsilon_{21} + C_{ij13}\varepsilon_{31}) \\
 &\quad + C_{ij21}\varepsilon_{21} + C_{ij22}\varepsilon_{22} + C_{ij23}\varepsilon_{23} \\
 &\quad + C_{ij31}\varepsilon_{31} + C_{ij32}\varepsilon_{32} + C_{ij33}\varepsilon_{33} \\
 \sigma_{ij,j} &= \sigma_{i1,1} + \sigma_{i2,2} + \sigma_{i3,3}
 \end{aligned}$$

2. Kronecker delta

$$\delta_{ij} = \begin{cases} 1 & \text{for } i = j, \\ 0 & \text{for } i \neq j. \end{cases}$$

$$\delta_{11} = \delta_{22} = \delta_{33} = 1$$

$$\delta_{12} = \delta_{23} = \delta_{31} = \delta_{21} = \delta_{32} = \delta_{13} = 0$$

$$\delta_{ii} = 3$$

3. Permutation tensor

$$\epsilon_{ijk} = \begin{cases} 1 & \text{for the even permutation of } 1, 2, 3, \\ -1 & \text{for the odd permutation of } 1, 2, 3. \\ 0 & \text{for other cases.} \end{cases}$$

$$\begin{aligned}
 \epsilon_{123} &= \epsilon_{231} = \epsilon_{312} = 1, \\
 \epsilon_{132} &= \epsilon_{213} = \epsilon_{321} = -1, \\
 \epsilon_{112} &= \epsilon_{223} = \epsilon_{333} = \dots = 0. \\
 \epsilon_{ijk} &= \epsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}
 \end{aligned}$$

Appendix B

$q_1 - q_4$ and r_{ijk} :

$$q_1 = C_{11}(C_{44}\kappa_{11} + e_{15}^2)$$

$$q_2 = -C_{13}^2\kappa_{11} + C_{11}(C_{33}\kappa_{11} + C_{44}\kappa_{33} + 2e_{15}e_{33}) - 2C_{13}(C_{44}\kappa_{11} + e_{15}^2) \\ - e_{31}(2C_{13}e_{15} - C_{44}e_{31})$$

$$q_3 = C_{33}C_{44}\kappa_{11} - (C_{13}^2\kappa_{11} - C_{11}C_{33} + 2C_{13}C_{44})\kappa_{33} + C_{33}(e_{15} + e_{31})^2 \\ - e_{33}(2C_{13}e_{15} - C_{11}e_{33}) - 2(C_{13} + C_{44})e_{31}e_{33}$$

$$q_4 = C_{44}(C_{33}\kappa_{33} + e_{33}^2)$$

$$\eta_{111} = -\frac{C_{11} - C_{12}}{C_{11}}q_1$$

$$\eta_{112} = -2q_1$$

$$\eta_{113} = -(C_{11} - C_{12})(C_{33}\kappa_{11} + C_{44}\kappa_{33} + 2e_{15}e_{33}) - 2C_{44}(C_{44}\kappa_{11} + e_{15}^2)$$

$$\eta_{114} = -2q_2$$

$$\eta_{115} = -(C_{11} - C_{12})(C_{33}\kappa_{33} + e_{33}^2) - 2C_{44}(C_{33}\kappa_{11} + C_{44}\kappa_{33} + 2e_{15}e_{33})$$

$$\eta_{116} = -2q_3$$

$$\eta_{117} = -2q_4$$

$$\eta_{121} = \frac{C_{11} + C_{12}}{C_{11}}q_1$$

$$\eta_{122} = (C_{11} + C_{12})(C_{33}\kappa_{11} + C_{44}\kappa_{33} + 2e_{15}e_{33}) - 2\kappa_{11}(C_{13} + C_{44})^2 \\ - 2(e_{15} - e_{31})[2C_{13}e_{15} + C_{44}(e_{15} - e_{31})]$$

$$\eta_{123} = (C_{11} + C_{12})(C_{33}\kappa_{11} + e_{33}^2) - 2\kappa_{33}(C_{13} + C_{44})^2 \\ + 2C_{33}(e_{15} + e_{31})^2 - 4e_{33}(C_{13} + C_{44})(e_{15} + e_{31})$$

$$\eta_{131} = (C_{13} + C_{44})\kappa_{11} + (e_{15} + e_{31})e_{15}$$

$$\eta_{132} = (C_{13} + C_{44})\kappa_{33} + (e_{15} + e_{31})e_{33}$$

$$\begin{aligned}
\eta_{141} &= C_{13}e_{15} - C_{44}e_{31} \\
\eta_{142} &= -C_{33}(e_{15} + e_{31}) + e_{33}(C_{13} + C_{44}) \\
\eta_{221} &= -2q_1 = \eta_{112} \\
\eta_{222} &= -\frac{C_{11} - C_{12}}{C_{11}}q_1 = \eta_{111} \\
\eta_{223} &= -2q_2 = \eta_{114} \\
\eta_{224} &= \eta_{113} \\
\eta_{225} &= -2q_3 = \eta_{116} \\
\eta_{226} &= \eta_{115} \\
\eta_{227} &= -2q_4 = \eta_{117} \\
\eta_{231} &= \eta_{131} \\
\eta_{232} &= \eta_{132} \\
\eta_{241} &= \eta_{141} \\
\eta_{242} &= \eta_{142} \\
\eta_{331} &= -C_{11}\kappa_{11} \\
\eta_{332} &= -C_{44}\kappa_{11} - C_{11}\kappa_{33} - (e_{15} + e_{31})^2 \\
\eta_{333} &= -C_{44}\kappa_{33} \\
\eta_{341} &= -C_{11}e_{15} \\
\eta_{342} &= C_{13}(e_{15} + e_{31}) + C_{44}e_{31} - C_{11}e_{33} \\
\eta_{343} &= -C_{44}e_{33} \\
\eta_{441} &= C_{11}C_{44} \\
\eta_{442} &= -C_{11}C_{33} - C_{13}^2 - 2C_{13}C_{44} \\
\eta_{443} &= C_{33}C_{44}
\end{aligned}$$

Appendix C

1. The values of constants l_i , m_i , l'_i and m'_i (i ranges from 1 to 5):

$$A_1 = (1 - \sqrt{v_f})C_{11}^f + \sqrt{v_f}C_{11}^m, A_2 = (1 - \sqrt{v_f})C_{13}^f + \sqrt{v_f}C_{13}^m,$$

$$A_3 = (1 - \sqrt{v_f})C_{33}^f + \sqrt{v_f}C_{33}^m$$

$$B_1 = (1 - \sqrt{v_f})(C_{12}^m - C_{12}^f), B_2 = C_{11}^m, B_3 = C_{13}^m$$

$$B_4 = (1 - \sqrt{v_f})(e_{31}^m - e_{31}^f), B_5 = (1 - \sqrt{v_f})(C_{13}^m - C_{13}^f), B_6 = C_{33}^m$$

$$B_7 = (1 - \sqrt{v_f})(e_{33}^m - e_{33}^f), B_8 = (1 - \sqrt{v_f})(\beta_{22}^m - \beta_{22}^f),$$

$$B_9 = (1 - \sqrt{v_f})(\beta_{33}^m - \beta_{33}^f)$$

$$l_1 = \frac{B_1 A_3 - B_5 A_2}{A_1 A_3 - A_2^2}, l_2 = \frac{B_2 A_3 - B_3 A_2}{A_1 A_3 - A_2^2}, l_3 = \frac{B_3 A_3 - B_6 A_2}{A_1 A_3 - A_2^2},$$

$$l_4 = \frac{B_4 A_3 - B_7 A_2}{A_1 A_3 - A_2^2}, l_5 = \frac{B_8 A_3 - B_9 A_2}{A_1 A_3 - A_2^2}$$

$$m_1 = \frac{B_1 A_2 - B_5 A_1}{A_2^2 - A_1 A_3}, m_2 = \frac{B_2 A_2 - B_3 A_1}{A_2^2 - A_1 A_3}, m_3 = \frac{B_3 A_2 - B_6 A_1}{A_2^2 - A_1 A_3},$$

$$m_4 = \frac{B_4 A_2 - B_7 A_1}{A_2^2 - A_1 A_3}, m_5 = \frac{B_8 A_2 - B_9 A_1}{A_2^2 - A_1 A_3}$$

$$l'_1 = \frac{\sqrt{v_f}}{1 - \sqrt{v_f}} l_1, l'_2 = \frac{(\sqrt{v_f} l_2 - 1)}{1 - \sqrt{v_f}}, l'_3 = \frac{\sqrt{v_f}}{1 - \sqrt{v_f}} l_3, l'_4 = \frac{\sqrt{v_f}}{1 - \sqrt{v_f}} l_4, l'_5 =$$

$$\frac{\sqrt{v_f}}{1 - \sqrt{v_f}} l_5$$

$$m'_1 = \frac{\sqrt{v_f}}{1 - \sqrt{v_f}} m_1, m'_2 = \frac{(\sqrt{v_f} m_2 - 1)}{1 - \sqrt{v_f}}, m'_3 = \frac{\sqrt{v_f}}{1 - \sqrt{v_f}} m_3,$$

$$m'_4 = \frac{\sqrt{v_f}}{1 - \sqrt{v_f}} m_4, m'_5 = \frac{\sqrt{v_f}}{1 - \sqrt{v_f}} m_5$$

2. Electroelastic Eshelby tensors for long fibers

For circular cylindrical inclusion ($a_3 \rightarrow \infty, \alpha = a_2/a_1, \alpha = 1$), the Eshelby tensor \mathbf{S} in a transversely isotropic matrix in which axis of symmetry being x_3 -axis is given as

$$S = \begin{bmatrix} S_{1111} & S_{1122} & S_{1133} & 0 & 0 & 0 & 0 & 0 & S_{1143} \\ S_{2211} & S_{2222} & S_{2233} & 0 & 0 & 0 & 0 & 0 & S_{2243} \\ C_{13}^x & C_{13}^x & C_{33}^x & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2S_{2323} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2S_{3131} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2S_{1212} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{4141} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & S_{4242} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The non-zero components of the Eshelby tensor are explicitly obtained as

$$S_{1111} = \frac{\alpha}{2(\alpha+1)^2} \left\{ \frac{2C_{11}+C_{12}}{C_{11}} + \frac{\alpha+2}{\alpha} \right\}, S_{1122} = \frac{\alpha}{2(\alpha+1)^2} \left\{ \frac{(2+\alpha)C_{12}}{\alpha C_{11}} - 1 \right\},$$

$$S_{1133} = \frac{C_{13}}{(\alpha+1)C_{11}}, S_{1143} = \frac{e_{31}}{(\alpha+1)C_{11}}, S_{1133} = \frac{C_{13}}{(\alpha+1)C_{11}},$$

$$S_{1143} = S_{2243} = \frac{e_{31}}{2C_{11}}, S_{2211} = \frac{\alpha}{2(\alpha+1)^2} \left\{ \frac{(1+2\alpha)C_{12}}{\alpha C_{11}} - 1 \right\},$$

$$S_{2222} = \frac{\alpha}{2(\alpha+1)^2} \left\{ 2\alpha + \frac{3C_{11}+C_{12}}{C_{11}} \right\}, S_{2233} = \frac{\alpha C_{13}}{(\alpha+1)C_{11}}$$

$$S_{1212} = S_{1221} = S_{2112} = S_{2121} = \frac{\alpha}{2(\alpha+1)^2} \left\{ \frac{\alpha^2+\alpha+1}{\alpha} - \frac{C_{12}}{C_{11}} \right\},$$

$$S_{3343} = \frac{\alpha e_{31}}{(\alpha+1)C_{11}},$$

$$S_{1313} = S_{1331} = S_{3131} = S_{3113} = \frac{1}{2(\alpha+1)},$$

$$S_{2323} = S_{2332} = S_{3232} = S_{3223} = \frac{\alpha}{2(\alpha+1)},$$

$$S_{4141} = \frac{1}{(\alpha+1)}, S_{4242} = \frac{\alpha}{(\alpha+1)}$$

All other components are zero.

List of Publications

1. S. K. Singh and S. K. Panda, “Determination of effective properties of piezoelectric composites containing short fibers”, *Emerging Materials Research*, 2020, vol. 9(4), pp. 1195-1208.
2. S. K. Singh and S. K. Panda, “An analytical approach to evaluate the effective coefficients of piezoelectric composites”, *Emerging Materials Research*, 2021, vol. 10(2), pp. 178-193.
3. S. K. Singh and S. K. Panda, “An analytical approach to study performance parameters of 1-3 piezoelectric composite”, *AIP Conference Proceedings*, 2021, vol. 2327, 020020.
4. S. K. Singh and S. K. Panda, “An analytical approach to evaluate effective coefficients of 1-3 piezoelectric composite”, *International Journal of Computational Materials Science and Engineering*, 2022, vol. 11(1), 2150026.
5. S. K. Singh and S. K. Panda, “A comparative study of micromechanics models to evaluate effective coefficients of 1-3 piezoelectric composite”, *Mechanics of Advanced Materials and Structures*, 2022. (Accepted for Publication)
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