# CHAPTER – V

# DESIGN OF FRACTIONAL ORDER PD CONTROLLER FOR NON-MONOTONIC PHASE SYSTEMS

# INTRODUCTION

The concept of extending an integer order calculus to non-integer order calculus by no means is a new topic. Fractional calculus having a history of over 300 years, but its research and applications in engineering has been developed in recent years. By early sixties, the theoretical research as well as applications in the field of fractional derivatives and integrals was started in the various fields of control engineering. Oustaloup et al (1991) is the first person to introduce the fractional order controller in feedback control systems. He made the so called commande robuste d ordre non entier (CRONE), controller which is used in different control systems fields. In recent years, Podlubny et al (1994) proposed a generalisation of PID controller called fractional $PI^{\lambda}D^{\mu}$ controller, involving $\lambda$ (fractional order integration action) $\mu$ (fractional order differentiation action). In fractional $PI^{\lambda}D^{\mu}$ controller, the fractional I and D actions being fractional have wider scope of design. In the literature (Chen et al., 2009; Kesarkar et al 2011; Luo et al 2009b; Zhao et al., 2005) several attempts has been found on tuning of the fractional order controller for specific class of plants. In frequency response compensation of an LTI system, the frequencies which are amplified in open loop can be used to make control action in closed loop system with negative feedback (Skogestad et al 2005).

The phase margin of a system is defined as the distance between the open loop phase and –180° line, at the frequency where the gain crosses 0 dB line. For stability, the system should have positive phase margin. Thus, the phase margin is an important robustness indicator, which shows up to what extent the open loop phase can be varied without degrading the stability of the closed loop system. It is also in relation with the damping ratio of a second order system. If the damping ratio is higher, the peak overshoot will be smaller (Philips et al 1999; Ogata et al 2009). The block diagram of a feedback control system with the plant and the controller is shown in Figure 5.1. The bandwidth will be defined as range of frequency from zero to the gain crossover frequency.

The phase margin is defined at the gain crossover frequency, but it is not always valid since if the left half plane zero is present near the left half plane dominant pole then the phase curve will exhibit a non-monotonic behavior in the frequency response.

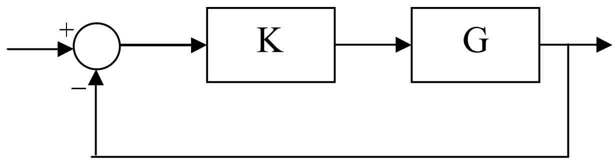



Figure 5.1- The block diagram of plant with the controller

Monotonicity means consistently increasing or consistently decreasing. So a non monotonic system has increasing phase somewhere and decreasing phase somewhere. In the frequency response of the non-monotonically decreasing phase system we will observe a point of minima in the phase curve the undamped frequency, which is a point of minima in phase curve. This is the worst-case phase of the system and the frequency where this point of minima is located is the worst-case phase frequency. This type of phase results because the left half plane zero is not located far from the left half pole (de Paula et al 2012).

Since the worst case frequency is in the bandwidth of the system, it will be always smaller than the gain crossover frequency and similarly the worst-case phase margin will be always less than the phase margin. Therefore, if there is positive worst-case phase margin then the phase margin will also be positive. In this chapter, a fractional order proportional derivative (FOPD) controller has been proposed to control a monotonic and non-monotonic phase system.

The chapter is organized as follows: Section 5.1 presents a brief description of non-monotonic phase system. Section 5.2 demonstrates the basic idea about the fractional order PD controller parameter setting with specified phase, gain margins and robustness requirement on the variations of the loop gain. The proposed method has been discussed in Section 5.3. Section 5.4 describes the results by considering an example. Finally, the conclusions are made in Section 5.5.

## 5.1 SYSTEM MODELLING

Consider the following Equation (5.4), which will be used for the modeling of the continuous LTI dynamical system that will be studied in this brief.

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_1 \dot{u}(t) + b_0 u(t) \qquad (5.1)$$

The systems that can be modeled by above differential Equation (5.1) are, an LC low pass filter, or a two ambient-coupled heat tank. A synchronous buck regulator with linearised model is considered. The buck regulator is a combination of the power stage, which is an LC low pass filter, and a pulse width modulation (PWM) controller (International Rectifier, 2002). The transfer function of the considered buck converter is

$$P(s) = \frac{V_{IN}}{V_{OSC}} \frac{1 + sR_C C}{LCs^2 + s(R_C C + \frac{L}{R}) + 1} \qquad (5.2)$$

Where

C=output capacitance

L=output inductance

R= load resistance

$R_C$ = output Capacitor intrinsic resistance

$V_{OSC}$ = PWM oscillator reference

$V_{IN}$ = power stage input voltage

A buck regulator which is suggested in (International Rectifier, 2002) yields the following transfer function with $R_C$ = 40-mΩ and other corresponding values

$$P(s) = \frac{4(1+1.2\times10^{-5}s)}{3\times10^{-9}s^2 + 3.6\times10^{-5}s + 1} \tag{5.3}$$

The transfer function revels that the undamped frequency is close to 18 krad/sec and the damping ratio is equal to 0.33. There is a left half plane zero at 83 krad/sec which is nearly five times greater than undamped frequency. The Bode plot of this system is shown in Figure 5.2 where a valley can be noticed in the phase curve which is due to the fact that the left half plane zero which is due to the output capacitor intrinsic impedance, is located near the poles. Thus, if there is any zero frequency after the bandwidth, it would require non-monotonic type of compensation. The transfer function will remain minimum phase and the Bode stability criterion could be used.

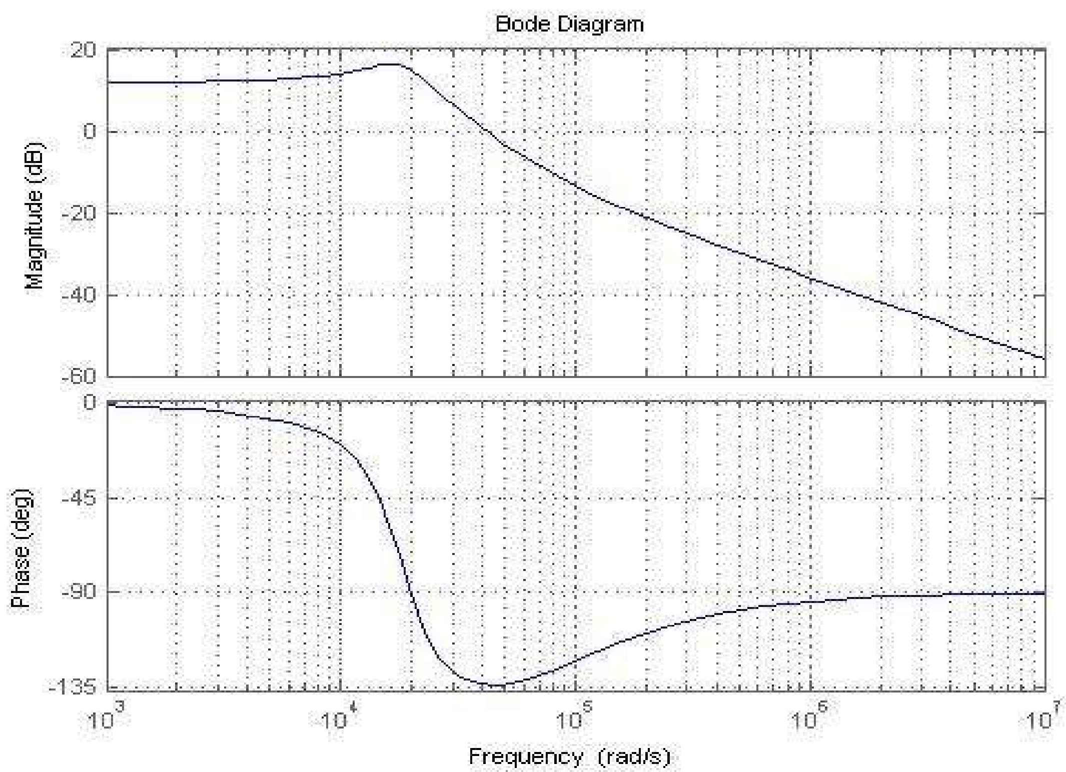



Figure 5.2- The Bode plot of the system

## 5.2 BASIC IDEAS OF FRACTIONAL ORDER PD CONTROLLER AND ITS DESIGN SPECIFICATIONS

The motivation for using fractional order PID controller (FOPID) is to take the advantage of two fractional power parameters to meet additional design specifications of the system (Podlubny et al 1994). Since integer order PID controller consists only three parameters, i.e., $K_p$, $K_i$ and $K_d$ . Thus, we can use three control specifications to design the controller. However, in the case of the fractional order PID controller, we have five parameters and we can achieve five different design specifications of the system.

In literature various techniques for tuning of FOPD controller has been developed by many researchers. FOPI controller has been formulated by Luo et al. (2010) by means of fractional M-constrained integral gain optimisation (MIGO) tuning rule (Astrom et al 2004). Emphasis on phase margin ($\varphi_m$), gain cross over frequency ($\omega_{gc}$), iso-damping property/robustness criteria (flat phase curve around $\omega_{gc}$) ensued a tuning algorithm to design FOPI/FOPD controller for stabilising integer order system (Monje et al., 2004a; Li et al 2008; Ho et al., 1995; Das et al 2008; Chen et al 2005) which was extended by Luo et al (2009a) and Luo et al. (2010) for a class of fractional order models. Monje et al. (2008, 2009) and Dorcak et al. (2007) has proposed an optimization-based tuning of FOPID in frequency domain with two extra specification sensitivity and complementary sensitivity function along with the parameters presented in Monje et al. (2004a) and Li et al (2008).

The generalized form of the FOPD is

$$C_{FOPD}(j\omega) = K_p(1 + K_d s^{\mu}) \tag{5.4}$$

The fractional order $PD^{\mu}$ controller described by Equation (5.4) can be written has

$$C_{FOPD}(j\omega) = K_p(1 + K_d (j\omega)^{\mu}) \tag{5.5}$$

$$= Kp[(1 + Kd\omega^{\mu} \cos\frac{\mu\pi}{2}) + jKd\omega^{\mu} \sin\frac{\mu\pi}{2}] \tag{5.6}$$

The phase of the FOPD controller is given by

$$\angle[C_{FOPD}(j\omega)] = \tan^{-1}\left(\frac{\sin\frac{(1-\mu)\pi}{2} + K_d\omega^{\mu}}{\cos\frac{(1-\mu)\pi}{2}}\right) - \frac{(1-\mu)\pi}{2} \tag{5.7}$$

The phase of the FOPD controller is given by

$$\left|C_{FOPD}(j\omega)\right| = K_p\sqrt{(1+K_d\omega^{\mu}\cos\frac{\mu\pi}{2})^2 + (\sin\frac{\mu\pi}{2})^2} \tag{5.8}$$

The derivative of the phase of the FOPD controller with respect to frequency ω is given by

$$\frac{d\angle C_{FOPD}(j\omega)}{d\omega} = \frac{\mu K_d\omega^{\mu-1}\cos\frac{(1-\mu)\pi}{2}}{\cos^2\frac{(1-\mu)\pi}{2} + \left(\sin\frac{(1-\mu)\pi}{2} + K_d\omega^{\mu}\right)^2} \tag{5.9}$$

By using fractional order PD [FOPD] controller, we can fulfill specifications regarding phase margin, gain crossover frequency and robustness specification.

- *Phase margin and the gain crossover specification:*

As discussed earlier the phase margin and the gain crossover frequency is a very powerful robustness indicator. The gain crossover frequency and the phase margin are given Equation (5.10) and Equation (5.11).

$$\left|C_{FOPD}(j\omega_{cg})P(j\omega_{cg})\right|_{dB} = 1dB, \tag{5.10}$$

$$\arg(C_{FOPD}(j\omega_{cg})P(j\omega_{cg})) = \pi + \phi_m \tag{5.11}$$

- *Robustness to variation of the gain of the plant:*

This constraint forces the phase of the open loop system to be flat at the crossover frequency and thus the phase remains almost constant in this interval around crossover frequency.

$$\left(\frac{d\arg(C_{FOPD}(j\omega)P(j\omega))}{d\omega}\right)_{\omega=\omega_{cg}} = 0 \tag{5.12}$$

## 5.3 PROPOSED TECHNIQUE

### 5.3.1 COMPENSATION METHODOLOGY

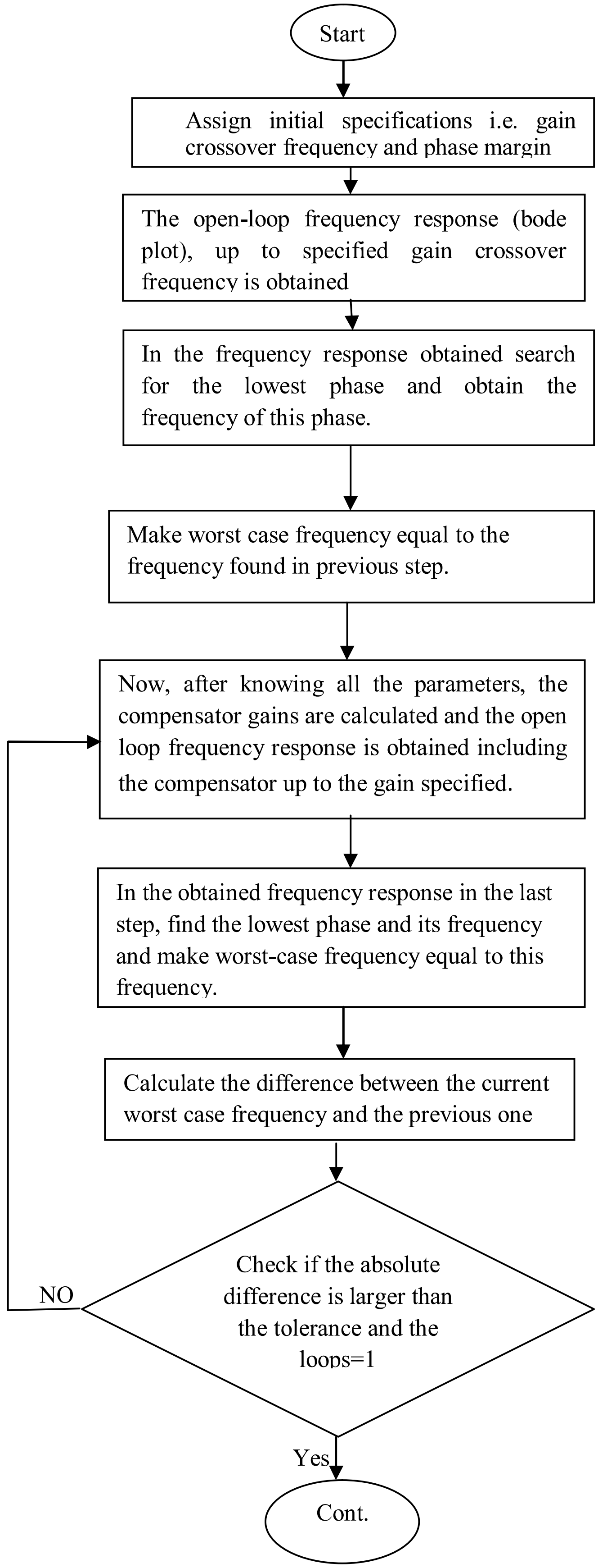
The proposed method is an analytical design procedure for the fractional order PD controller which will give a worst case phase margin specification throughout the bandwidth for a stable minimum phase open loop control system. Since phase margin and the gain crossover frequency are the powerful robustness measure and are in relation to the damping ratio and the undamped frequency. Here we assume that the negative feedback control system will behave as a low pass system in the open loop frequency response because of the action of the controller.

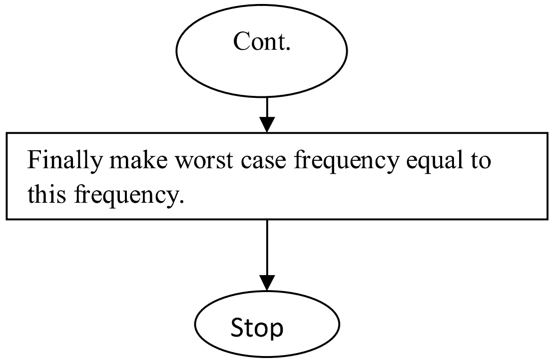
Here we need to define four parameters, i.e., gain crossover frequency ($\omega_u$), phase margin ($\phi_u$), worst case frequency ($\omega_m$) and the worst case phase margin ($\phi_m$). The gain crossover frequency ($\omega_u$) is the characteristic equation solution of the transfer function in the frequency domain, phase margin ($\phi_u$) is the difference between the phase at gain crossover frequency and 180 degrees. The worst case frequency ($\omega_m$) is the frequency where the system exhibits minima in phase curve inside the bandwidth and the worst case phase ($\phi_m$) is the difference between the 180$^{o}$ line and the phase in degrees at the worst case frequency. The worst case frequency is always smaller or equal to the gain crossover frequency, and thus the worst case phase margin is smaller or equal to the phase margin. So a positive worst case phase margin means the phase margin will automatically be positive and the system will be stable.

$$\phi_m \leq \phi_u \tag{5.13}$$

$$\omega_m \leq \omega_u \tag{5.14}$$

The gain crossover frequency and the worst case phase margin will be free specifications but as the open loop system will be modified by the compensator the worst case frequency cannot be a free specification. To find out the worst case frequency a recursive search procedure has to be followed (de Paula et al 2012). This search procedure is explained by the help of the flowchart.


Start
Assign initial specifications i.e. gain crossover frequency and phase margin
The open-loop frequency response (bode plot), up to specified gain crossover frequency is obtained
In the frequency response obtained search for the lowest phase and obtain the frequency of this phase.
Make worst case frequency equal to the frequency found in previous step.
Now, after knowing all the parameters, the compensator gains are calculated and the open loop frequency response is obtained including the compensator up to the gain specified.
In the obtained frequency response in the last step, find the lowest phase and its frequency and make worst-case frequency equal to this frequency.
Calculate the difference between the current worst case frequency and the previous one
Check if the absolute difference is larger than the tolerance and the loops=1
NO
Yes
Cont.




The characteristic equation is given by

$$\left|K(j\omega_u)P(j\omega_u)\right|\angle K(j\omega_u)P(j\omega_u)=1\angle-180^0+\phi_u \tag{5.15}$$

The Equation (5.15) has a magnitude part and a phase part. This equation defines the phase margin and the gain crossover frequency. As we have discussed that $\omega_u$ is always greater than $\omega_m$, and also $\phi_u$ is greater than $\phi_m$. So if we make $\phi_m$ positive then $\phi_u$ will also be positive. Thus, in the phase part of the Equation (5.15) we can replace $\omega_u$ by $\omega_m$ which will give the system the worst case stability and the system will remain stable throughout the bandwidth.

$$\left|K(j\omega_u)\right|=\frac{1}{\left|G(j\omega_u)\right|} \tag{5.16}$$

$$\angle K(j\omega_u)=-180^0+\phi_m-\angle G(j\omega_m) \tag{5.17}$$

### 5.3.2 FRACTIONAL ORDER PD CONTROLLER DESIGN

To justify the superiority of the proposed method over the conventional method, we need to design the FOPD controller for the buck regulator by both methods. The generalised transfer function of FOPD controller is

$$C_2(s)=K_p(1+Ks^{\mu}) \tag{5.18}$$

In order to design the fractional order PD controller, calculate the worst case frequency by the procedure explained in the flowchart. As discussed in the previous section, assign the initial specification i.e. the gain crossover frequency and the desired phase margin. Here the desired phase margin is $40^o$. As seen from the transfer function, we observe that the undamped frequency is closely 18krad/sec and the left half plane zero is about 83krad/sec. The switching frequency of the converter is found to be 200 kHz (approximately 1250 krad/sec). So, it is suggested that the gain crossover frequency should be nearly one fifth of switching frequency. So the gain crossover frequency is 250krad/sec. Now, with these initial specifications, we begin the search for worst case frequency adopting the flowchart given in the previous section. As the search is complete the worst case frequency comes out to be 40 krad/sec. For designing the FOPD controller we need three specifications since there are three variables in the transfer function of the controller Li et al (2008). The three specifications that we will be considering are

- *Phase Margin Constraint*

$$\arg(G(j\omega_c)) = \arg(C(j\omega_c)P(j\omega_c)) = \pi + \phi_m \qquad (5.19)$$

- *Gain crossover frequency constraint*

$$\left|G(j\omega_c)\right|_{dB} = \left|C(j\omega_c)P(j\omega_c)\right|_{dB} = 0 \qquad (5.20)$$

- *Robustness to loop gain variation constraint*

$$\left(\frac{d(\arg(G(j\omega)))}{d\omega}\right)_{\omega=\omega_c} = 0 \qquad (5.21)$$

Now, we design the $PD^{\mu}$ controller firstly by the monotonic method, or the conventional method. The steps are given below.

I. Given, the $\omega_c = \omega_u$ =250 krad/sec and $\phi_m = 40^o$

II. With these values we write Equation (5.19) and Equation (5.21), it will give us two equations with two unknowns i.e. $\mu$ and $K_d$.

III. Determine the values of these two parameters by fsolve command in MATLAB. The values comes out to be, $\mu$=-0.4938 and $K_d$=454.02

IV. Put these two values in Equation (5.20) to determine the value of $K_p$. The value of $K_p$ comes out to be 8.04.

V. With the obtained values of $\mu$, $K_d$ and $K_p$, obtain the transfer function of the FOPD controller.

VI. Approximate the FOPD controller to integer order by using oustapp command in fomcon toolbox of MATLAB (take frequency band 0.0001 to 1000000).

Now, we will design the FOPD controller by the non-monotonic method, by using the same procedure but with worst case values.

I. Given, the $\omega_c$=$\omega_m$=40 krad/sec and $\varphi_m$= 40$^o$.

II. With these values we write Equation (5.19) and Equation (5.21), it will give us two equations with two unknowns i.e. $\mu$ and $K_d$.

III. Determine the values of these two parameters by fsolve command in MATLAB. The values comes out to be, $\mu$=0.432 and $K_d$= -0.0006.

IV. Put these two values in Equation (5.20) to find out the value of $K_p$. The value of $K_p$ comes out to be 18.32.

V. With the obtained values of $\mu$, $K_d$ and $K_p$, obtain the transfer function of the FOPD controller.

VI. Approximate the FOPD controller to integer order by using oustapp command in fomcon toolbox of MATLAB (take frequency band 0.0001 to 1000000).

Thus, by this procedure we are able to design controllers for both monotonic and non-monotonic method.

## 5.4 ILLUSTRATIVE EXAMPLE

This section presents a design example considering the proposed technique and buck regulator presented in section 2. By the procedure given in previous section the controllers by a monotonic and non-monotonic method has designed. Table.1 shows the performance of the Monotonic compensation and Non-Monotonic Compensation.

| Compensation type | Values of controller parameters | Peak over shoot | Settling time |
|---|---|---|---|
| Monotonic compensation (FOPD) | $K_p$=8.04, $K_d$=454.02, μ=-0.493. | 1.26 | 56.4μs |
| Non-Monotonic Compensation (FOPD) | $K_p$=18.32, $K_d$=-0.0006. μ= 0.432 | 1.16 | 31.2μs |

Table 5.1 Performance of the monotonic compensation and non-monotonic compensation of FOPD controllers

Bode plots referred as IOPID-MONO (integer order proportional integral derivative Monotonic compensation) and IOPID-NON MONO (integer order proportional integral derivative Non-monotonic compensation) in Figure 5.3 are earlier obtained results taken from de Paula et al (2012).

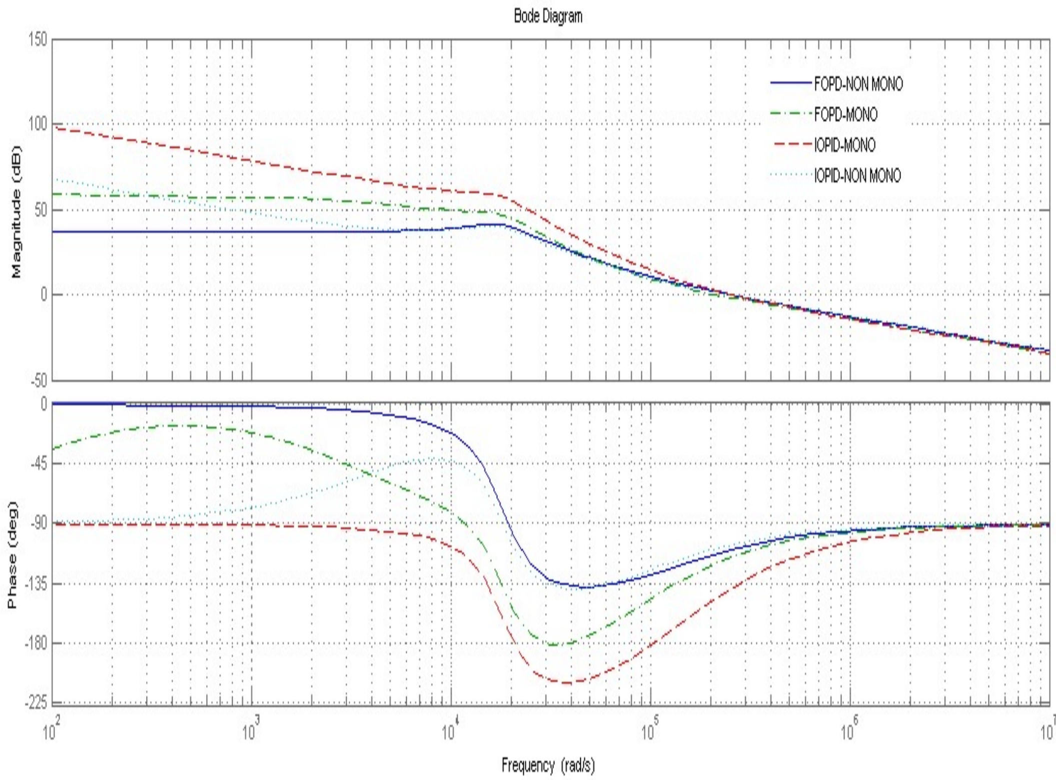



Figure 5.3- Bode plots of monotonic compensation and non-monotonic compensation.

In Figure 5.3 we can observe that, the bandwidth is almost same for both types of controllers. The non-monotonic compensation assures $40^{o}$ of phase margin throughout the bandwidth to all frequencies which the monotonic method is unable to provide. In the bode plot we can see that the monotonic compensation has crossed the stability limit of $-180^{o}$ two times, thus its stability can't be analyzed by the bode plot, so we will draw its nyquist plot shown in Figure 5.4 to analyze the stability of both the compensation.

Nyquist plots referred as IOPID-MONO and IOPID NON-MONO in Figure 5.4 are earlier obtained results taken from de Paula et al (2012).

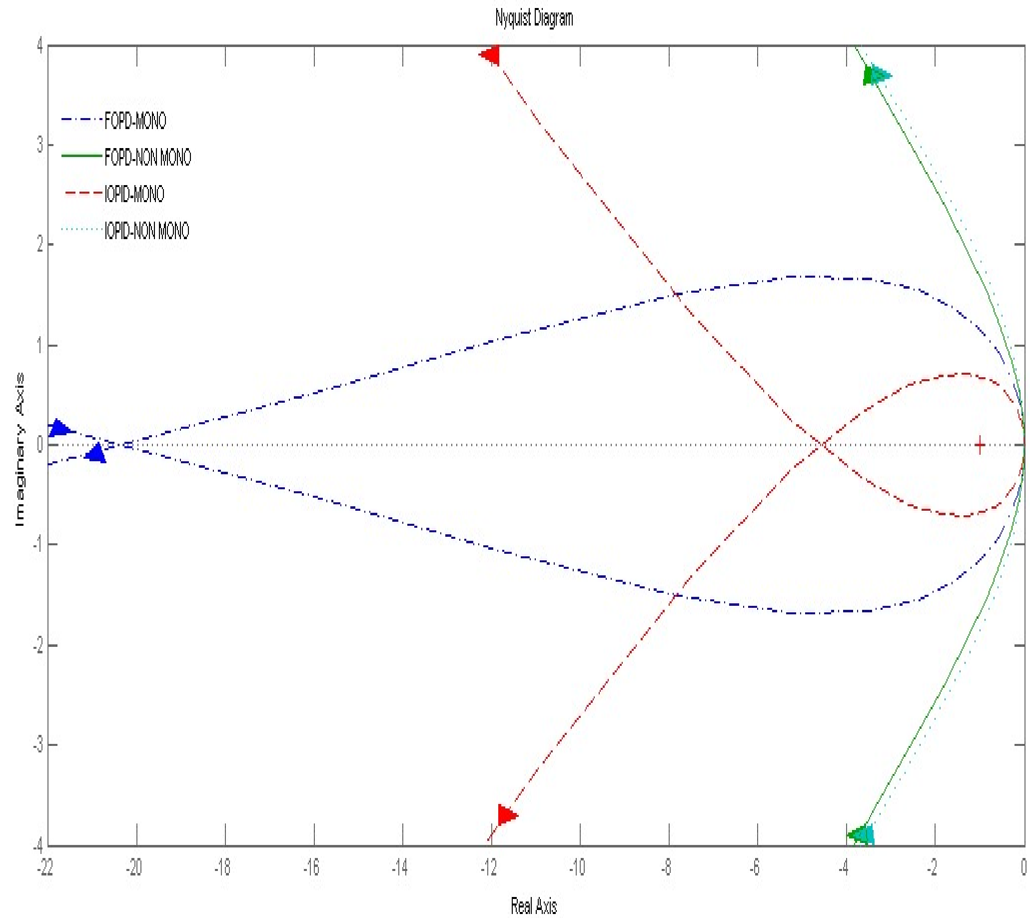



Figure 5.4- Nyquist plot of non-monotonic compensation and monotonic compensation.

From the Figure 5.4, it can be inferred that the monotonic compensation does not assure the stability, as there is one encirclement of the critical point i.e. (-1, 0). But in the non-monotonic we can see that there is no encirclement of the critical point, thus it can be concluded that the system is stable in non-monotonic compensation.

Step response referred as IOPID-MONO and IOPID NON MONO in Figure 5.5 are earlier obtained results taken from de Paula et al (2012).

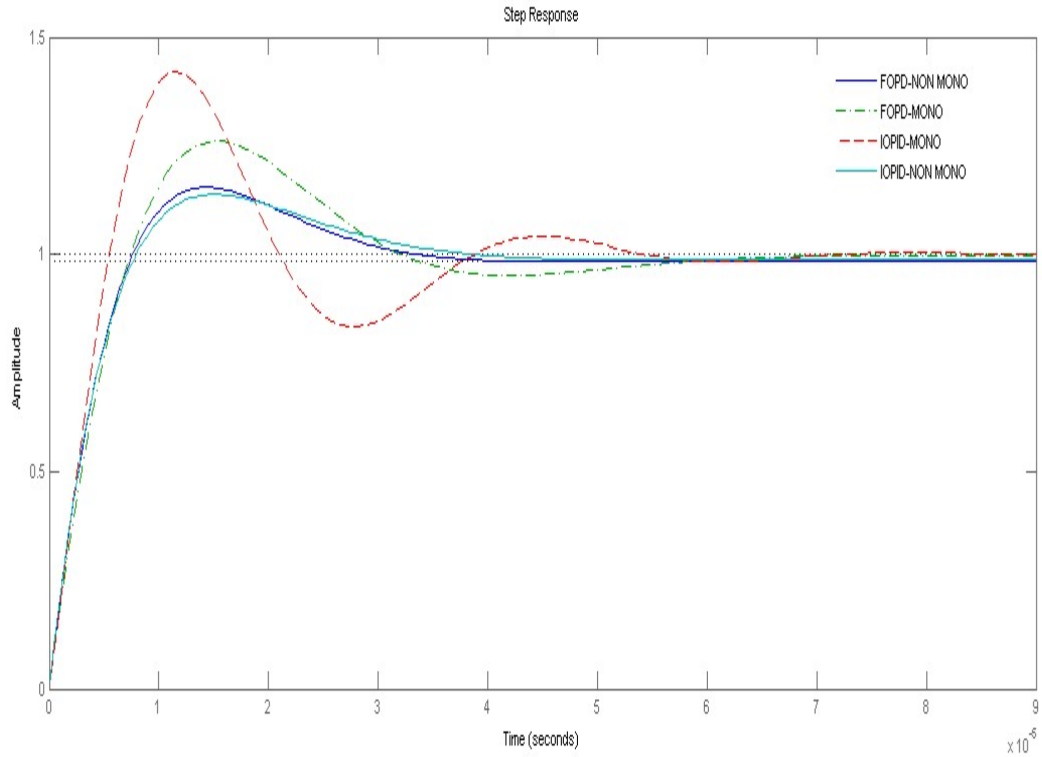



Figure 5.5- step response of non-monotonic compensation and monotonic compensation

The Figure 5.5 displays that the settling time for the monotonic compensation is 56.4 micro seconds and for non-monotonic compensation, it is about 32 micro seconds. Also the peak overshoot of the monotonic compensation is 1.26 and for non-monotonic compensation it is about 1.16. Thus it is clear from the response of both the systems that peak overshoot of the non-monotonic system is noticeably belittled than the monotonic system. This is because the system assures a minimum amount of phase throughout the bandwidth. Also, the smaller overshoot of the non-monotonic compensation system shows the robustness of the system which has been discussed in the Nyquist plot. So, the proposed non-monotonic method of designing of the FOPD controller offers much improved design than the conventional monotonic method.

## 5.5 CONCLUSION

In this chapter, an improved technique for designing of the fractional order PD controller has been discussed for non-monotonically decreasing phase system. The proposed Technique offers some desirable improvement which has not been achieved by the conventional monotonic method and it is applied to the systems with monotonic process plant but the open loop is non-monotonic. The design has been carried out in terms of gain crossover frequency and the phase margin. The concept of the phase margin has been redefined. The proposed method has some improvements like minimum phase margin throughout the bandwidth, improved closed loop performance and superior robustness properties. The peak overshoot of the system is considerably smaller and the steady state comes much sooner than the classical method. The bandwidth for the proposed technique is almost the same as the classical method but the system is more stable and robust.