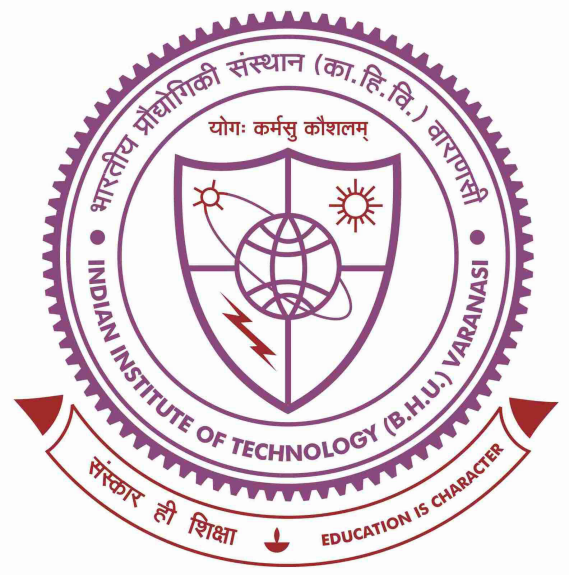
STABLE NUMERICAL SCHEMES FOR FRACTIONAL AND VARIABLE ORDER MATHEMATICAL MODELS



Thesis submitted in partial fulfillment for the

award of degree

DOCTOR OF PHILOSOPHY

By

Priyanka Rajput

DEPARTMENT OF MATHEMATICAL SCIENCES
INDIAN INSTITUTE OF TECHNOLOGY
(BANARAS HINDU UNIVERSITY)
VARANASI-221005, INDIA

Roll No. : 19121014 2025

Dedicated to

My Parents

"Shri Magan Singh, Smt. Usha Devi"

CERTIFICATE

It is certified that the work contained in this thesis titled **"Stable Numerical Schemes for Fractional and Variable Order Mathematical Models"** by **Priyanka Rajput** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of the Comprehensive Examination, Candidacy, and SOTA seminar for the award of Ph.D. Degree.

22/8/2025

Prof. Vineet Kumar Singh

(Supervisor)

Professor

Department of Mathematical Sciences

Indian Institute of Technology

(Banaras Hindu University)

Varanasi-221005

पर्यवेक्षक /Supervisor
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी /Varanasi-221005

DECLARATION BY THE CANDIDATE

I, **Priyanka Rajput**, certify that the work embodied in this thesis is my own bona fide work and carried out by me under the supervision of **Prof. Vineet Kumar Singh** from **July 2019** to **August 2025** at the Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

Date: 22.08.2025

Place: Varanasi

Priyanka Rajput

(Priyanka Rajput)

CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my knowledge.

22/8/2025

(Prof. Vineet Kumar Singh)
Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

पर्यवेक्षक/Supervisor
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी/Varanasi-221005

Subir Das

(Prof. Subir Das)
Professor and Head
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

विभागाध्यक्ष/HEAD
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी/Varanasi-221005

COPYRIGHT TRANSFER CERTIFICATE

Title of the Thesis : Stable Numerical Schemes for Fractional and Variable Order Mathematical Models

Name of the Student : Priyanka Rajput

Copyright Transfer

The undersigned hereby assigns to the Indian Institute of Technology (Banaras Hindu University) Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the Ph.D. degree.

Priyanka Rajput

Date: 22.08.2025

Place: Varanasi

(Priyanka Rajput)

Note: However, the author may reproduce or authorize others to reproduce material extracted verbatim from the thesis or derivative of the thesis for author's personal use provided that the source and the Institute copyright notice are indicated.

Acknowledgment

I have completed this thesis with the grace of **Baba Vishwanath** and the blessings of my family and pedagogues. Only my name appears on the cover page of this thesis, but it has been completed with encouragement, support and efforts of numerous people, including my friends, well-wishers, family members, relatives, and supervisor. So, I would like to thank all of them.

First and foremost, I would like to express my profound gratitude and heartfelt thanks to my thesis supervisor, **Prof. Vineet Kumar Singh, Indian Institute of Technology (Banaras Hindu University), Varanasi**, for his guidance during my research work. I am deeply indebted to him for his invaluable advice, immense patience, utmost care, and constant encouragement throughout this work. Without his guidance and persistent help, this work could not have been accomplished. The confidence shown on me by him was the most significant source of inspiration for me. It has been a privilege working with him for several years. I want to extend my gratitude to **Mrs. Priya Singh** for her care and love throughout my Ph.D. journey.

Besides my supervisor, I want to convey my sincere thanks to the rest of my RPEC (Research Progress Evaluation Committee) members **Dr. Sunil Kumar** and **Dr. Kishor P. Sarawadekar, Department of Electronics** for their cooperation and

useful comments during the progress of my research work. I owe my special thanks to **Prof. Subir Das, Head of the Department, and Dr. Anuradha Banerjee, Convener, DPGC of Department** for their support throughout my research work. I also convey my gratitude to all the faculty members who have offered their help at different stages during my research time.

I sincerely acknowledge my Institute, IIT (BHU), for providing necessary resources throughout my Ph.D. program. I am most grateful to the Ministry of Human Resource Development, Government of India, for providing me the fellowship in the form of Junior Research Fellowship and Senior Research Fellowship.

I express my deepest gratitude to my PhD senior, **Dr. Nikhil Srivastava, Assistant Professor & Head, the Department of BS&HSS, NIT Mizoram**, for their unwavering guidance, insightful feedback, and constant encouragement throughout my research journey. His expertise and mentorship have been invaluable in shaping this thesis and my academic growth. I am deeply thankful for his patience and mentorship, which have made this challenging endeavor truly rewarding.

I am eternally grateful to my best friends, Kriti Yadav and Amit Sharma, whose unwavering support and infectious spirit transformed my PhD journey into a tapestry of cherished memories. This thesis stands as a tribute to their friendship, which made every step of this path unforgettable. Also, I would like to acknowledge my colleagues from my batch for their help and support during my Ph.D. work. I take this opportunity to convey my gratitude to my labmate Dr. Anant Pratap Singh. I am grateful to him for bearing me for so many years and helping me during this time.

I would always remember my lab mates for the fun times spent together, which are unforgettable moments. It was a wonderful experience working with Dr. Vinita Devi, Dr. Rahul Kumar Maurya, Dr. Yashveer Kumar, Dr. Aman Singh, Dr. Nikki

Kedia, Dr. Poonam Yadav, Dr. AP Singh, Shivani Gupta, Vaibhav Agarwal, Sunil Kumar, Priya Singh, Vikash, Akash, Gauri, Priyanka and Mayank of my research group and I hope that our collaboration will continue in future also.

I am immensely grateful to my seniors Dr. Aakansha, Assistant Professor, GEC Jehanabad (Bihar) and Dr. Aman Singh for their support during the ups and downs in my PhD journey. I am also grateful to my lovely junior Shivani Gupta, for her love and support. Their energy and endless masti made my Ph.D. journey so much enjoyable, your presense was constant source of joy and motivation. Thank you all, for being such a wonderful companion and for making even the toughest days feel lighter with your vibrant spirit.

I would like to thank my friends Dr. Akansha Verma, Assistant professor, Dayal Singh college (Delhi University) and Dr. Swati Saxena, Neha Singh Rajput, Dr Sanju Yadav, Vaishali Rajput, Akshay Kumar, and all the Ph.D. colleagues of the department.

I am grateful to my best friend Abhishek Kumar for being my pillar of strength and constant inspiration. Your unwavering support made this Ph.D. journey possible.
My special thanks go to my parents **Shri Magan Singh** and **Smt. Usha Devi** for always believing in me, supporting me through every challenge, and showering me with their endless love and motivation. I would like to thank my elder brother **Sumit Rajput**, my sister in law **Shalu Rajput**, my loving nephew **Madhavan Rajput**, my younger cousins **Priyanshu Rajput**, **Shivika Rajput** and all others my family members for his/her continuous encouragement and support during not only my Ph.D. but at every step of my life.

This acknowledgment would be incomplete if the name of great visionary "**Pt. Madan Mohan Malaviya**" is not mentioned, who made this divine center of knowledge. Deepest regards to him.

Finally, I owe **my family** all my thanks for their most profound love, endless patience and continued support in the pursuit of my aspirations through all of the years of my study. I bow down my head to "**Kashi Vishwanath and Sankat Mochan**" for showers of blessings throughout my research work to complete it successfully.

Date: 22.08.2025

Place: Varanasi **Priyanka Rajput**

List of Figures

List of Tables

List of Symbols

0.1 Nomenclature

$\mathbb{R}$	Set of real numbers
$\mathbb{Z}$	Set of integers
$\mathbb{N}$	Set of natural numbers
$\mathbb{R}^+$	Set of non-negative real numbers
$\mathbb{Z}^+$	Set of non-negative integers
$\otimes$	Kronecker product
Γ	Gamma function
$\nabla^2,\ \Delta^x$	Laplace operator
${}_0^C D_t^{\alpha(x,t)} u(t)$	Variable order Caputo derivative of function $u(t)$ w.r.t. variable order $\alpha(x,t)$
${}_0^C D_t^{\alpha} u(t)$	Caputo fractional derivative of function $u(t)$ w.r.t. fractional order α
$h, \Delta x$	Space step-size
$\tau,\ \Delta t$	Time step-size
$\|.\|,\ \|.\|_2$	L_2- norm
$\|.\|_\infty$	$L_\infty-$ norm
Φ	Basis function w.r.t. shifted Legendre polynomial