

# CHAPTER V

## OPTIMAL COST WAMS INCORPORATING COMMUNICATION INFRASTRUCTURE

### 5.1. INTRODUCTION

Various methods have been employed in the past to determine the optimal locations of devices to retain the observability of the system which also minimize the installation cost of PMUs. The installation cost of PMUs is not only depends upon the cost of PMUs, but it also depends upon the cost of communication infrastructure (CI) such as cost of fiber optics, cost of switches, cost of labors etc. However, the costs of CI from Phasor Data Concentrator (PDC) to PMUs have not been given proper attention. Taking cognigence of this issue, recently *Shahraeini et al.* [129] and *Mohammadi et al.* [130] incorporated the cost of CI in their formulation of PMU placement method. However, maximum observability (MO) has not been considered in above work. Figure 5.1 shows the CI in WAMS where PDC collects the data from all the PMUs which are directly connected to the PDC through communication links.

In present chapter, a new meta-heuristic BGSA with a new multi-objective function has been proposed to minimize the overall cost of installation of PMUs by minimizing the number of PMUs with MO while minimizing the path of communication infrastructure in the system. These problems deal with real

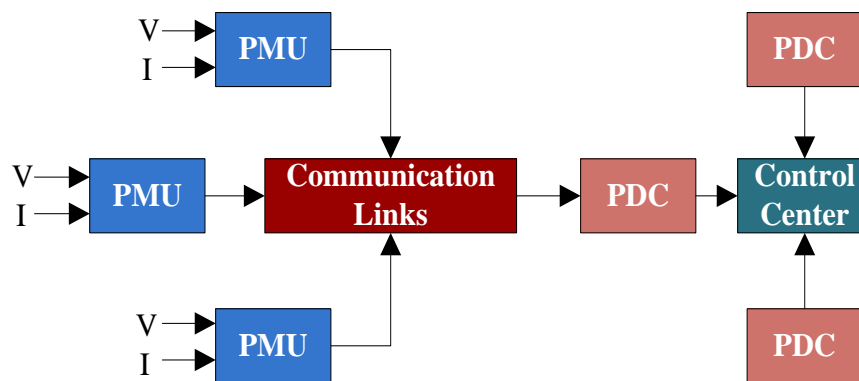


Figure 5.1. Communication infrastructure in WAMS

numbers. Since the OPP problem deals with the binary variables, basic concepts of GSA have been extended in this investigation to incorporate discrete binary search. The resulting algorithm is called BGSA.

The performance of proposed method has been tested on various sizes of the systems under various conditions. The results have also been compared with the methods reported in the literature. The results reveal that the proposed method required a lesser number of switches requiring lesser distance path of the communication link and least number of PMUs with higher observability.

The main contributions of this chapter are as under:

- i) The entire system is observable to the maximum extent to produce better reliability with a minimum number of PMUs obtained incorporating the cost of CI,
- ii) It yields PMU locations following least path of the communication link. In turn, it requires the least number of switches,
- iii) Single line outage or single PMU outage has been considered as the contingency in the power network. Presence of zero injection bus (ZIB) has been considered in all the case studies.

## 5.2. FORMULATION OF THE PROPOSED METHOD

In this chapter, the multi-objective function has been proposed to optimize the number of PMUs and minimize the cost of CI with maximum observability. All the proposed results include ZIB in the system. ZIB are buses with neither generation nor load. Its presence may reduce the number of PMUs.

### 5.2.1. PMU Placement in normal case

The proposed objective function can be written as follows:

$$\text{Minimize } (F_1 + F_2) \quad (5.1)$$

$$\text{where } F_1 = \sum_{i=1}^n w_i c_i = \sum_{i=1}^n \frac{c_i}{(c_{cl} * NC_i) + c_p} \quad (5.2)$$

$$\text{and } F_2 = c_{fb} \sum_{i=1}^p l_i + c_{sw} \sum CI \quad (5.3)$$

$$\text{subject to: } AC \geq 1 \quad (5.4)$$

where,  $c_i$  is the elements of column vector  $C$ , which represents the status of the installation of a PMU at bus  $i$ . If  $c_i = 1$ , it means PMU is installed at bus  $i$ ,

otherwise  $c_i = 0$ . Equation (5.2) is to optimize the number of PMUs with MO. In previous references [22], [20], [38] and [39] authors used the multi-objective function to optimize the number of PMUs having MO. In this chapter, Equation (5.2) provides both objectives in a single objective function which is the advantage over previous methods. In Expression (5.2),  $w_i$  is the element of column vector  $W$ , which represents the inverse of the cost of PMU with respect to a number of channels (branches) connected to bus  $i$ .  $w_i$  is defined as follows:

$$w_i = \frac{1}{(c_{cl} * NC_i) + c_p} \quad (5.5)$$

Equation (5.3), define the minimum path with minimum switches for optimal PMUs from Equation (5.2). Both the Equations (5.2) & (5.3) run simultaneously in Equation (5.1). Every switch in CI represents the system bus. Connectivity matrix  $A$ , defines the interconnection of system buses by transmission lines. To inject the concept of ZIB, modify the Expression (5.4) as given in Expressions (4.8) and (4.9).

### 5.2.2. PMU placement considering single PMU outage or single line outage

In this chapter, line outages or loss of a PMU have been included. However, which of the contingencies has occurred may not be known a priori. This necessitates formulation of the PMU placement problem such that none of the buses is left unobservable in such contingency. The optimal number of PMUs increases as further line outages are considered. If the outage of a radial line occurs, observability of the radial bus on this line will be lost. However, this loss of observability will not affect the observability of the remaining system as the radial bus will be isolated from rest of the system. This can be incorporated by eliminating the radial lines from the set of lines,  $L$ .

The case of line outage can mathematically be expressed by modifying the relation for connectivity matrix expressed by Expression (5.4). For example, if a line between bus  $i-j$  gets disconnected, then the elements of connectivity matrix  $a_{ij} = a_{ji} = 0$ . So, Expressions (5.4, 4.8 and 4.9) need to be updated to incorporate this change for each line outage. This process should be repeated until the outages of all lines have been incorporated. The expressions for constraints, in this case, can be expressed as Expressions (4.11)-(4.13). Expression (4.11), is written on the premise that all the buses must be observed at least once by a PMU for full system observability. Therefore, the entries of vector  $b$  are all ones.

However, for a single PMU outage, all the buses must be observed at least twice by PMUs. In order to do this, the right-hand side inequality constraint of Expression (4.11) has been multiplied by 2 to provide the observability of each bus at least twice by the PMUs so that loss of a PMU does not affect the system observability. Hence, Expression (4.11) can be modified to Expression (5.6), [19];

$$f_i^k = \sum_{j=1}^n a_{ij}^k c_j \geq 2 \quad \forall i \in I \quad (5.6)$$

From Expression (5.6), it can be seen that a single PMU outage cannot affect the full system observability. Expression (5.6) indicates that all the buses are observed at least twice by a PMU. Now, the resulting Expressions (4.12, 4.13 and 5.6) should be repeatedly solved until the outage of all lines has been incorporated. Thus the system would be fully observable irrespective of a line outage or a PMU outage.

### 5.2.3. PMU Placement in case of preinstalled PMUs

In this case, it is assumed that the PMUs are already placed at some of the buses. The constraint incorporating this case can be expressed as follows:

$$c_t = 1 \quad \forall t \in PP \quad (5.7)$$

where  $PP$  is the set of buses at which PMUs are already installed. Equation (5.7) must be added to the OPP formulation described by Expressions (5.1) to (5.5) and by Expressions (4.8) and (4.9).

Finally, the system observability can be computed as,

$$Obs. = \sum_{q=1}^p A_{Lp}(q) \quad (5.8)$$

In this chapter, Dijkstra Algorithm (DA) has been used to find out the shortest paths from PDC to all PMUs. Details of this method can be found in [131], [132]. Shortest path provides the minimum length of fiber optics which reduces the cost of communication infrastructure. For this case, the computational complexity of DA is comparatively low as compared to other algorithms [130]. Figure 5.2 (a) represents the situation before applying DA and Figure 5.2 (b) to Figure 5.2 (e) indicate the successive iteration of the algorithm to find out the shortest path from source node to each node. Dark lines show the shortest path from source ( $S_s$ ) to another node where the value in dark circle represents the distance of that node from the source node.

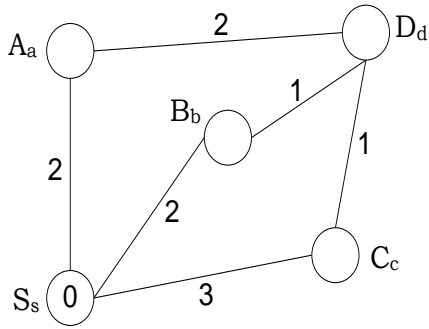


Figure 5.2 (a)

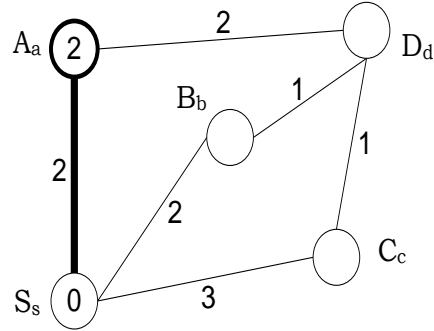


Figure 5.2 (b)

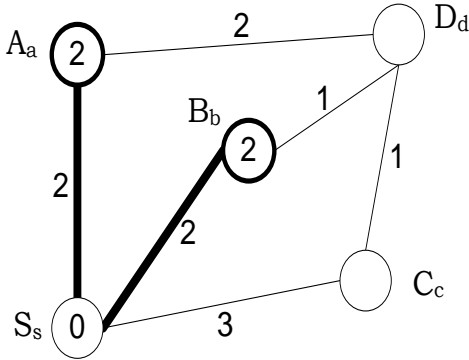


Figure 5.2 (c)

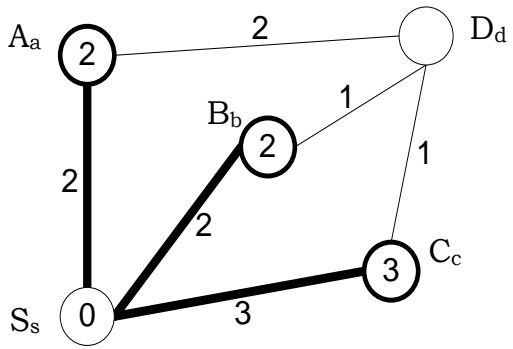


Figure 5.2 (d)

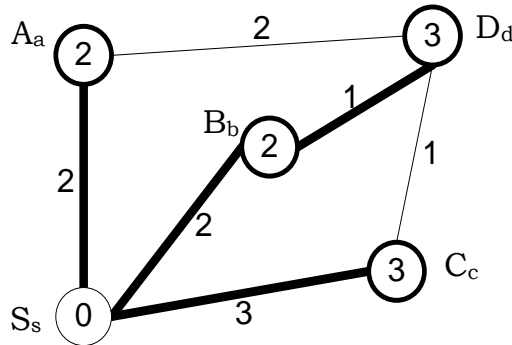


Figure 5.2 (e)

Figure 5.2. Shortest path from source node (S) to all other nodes

(a) Situation before applying DA (b) Shortest path from  $S_s$  to  $A_a$  (c) Shortest path from  $S_s$  to  $B_b$  (d) Shortest path from  $S_s$  to  $C_c$  (e) Shortest path from  $S_s$  to  $D_d$

### 5.3. IMPLEMENTATION OF BGSA FOR OPP PROBLEM

This chapter proposes an algorithm for optimal PMU placement with MO and minimum CI. MO is the additional advantage over [129] and [130]. To minimize the cost of communication links, DA has been used in this chapter. By using the DA, a distance matrix ( $dm$ ) of lesser distance path with minimum switches is calculated.

The detailed procedure to apply the BGSA based on Newton's Law of Gravity and Mass interactions for solving the OPP problem including the CI is as

follow:

- Step 1. Read bus data and line data of the test system.
- Step 2. Determine the connectivity matrix ( $A$ ) and ZIB.
- Step 3. Determine the set of constraints using Expressions (4.11)-(4.13) and Expression (5.6).
- Step 4. A distance matrix ( $dm$ ) of lesser distance path with minimum switches is prepared using Dijkstra Algorithm.
- Step 5. Identify the search space.
- Step 6. Initialize BGSA parameters such that  $T$ ,  $Q$ ,  $G_0$  and  $\beta$ .
- Step 7. Calculate the fitness values of each agent in the population for the OPP problem.
- Step 8. Update  $G(t)$ ,  $best(t)$ ,  $worst(t)$  and  $M_i(t)$  for  $i=1, 2, \dots, Q$  based on fitness value.
- Step 9. Calculate total force in different directions using Equation (3.15).
- Step 10. Modify acceleration of each agent using Equation (3.16).
- Step 11. Update velocity and position of each agent using Equations (3.25) and (3.26) respectively.
- Step 12. Repeat steps 8-12 until the termination criterion is reached.
- Step 13. Save the results.
- Step 14. Stop

#### **5.4. CASE STUDY AND RESULTS**

This investigation has proposed a method for optimal placement of PMUs incorporating the cost of CI. The solution of this problem has been obtained by BGSA. MO and minimum CI is the main objective of this chapter. Following three case studies have been considered in this chapter:

Case 1: Normal case

Case 2: Single PMU outage or single line outage case

Case 3: PMU placement in case of preinstalled PMUs

The performance of proposed method has been tested on IEEE 14-bus, 30-bus and 118-bus test systems. The length of the transmission lines for the considered test systems have been given in Appendix II. Results of proposed

method have also been compared with existing methods to check the effectiveness of proposed method. The cost of a PMU and cost of optic fiber has been taken same as in [130] for a fair comparison of proposed method with [130]. In this chapter, the cost of a PMU ( $c_p$ ) with double channel is assumed 40000\$ and extra channel cost ( $c_{EC}$ ) is 4000\$ per channel. Also, the cost of a switch ( $c_{sw}$ ) and the cost of per kilometer fiber optics ( $c_l$ ) are assumed 4000\$. Therefore, the expression of total cost ( $TC$ ) of installation of PMUs is as follows:

$$TC = (p \times c_p) + (EC \times c_{EC}) + (sw \times c_{sw}) + (l \times c_l) \quad (5.9)$$

#### 5.4.1. Case 1: Normal case

The intact system i.e. no contingency is considered as a normal case. Table 5.1 shows the optimal number of PMUs and their locations for the normal case. Table 5.2 depicts the comparison of results obtained by proposed method for case 1 with previously reported results [19], [25], [32], [38], [130]. The total cost of installation of IEEE 14-bus and 118-bus systems is less with higher observability as compared to other methods. In IEEE 30-bus test system, the total cost of installation of PMU is same as reported in [130] due to the similar locations of PMUs in [130]. It means proposed method provides the MO under economic consideration. To get the MO of the system, the number of EC increases as compared to others which increase the per channel cost. However, the total cost of installation of PMUs is less. In IEEE 14-bus & 30-bus systems, the location of PDC has the choice to install the PDC at the power system bus with the smaller path and lower number of switches.

#### 5.4.2. Case 2: Single line outage or single PMU outage

In this case, locations of PMUs are determined such that any one of the single line outage or single PMU outage does not affect the observability of the

Table 5.1. Optimal no. of PMUs and locations for IEEE test systems (Case 1)

Test System	No. of PMUs	Location of PMUs
IEEE 14-bus	4	4,5,6,9
IEEE 30-bus	7	3,7,10,12,15,20,27
IEEE 118-bus	30	2,8,11,12,17,21,24,27,31,32,34,37,40,43,47,49,52,56,62,71,75,77,80,85,86,90,94,102,105,110

Table 5.2. Test results of proposed method and its comparison in Case 1

<b>Test System</b>	<b>Method</b>	<b>No of PMUs</b>	<b>MO</b>	<b>PDC location</b>	<b>Sw</b>	<b>EC</b>	<b>Total length</b>	<b>Total cost (millions\$)</b>
<b>IEEE 14-bus</b>	<b>proposed</b>	4	21	4/5/6/9	4	9	21	0.296
	<b>Ref. [19]</b>	3	15	5/6	5	6	108.6	0.5984
	<b>Ref. [25]</b>	3	15	5/6	5	6	108.6	0.5984
<b>IEEE 30-bus</b>	<b>Proposed</b>	7	32	3/4/6/7/10	10	11	129.3	0.8812
	<b>Ref. [19]</b>	7	31	5/6/7/10/20	15	10	218.7	1.255
	<b>Ref. [25]</b>	7	36	4/12/15	9	15	157	1.004
	<b>Ref. [32]</b>	7	29	6	17	8	337.2	1.728
	<b>Ref. [38]</b>	7	31	6/7/10/20/21	14	10	223.2	1.2688
	<b>Ref. [130]</b>	7	32	3	10	11	129.3	0.8812
<b>IEEE 118-bus</b>	<b>Proposed</b>	30	154	21	58	64	2386.8	11.2352
	<b>Ref. [19]</b>	28	141	50	61	57	2641	12.156
	<b>Ref. [32]</b>	28	138	50	61	54	2570.5	11.860
	<b>Ref. [38]</b>	29	148	21	58	61	2509.4	11.6736
	<b>Ref. [130]</b>	30	150	21	60	60	2428	11.392

system. Based on this premise BGSA has been applied for such cases. Table 5.3 shows the optimal number of PMUs and their locations in case (2) single line or single PMU outage case. A comparison of these results with existing results is shown in Table 5.4. For all the test systems, the total cost of installation of PMUs in proposed method is least as compared to others. Observability is highest in proposed method with maximum EC and the lowest number of switches as compared to the number of PMUs in others methods. Overall, installation cost of PMUs and CI are less as compared to [19], [25], [28].

#### 5.4.3. Case 3: PMU placement in presence of preinstalled PMUs

In this section, some preinstalled PMUs have been taken with proposed problem. Further locations have been obtained using proposed method considering presence of existing PMUs. The results are tabulated in Table 5.5 in which buses with existing PMUs, final number of PMUs produced by proposed method and their locations are given in this table along with other method. Two sets of preinstalled PMUs, A1 and A2 have been assumed for each system as shown in the second column of Table 5.5. For example, A1-30 and A2-30 are the two sets of preinstalled PMUs in IEEE 30-bus system. The buses 2 & 12 and 2 & 14 are the two locations in each set A1-30 and A2-30 respectively. The same preinstalled PMUs locations have been taken in [130]. Similarly, identical preinstalled PMUs locations have been taken for IEEE 118-bus system also as in [130] so that performance of proposed method can be compared with this method.

Table 5.3. Optimal no. of PMUs and locations for IEEE test systems (Case 2)

<b>Test System</b>	<b>No. of PMUs</b>	<b>Location of PMUs</b>
<b>IEEE 14-bus</b>	7	2,4,5,6,9,10,13
<b>IEEE 30-bus</b>	15	2 ,3 ,4 ,5,6,10,12,13,15,17,18,20,24,27,29
<b>IEEE 118-bus</b>	61	2, 3, 6, 8, 9, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 31, 32, 34, 36, 40, 42, 44, 45, 46, 49, 51, 53, 54, 56, 57, 59, 62, 66, 68, 70, 72, 75, 77, 78, 80, 83, 85, 86, 87, 89, 91, 92, 94, 96, 100, 101, 105, 106, 109-112, 115, 117, 118

Table 5.4. Test results of proposed method and its comparison in Case 2

<b>Test Results</b>	<b>Method</b>	<b>No of PMUs</b>	<b>MO</b>	<b>PDC location</b>	<b>Sw</b>	<b>EC</b>	<b>Total length</b>	<b>Total cost (millions\$)</b>
<b>IEEE 14-bus</b>	<b>proposed</b>	7	33	5/6/11/13	7	12	129.4	0.8736
	<b>Ref. [19]</b>	8	33	1	11	9	177.6	1.1104
	<b>Ref. [25]</b>	8	35	5/6/11	9	11	129.6	0.9184
<b>IEEE 30-bus</b>	<b>Proposed</b>	15	65	5	21	20	361.6	2.210
	<b>Ref. [19]</b>	17	60	24	23	9	397.8	2.399
	<b>Ref. [25]</b>	16	61	1	23	13	424.6	2.482
	<b>Ref. [130]</b>	16	58	1	22	10	388.4	2.322
<b>IEEE 118-bus</b>	<b>proposed</b>	61	279	50	82	96	3706.2	17.977
	<b>Ref. [19]</b>	65	289	21	89	94	3869.3	18.809
	<b>Ref. [25]</b>	62	278	58	86	92	3847.4	18.582
	<b>Ref. [130]</b>	64	282	21	88	90	3799.7	18.471

A comparison of the performance of proposed method with above mentioned method on the basis of number of PMUs, observability and total installation cost have been tabulated in Table 5.6. Results of IEEE 14-bus system are not reported in [130]. It can be substantiated from this table that the numbers of PMUs are same in IEEE 30-bus and 118-bus systems except for set A1-118 of IEEE 118-bus system in which proposed method suggests lower number of PMUs, as for as observability is concerned, it has been found to be the same for IEEE 30-bus system for both the sets. However, the proposed method produced higher observability for set A2 of IEEE 118-bus system. For set A1-118, observability is lower because numbers of PMUs are also lower, suggesting greater benefit. The installation cost is also same for IEEE 30-bus system, but it is lower in case of both the sets of IEEE 118-bus system. This indicated that the benefit can be observed in larger systems. Moreover, the cost of installation in case of preinstalled PMUs is higher for IEEE 14-bus system as compared to a fresh location as shown in Table 5.2. This has occurred due to binding of one predefined location.

Table 5.5. Results in case of preinstalled PMUs at some buses (Case 3)

<b>Test System</b>	<b>Assump-tion</b>	<b>Buses with available PMUs</b>	<b>No. of PMUs</b>	<b>Location of PMUs</b>
<b>IEEE 14-bus</b>	<b>A1-14</b>	13	4	4,5,11,13
	<b>A2-14</b>	1	4	1,4,6,9
<b>IEEE 30-bus</b>	<b>A1-30</b>	2,12	7	2,4,10,12,15,20,27
	<b>A2-30</b>	2,24	7	2,4,10,12,19,24,27
<b>IEEE 118-bus</b>	<b>A1-118</b>	2,15,37,49,60, 100	31	2,5,8,12,15,17,21,27,31,32,34, ,37,40,45,49,52,56,60,66,72, 75,77,80,85,86,89,92,96, 100,105,110
	<b>A2-118</b>	15,32,51,60,89, 100	31	3, 5, 8, 12, 15, 17, 21, 27, 31, 32, 34, 40, 45, 49, 51, 54, 56, 60, 66, 72, 75, 77, 80, 85, 86, 89, 92, 96, 100, 105, 110

Table 5.6. Comparison of results in case of preinstalled PMUs placed at some buses (Case 3)

<b>Test System</b>	<b>Method</b>	<b>Assumption</b>	<b>No. of PMUs</b>	<b>MO</b>	<b>PDC location</b>	<b>Sw</b>	<b>EC</b>	<b>Total length</b>	<b>Total cost (millions\$)</b>
<b>IEEE 14-bus</b>	<b>proposed</b>	A1-14	4	18	4/5/6	5	6	41.7	0.371
	<b>proposed</b>	A2-14	4	19	1/4/5/6/9	5	7	115.4	0.6696
<b>IEEE 30-bus</b>	<b>proposed</b>	A1-30	7	36	4/12	9	15	155.9	0.9996
	<b>Ref. [130]</b>	A1-30	7	36	4/12	9	15	155.9	0.9996
	<b>proposed</b>	A2-30	7	35	6/10/20/21	12	14	172.7	1.0748
	<b>Ref. [130]</b>	A2-30	7	35	6/10/20/21	12	14	172.7	1.0748
<b>IEEE 118-bus</b>	<b>proposed</b>	A1-118	31	172	21	58	79	2405.4	11.4096
	<b>Ref. [130]</b>	A1-118	33	175	21	61	76	2426.2	11.573
	<b>proposed</b>	A2-118	31	173	21	57	80	2422	11.476
	<b>Ref. [130]</b>	A2-118	31	168	21	58	75	2438.6	11.526

NOS: No Optimal Solution

#### 5.4.4. Convergence characteristics and time

Installation cost versus iteration has been plotted in Figures 5.3 (a), (b) and (c) in order to observe the convergence behavior of the proposed method. Three different plots for different cases studied in this investigation have been obtained. Convergence behavior of all the three test systems for a particular case has been plotted on a single graph for fair comparison of their convergence behavior. It can be observed that the convergence behavior of all the three test systems is almost similar for all the three cases. The IEEE -14, -30, and 118-bus systems converged within 41 to 47, 48 to 53 and 66 to 79 iterations respectively. Thus all the three test systems converged well within 150 iterations, the chosen maximum number iterations. The similar convergence behavior of the tested systems also authenticates the proper choice of BGSA parameters. In Figure 5.4, the computational time versus various cases of the proposed work has been plotted. It is clear from the Figure 5.4 that the computational time increases with increase in size of the system and varies according to the various cases.

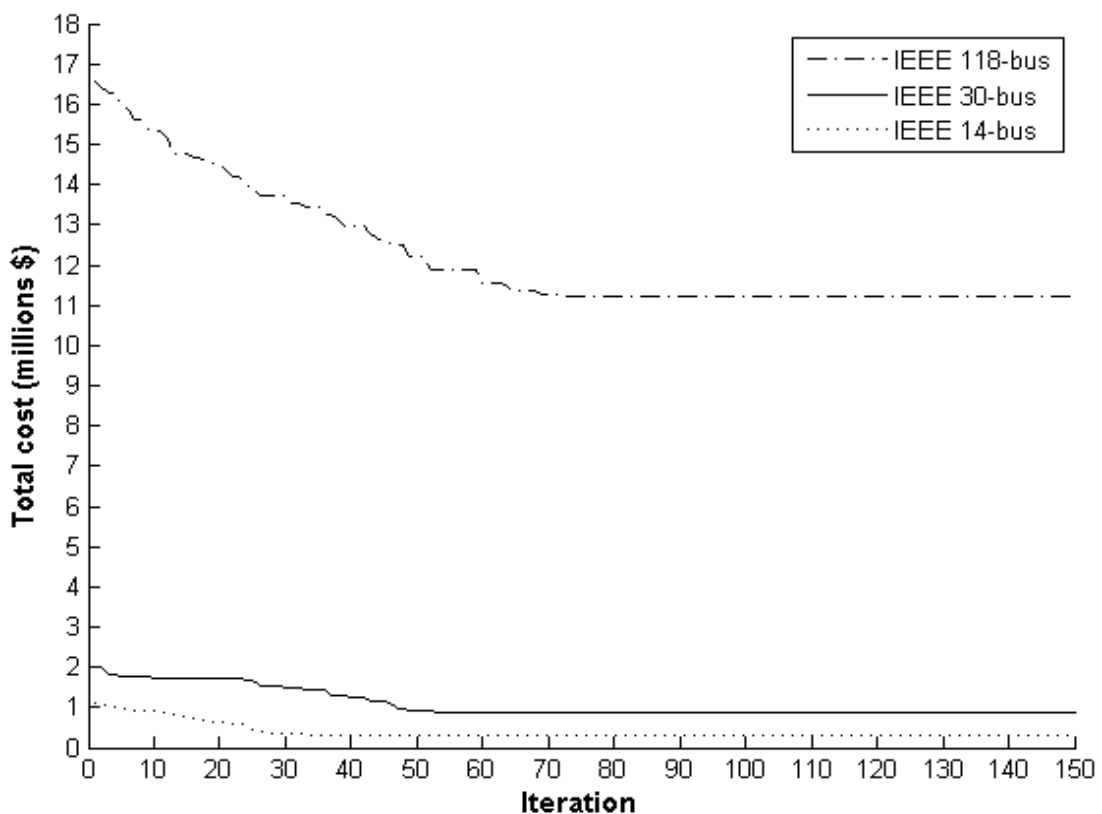


Figure 5.3 (a)

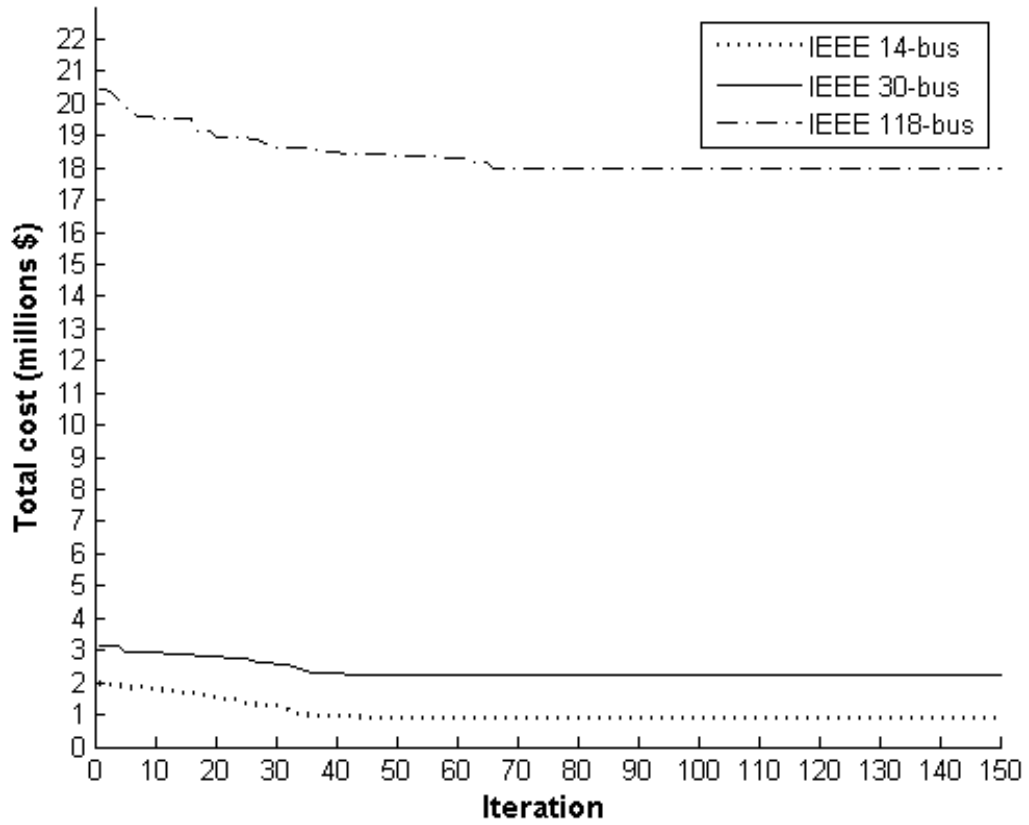


Figure 5.3 (b)

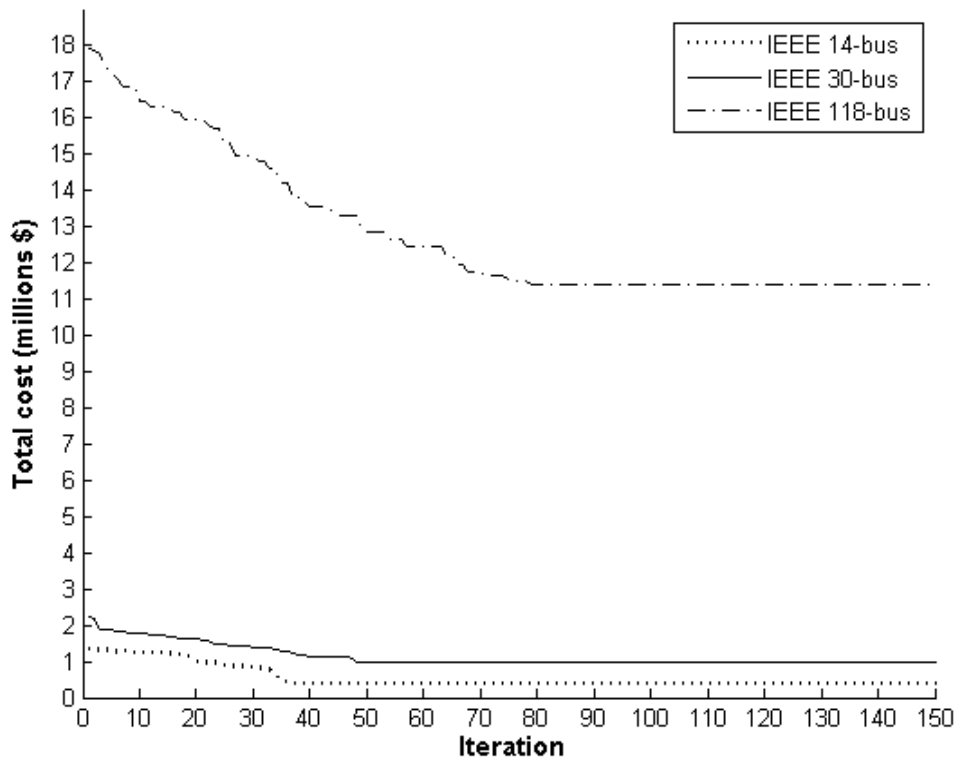


Figure 5.3 (c)

Figure 5.3. Convergence of BGSA for all the test systems considering (a) Case 1 (b) Case 2 and (c) Case 3

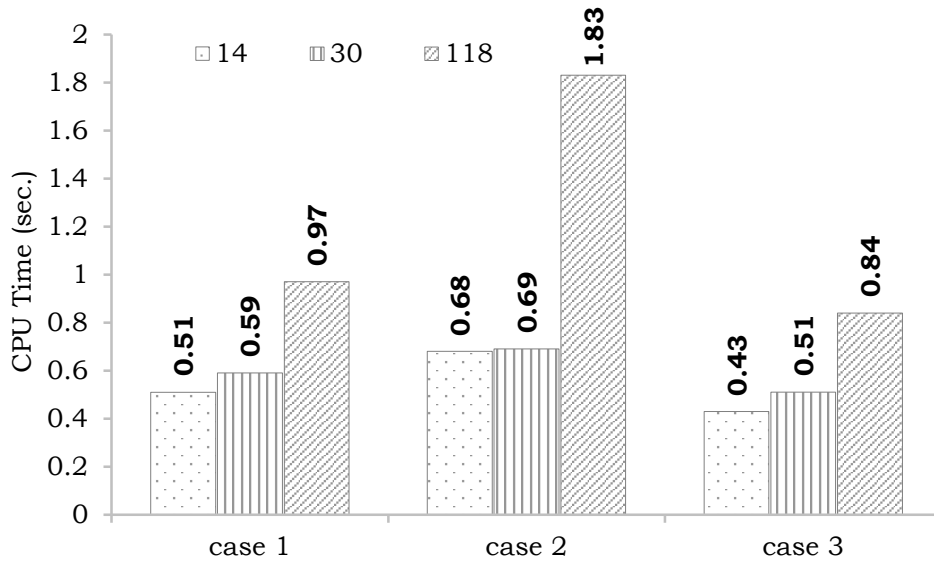


Figure 5.4. Computational time analysis for various cases

## 5.5. CONCLUSION

In this chapter, BGSA has been suggested to insure maximum observability of power system while minimizing the number of PMUs and cost of communication infrastructure. The problem of allocation of minimum number of PMUs while achieving the maximum observability has been expressed by single objective function unlike earlier method which proposed two separate objective functions. The second objective of proposed method is to find out the optimal location of PDC, where from all the PMUs are at a shortest distance. To find out the shortest paths from PDC to all the PMUs, a well-known Dijkstra Algorithm has been used in this chapter. The proposed method has been tested on the IEEE 14-bus, 30-bus and 118-bus test systems for various cases in presence of ZIB, preinstalled PMUs and line outage. The results have been compared with previous reported results. The simulation results indicate that the proposed method produced either lower or equal number of PMUs, higher or equal observability and lower or equal cost.