PREFACE

Interpolation, a long-standing technique in applied mathematics, has traditionally relied on polynomials and other smooth functions. However, a relatively recent approach of Fractal Interpolation Functions (FIFs) introduces a novel method of interpolation capable of generating both smooth and non-smooth interpolants. Fractal interpolation has garnered significant attention within the fractal community over the past three decades and emerged as a prominent research area. It is widely recognized that theories of interpolation and approximation are closely intertvined. Nonetheless, in the realm of fractal interpolation, the relationship between these two theories takes on a subtler nature. The concept of α-fractal function plays a crucial role in exploring the intriguing connections between interpolation and approximation theories within the setting of fractal functions. This thesis aims to provide valuable insights into the approximation theory of fractal functions on Sierpinski Gasket (SG) and rectangular domain. The α-fractal function is employed as a powerful and effective tool for analysing these fractal functions. Special attention is given to the significance of fractal dimensions as fundamental indices in approximation theory. Moreover, a rigorous examination of the box dimension and Hausdorff dimension is conducted for the graph of these α- fractal functions on SG and rectangular domain. Through meticulous efforts, this research enhances our understanding of fractal approximation within these specific domains, contributing noteworthy findings to the field of approximation theory.

Chapter 1 serves as the introductory section of the thesis, providing an overview of the fundamental theory and relevant results that form the basis of our study in the subsequent chapters. This chapter also incorporates a concise review of the existing literature pertaining to the topics of interest.

Chapter 2 discusses the fractal interpolation function and its box dimension corresponding to a continuous function defined on SG. Furthermore, we investigate the so-called fractal operator, which is associated with the α-fractal function. We also illustrate certain important fractal operator aspects such as topological automorphism, Fredholm, and many more within certain restrictions. This chapter also explores some properties of fractal polynomials constructed through the perturbation of fractal polynomials defined by Strichartz on SG. We also provide some results on constrained approximation by fractal polynomials and study the best approximation properties of fractal polynomials defined on SG. Further, we discuss some interesting properties of the class of polynomials defined on SG. At the end, we try to estimate the fractal dimensions of the graph of α- fractal function using the method of oscillation of functions.

Chapter 3 is concerned with the approximation of functions by fractal functions with respect to $\mathcal{L}^p$-norm on SG. We define the α-fractal function in $\mathcal{L}^p$- space. The properties such as topological isomorphism and many others, which are closely associated with the fractal operator will be discussed in more detail. We also prove the existence of a non-trivial closed invariant subspace for the fractal operator. Additionally, we define set-valued mapping and discuss some useful properties.

Chapter 4 provides the construction of bivariate dimension preserving approximants, using the concept of fractal interpolation functions. Furthermore, some multi-valued fractal operators associated with bivariate α-fractal functions are defined and studied.

In Chapter 5, we study the continuous dependence of the bivariate α-fractal function on the parameters such as the scaling function α, net $\triangle$ of rectangular grid, and the base function s involved in its construction. Further, we establish some results regarding its dimension.