Available online at www.sciencedirect.com
ScienceDirect
IFAC Papers Online CONFERENCE PAPER ARCHIVE
IFAC-PapersOnLine 49-1 (2016) 236–240

Fractional Order PI Controller Design for Non-Monotonic Phase Systems

Santosh Kumar Verma*, Shekhar Yadav, Shyam Krishna Nagar*****

Department of Electrical Engineering, Indian Institute of Technology (BHU), Varanasi-221005, U.P., India
e-mail: santosh.rs.eee13@iitbhu.ac.in;*
*syadav.rs.eee@iitbhu.ac.in**; sknagar.eee@iitbhu.ac.in****

Abstract: In this paper, a fractional order proportional-integral (FOPI) controller is proposed for controlling the non-monotonic decreasing phase DC-buck regulator system. The parameters of FOPI controller are optimized by Nelder's-Mead (NM) method. The FOPI controller provides fast closed-loop performance as well as improves the robust properties of the system in time and frequency domain. The controller preserves the monotonic phase between the desired bandwidth and improves the characteristic performance of the system. The proposed method is validated by comparing the time domain as well as frequency domain characteristics with the integer order proportional-integral (IOPI) controllers tuned using various techniques.

Keyword: Fractional Order PI controller, non-monotonic phase system, DC-buck regulator, Nelder's-Mead optimizer.

1. INTRODUCTION

Feedback control system maintains a prescribed relationship between the reference input and the desired output. The difference between the input-output relationships of the system is noted as error of the system and minimized using controller. In frequency response compensation of a continuous (or discrete) linear time invariant (LTI) system is done by applying a negative feedback control system. In classical control design the phase margin of a closed-loop system having monotonically decreasing phase inside the bandwidth is the distance between the open-loop phase at the gain crossover frequency and the stability limit of -180°. However, the system having a left half-plane zero located near the dominant-poles (i.e minimum-phase system) shows non-monotonic phase behaviour inside the bandwidth (Caio F.de Paula et al. 2012). For a closed loop system to be stable the phase margin must be positive. In addition, the phase margin of a closed-loop system estimate the robustness and informs how much the open-loop system phase may vary while the closed-loop system remains stable (S. Skogestad and I. Postlethwaite 2005).

DC-buck regulator taken in this paper also shows the non-monotonic phase behaviour inside the bandwidth. Hence a controller is needed for non-monotonic system to get the wider bandwidth, faster time response and more sensitive to noise and parameter variations.

Since few decades, the fractional order controllers are being the part of the control application due to having extra degree of freedom. The integral and the derivative term of the fractional order controller are in fractions ($PI^{\lambda}D^{\mu}$) (I. Pan and S. Das 2012; Shantanu Das 2008; G. Q. Zeng et al. 2015; H. Shayeghi et al. 2015; I. Podlubny 1999a, b). These fractional terms increases the complexity of the controller but more powerful than the conventional integer order controllers and supple design methods to satisfy the controlled system specifications. In I. Podlubny (1999) two additional tuning knobs given which are able to make balance between settling time and maximum overshoot of the system.

In the last few years, fractional order control strategies have been successfully applied to many systems like controlling of smart wheel via internet with variable delay (Inés Tejado et al. 2015), Automatic voltage regulator system (H. Remezanian et al. 2013) and many other (D.Y. Xue et al. 2006; S. Das 2012; G. Q. Zeng et al. 2015; Guo-Qiang Zeng et al. 2015). The FOPI controller is designed for DC-buck regulator system in this work. The optimization of the FOPI parameters can be done by any optimization algorithm like Genetic Algorithm, Particle Swarm Optimization etc. In this paper Nelder-Mead algorithm has been used for this purpose (Nelder, John A. and R. Mead (1965). Before applying in the closed-loop system the fractional order terms of the FOPI controller are approximated into integer order using Oustaloup's approximation algorithm within a frequency range of $\omega \in (10^{+3} - 10^{+8})$ rad/sec (A. Oustaloup, 1991; A. Oustaloup, 1981).

The detailed organization of this paper is as follows: in Section 2 the circuit diagram of DC-buck regulator and its non- monotonically decreasing phase behaviour is addressed. Section 3 explains detailed study of FOPID controller design. Section 4 shows the results and comparative study of FOPI and other conventional IOPI techniques. Finally, the conclusion followed by the references.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.03.059

2. DC-BUCK REGULATOR SYSTEM

The DC-buck regulator is the most commonly used dc-dc converter topology and have a very large application area like in power management and microprocessor voltage regulator applications (Caio F.de Paula, and Luís H. C. Ferreira, 2012). This is very popular because of its smaller size and efficiency compared to the linear regulators. In this paper the design of controller is made on the simplest dc-dc converter circuit i.e. the DC-buck regulator circuit.

The DC-buck regulator system is a combination of the power stage (i.e a LC low-pass filter) and a pulse-width modulation (PWM)-based controller (International Rectifier, 2002). The circuit diagram of DC-buck converter with voltage controller is given in Fig. 1.

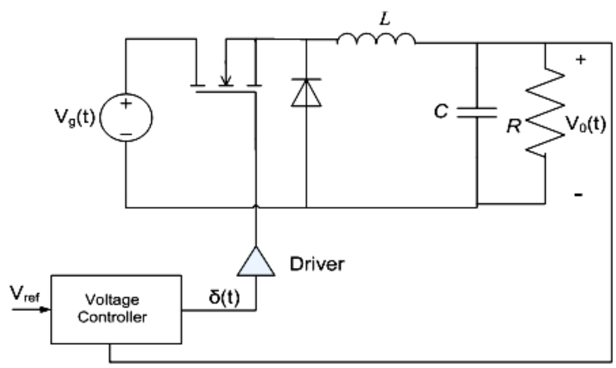

Fig. 1. The DC-Buck Converter with a voltage controller

The transfer function of DC-buck regulator system can be written as ratio of the output (regulated voltage) to the input (PWM modulator input voltage) as:

$$G = \frac{V_{in}(1+sR_C C)}{V_{OSC}LCs^2+s\left(R_C C+\frac{L}{R}\right)+1} \quad (1)$$

where C is output capacitance, L is output Inductance, R is load resistance, R_C is the output capacitor intrinsic resistance, V_{in} the power stage input voltage and V_{OSC} is the PWM oscillator reference voltage.

For a typical application of DC-buck regulator taken in International Rectifier, (2002) with $R_C = 40\ m\Omega$, the transfer function is given as:

$$G = \frac{4(1+1.2\times10^{-5}}{3\times10^{-9}s^2+3.6\times10^{-5}s+1} \quad (2)$$

The Bode plot of the DC-buck regulator is shown in Fig. 2, where the non-monotonic phase behaviour close to the undammed frequency can be seen in the phase plot. The closed-loop step response without any controller is also shown in Fig. 3, which shows that the system is under-damped. Hence a controller is needed to compensate the non-monotonic phase actions of the system.

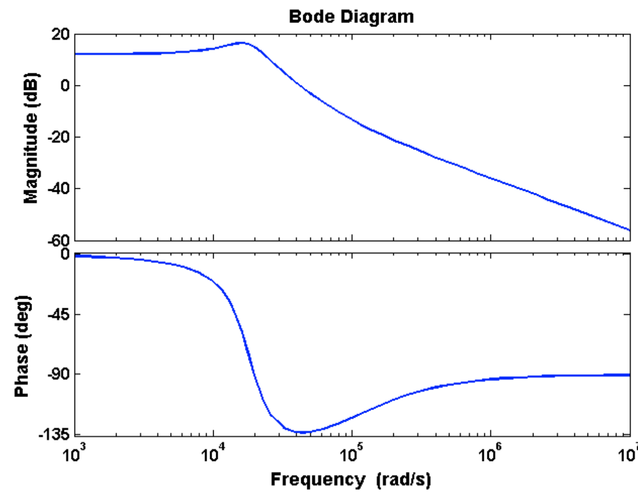

Fig. 2. Bode plot of DC-buck regulator system

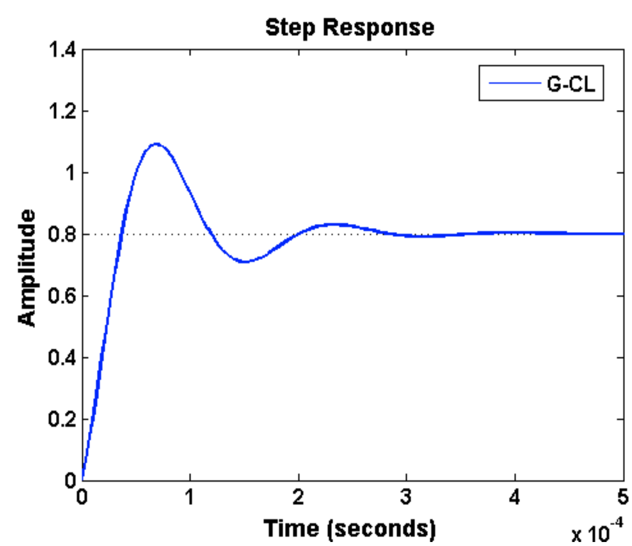

Fig.3. Step response of closed-loop DC-buck regulator

3. FRACTIONAL ORDER PID CONTROLLER

FOPID is presented in generalized form of IOPID controller and represented as $PI^{\lambda}D^{\mu}$ (I. Podlubny, 1999). The FOPID controller is very powerful and provides flexible control design approach. The transfer function for the FOPID controller is given as:

$$C_{FOPID}(s) = K_P + \frac{K_I}{s^{\lambda}} + Ds^{\mu} \quad (3)$$

where for λ and μ are the fractional power of integral and differential control respectively.

For FOPI controller the transfer function will be:

$$C_{FOPI}(s) = K_P + \frac{K_I}{s^{\lambda}} \quad (4)$$

Theses FOPI controller variables (K_P, K_I, and λ) are optimized by using Nelder-Mead algorithm (Nelder, John A. and R. Mead, 1965).

The fractional terms of this optimized FOPI controller is needed to approximate into integer order before applying in the closed-loop system. Oustaloup's approximation algorithm within a frequency range of $\omega \in (10^{+3} - 10^{+8})$ rad/sec is used in this paper.

There are mainly two methods in the literature for integer order approximation of fractional order terms in continuous domain as:

- Approximations using continued fraction expansions and interpolation techniques.

The techniques are based on this approximation are:

a) General CFE method for approximation of fractional integro-differential operators.

b) Carlson's method.

c) Matsuda's method.

- Approximations using curve fitting or identification techniques.

The techniques are based on this approximation are given as:

a) Oustaloup Recursive Approximations.

b) Chare's method.

c) Modified Oustaloup Filter.

3.1 Oustaloup's Approximation Algorithm

The Oustaloup's approximation algorithm is one of the most popular methods used for integer order approximation of fractional order systems within a specified frequency band (A. Oustaloup, 1991; A. Oustaloup, 1981). Suppose that the frequency range to be fit by an integer order filters to fractional order derivative are given by $[\omega_b, \omega_h]$, the term s/ω_u can be substituted with.

$$C_0 \frac{1+s/\omega_b}{1+s/\omega_h} \quad (5)$$

where $\sqrt{\omega_b \omega_h} = \omega_\mu$ and $C_0 = \frac{\omega_b}{\omega_u} = \frac{\omega_u}{\omega_h}$

The Oustaloup's approximation of a fractional order differentiator $s^\propto$ can be written as

$$G(s) = (C_0)^\alpha \prod_{k=-N}^{N} \frac{1+s/\omega'_k}{1+s/\omega_k} \quad (6)$$

where, $\omega'_k = \omega_b \left(\frac{\omega_b}{\omega_u}\right)^{\frac{k+N+\frac{1}{2}+\frac{\propto}{2}}{2N+1}}$ and $\omega_k = \omega_b \left(\frac{\omega_b}{\omega_u}\right)^{\frac{k+N+\frac{1}{2}-\frac{\propto}{2}}{2N+1}}$ are respectively the zeros and poles of rank k. The total number of zeros or poles are given as $(2N + 1)$.

3.2 Nelder-Mead Optimization Algorithm

Nelder and Mead presented a very simple method in 1965 to find a local minimum of a function of many variables (Nelder, John A. and R. Mead, 1965). The method is a pattern search which compares function values at the three vertices of a triangle for two variables. The worst vertex of the triangle, where $f(x, y)$ is largest, is rejected and replaced with a new vertex. Thus a new triangle is formed and the search is continued. The process generates a chain of triangles (it may have different shapes depending on number of variables), for which the function values at the vertices get lesser and lesser. The size of the triangles is reduced and the coordinates of the smallest point are obtained.

4. RESULT AND DISCUSSION

As shown in Fig. 3 the closed-loop response of the DC-buck regulator system is under-damped and shows the non-monotonic phase inside the bandwidth. Therefore, a controller is required to control the output voltage of the system. A FOPI controller is designed to improve the performance of the system. The parameters of the FOPI controller are optimized using NM-method and before implementing it into the system the fractional order terms are approximated into integer order by using Oustaloup's approximation algorithm. The performance criterion considered for designing the controller is ISE. The objective function is given by following equation:

$$J = \int_0^\infty e^2(t)\, dt \quad (7)$$

where e is the error of the system and t is the time period.

The parameters of the FOPI controller obtained using Nelder-Mead optimization are $K_P = 175, K_I = 97$ and $\lambda = 0.81$. The step response of the FOPI controller is compared with the closed-loop response of the system in Figure 4.

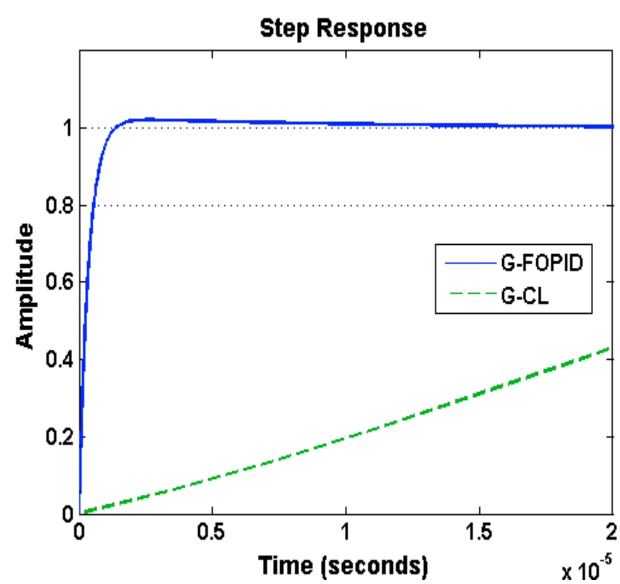

Fig. 4. Comparison of step response of FOPI controller and closed-loop system

The step response of the DC-buck regulator system with FOPI controller is also compared with PI controller designed by using various conventional methods like ZN (J. G. Ziegler and N. B. Nichols; 1943), Approximated M-constrained integral gain optimization (MIGO) (K.J. Astrom and T. Hagglund; 2004), Chen-Hrones-Reswick (Kun Li Chien et al. 1952) and Skogested Internal model control (IMC) (Rivera D.E. et al. 1986) in Figure 5. The

FOPI controller provides better response in terms of performance characteristics like Rise-time, Settling-time and Peak-overshoot as compared to the other controllers.

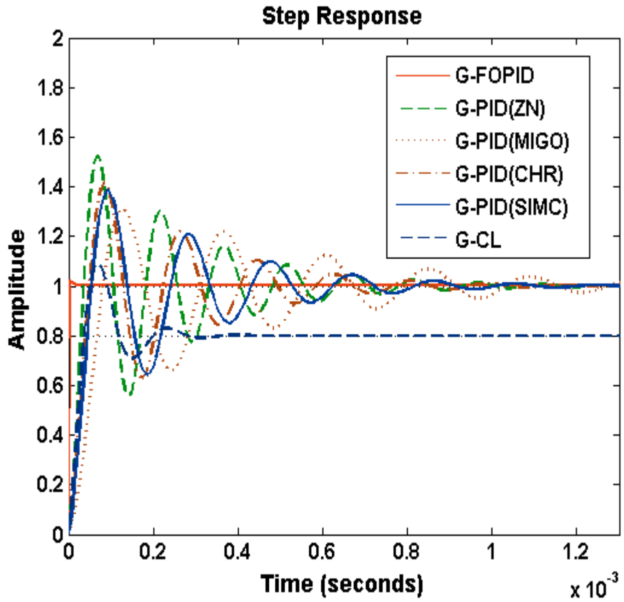

Fig. 5. Comparison of step response of FOPI controller and other conventional techniques

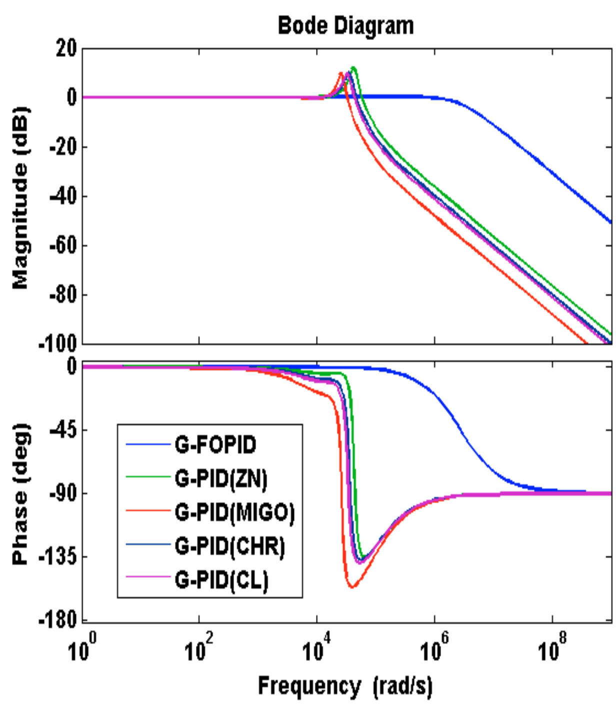

Fig. 6. Comparison of Bode plots of FOPI controller and other conventional techniques

Comparison of bode plot of FOPI with other conventional methods are shown in Figure 6. It is very clear that using FOPI controller we get monotonic phase of the system whether in case of other controller the non-monotonic behaviour of the system remains same.

The performance characteristics like Rise-time, Settling-time and Peak-overshoot of all the methods are compared in Table 1, which reports that the FOPI controller provides the fastest and better control as compared to the all other methods.

Table 1: Comparison of Performance characteristics

Control Techniques	Rise time (sec)	Settling time (sec)	Peak Overshoot (%)
NM-FOPI	7.2848×10^{-7}	4.5028×10^{-6}	2.2699
ZN-IOPI	2.7524×10^{-5}	8.2769×10^{-4}	52.0312
Approx-MIGO-IOPI	5.8402×10^{-5}	12×10^{-4}	30.8267
Chen-Hrones-Reswick-IOPI	3.6435×10^{-5}	8.1506×10^{-4}	41.4965
Skogested IMC-IOPI	4.0378×10^{-5}	8.6556×10^{-4}	38.4285
Closed-loop	2.8388×10^{-5}	2.6031×10^{-4}	36.2644
Open-loop	7.1775×10^{-5}	5.9789×10^{-4}	34.3293

5. CONLUSION

A FOPI controller is designed to control the non-monotonic decreasing phase actions and output voltage of the DC-buck regulator system. The parameters of FOPI controller are optimized using Nelder-Mead method and the approximation is done by the use of Oustaloup's approximation algorithm. The performance characteristics of FOPI are compared with various conventional techniques. The proposed technique offers enviable improvements which are not achievable when the classical design methods are used. Hence the proposed FOPI controller enhanced the system performance as compare to other conventional control techniques.

REFERENCES

[1] Caio F.de Paula, and Luís H.C.Ferreira (2012). An Improved Analytical PID Controller Design for Non-Monotonic Phase LTI Systems. *IEEE transaction on control system technology,* vol.20 No.5, 1328-1333.

[2] S. Skogestad and I. Postlethwaite (2005). Multivariable Feedback Control: Analysis and Design. New York: Wiley.

[3] I. Pan and S. Das (2012). Chaotic multi-objective optimization based design of fractional order $PI^{\lambda}D^{\mu}$ controller in AVR system. *Int. J. Electr. Power Energy Syst.,* vol. 43, 393–407.

[4] Shantanu Das (2008). Functional Fractional Calculus for System Identification and Controls. ISBN 978-3-540-72702-6 Springer Berlin Heidelberg New York.

[5] G. Q. Zeng, Jie Chen, Yu-Xing Dai, Li-Min Li, Chon-Wei Zheng, Min-Rong Chen (2015). Design of fractional order PID controller for automatic regulator voltage system based on multi-objective extremal optimization. *Neurocomputing,* vol. 160, 173–184.

[6] H. Shayeghi, A. Younesi, and Y. Hashemi (2015). Optimal design of a robust discrete parallel FP+FI+FD controller for the Automatic Regulator system. *Electrical Power and Energy Systems,* vol. 67, 66-75.

[7] I. Podlubny (1999). Fractional-order systems and $PI^{\lambda}D^{\mu}$ controllers. *IEEE Trans. Autom. Control,* vol. 44, 208–213.

[8] I. Podlubny (1999). Fractional differential equations: An introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications. *Academic press San Diego*.

[9] Inés Tejado, S. Hassan HosseinNia, Blas M. Vinagre, and YangQuan Chen (2013). Efficient control of a Smart Wheel via Internet with compensation of variable delays. *Mechatronics,* vol. 23, 821–827.

[10] H. Remezanian, S. Balochian, A. Zare, "Design of optimal fractional-order PID controllers using particle swarm optimization algorithm for automatic voltage regulator (AVR) system," *J. Control Autom. Electr. Syst.* 24 (5) (2013) 601–611.

[11] D.Y. Xue, C.N. Zhao, and Y.Q. Chen (2006). Fractional order PID control of a DC-motor with elastic shaft: a case study. *Proc. of the 2006 American Control Conference,* MN, USA.

[12] Guo-Qiang Zeng, Jie Chen, Yu-Xing Dai, Li-Min Li, Chon-Wei Zheng and Min-Rong Chen(2015). Design of fractional order PID controller for automatic regulator voltage system based on multi-objective extremal optimization. *Neurocomputing,* vol. 160, 173–184.

[13] Nelder, John A. and R. Mead (1965). A simplex method for function minimization. Computer Journal, vol. 7, 308–313.

[14] A. Oustaloup. La Commade CRONE (1991). Commande Robuste d'Order Non Entier. Hermes, Paris.

[15] A. Oustaloup (1981). Linear feedback control systems of fractional order between 1 and 2. Proc. of the IEEE Symposium on Circuit and Systems, Chicago, USA, 4.

[16] International Rectifier (2002). Synchronous PWM controller for termination power supply applications. 2nd Ed., 2002.

[17] J. G. Ziegler and N. B. Nichols (1943). Process lags in automatic control circuits. *Trans. Amer. Soc. Mechan. Eng.*, vol. 65, pp. 433–444.

[18] K.J. Astrom and T. Hagglund (2004). Revisiting the Ziegler-Nichols step response method for PID control. *Journal of Process Control,* vol. 14, 635–650.

[19] Kun Li Chien, J. A. Hrones, and J. B. Reswick (1952). On the Automatic Control of Generalized Passive Systems. *Transactions of the American Society of Mechanical Engineers.*, Bd. 74, Cambridge (Mass.), USA, Feb., S. 175-185.

[20] Rivera D.E., M. Morai and S. Skogestad, (1986) "Internal Model Control. 4. PID Controller Design", *Ind. Eng. Chem. Process Des*. Dev.25, 252.