3 Mathematical formulations

3.1 General

From the study of finite element analysis of solids and structures, the structure must be discretized into sub-domain and the sub-domain is analyzed to get the response of the continuum. The surface topology defines the element connectivity of the structure and in the context of the present analysis method, it is assumed that the structure is discretized into finite elements and the element connectivity is already defined. The surface geometry is defined by the nodal coordinates in finite element model and the shape function of the element. In this analysis, initially a trial surface geometry of the structure is assumed, which is not in equilibrium condition. The geometry boundary conditions are specified for the analysis and constraints are also introduced based on the symmetry and anti-symmetry requirements. Membrane structures exhibit high geometric nonlinearity, and hence proper element formulation is necessary to mimic the nonlinear behavior.

3.2 Finite element formulations for TMS

The most widely accepted finite element is a three node constant strain triangular (CST) element. The basis of this element is geometrically nonlinear cable resemblance procedure with an assumption of presence of small strains within the element. The very first step for finite element formulation for overall analysis of TMS is the discretization of the membrane surface into finite element of interest in order to define the surface topology of a TMS. The assortment of finite element for a to-be designed TMS should be based on the capability of finite element to accommodate the modeling of surface stresses. To facilitate the applicability of FE in TMS design the orientation of the CST elements can be embraced with respect to the orthogonally weaved yarns i.e. warp and fill directions and the principle stresses are assumed to be acting in the alignment of these woven yarn direction. The actual modeling of uniform surface stresses by CST element pre-stress depends a lot on the elements deviating from their perpendicular direction, such a deviation of finite elements will result to a false representation of element pre-stress values representing the uniform surface stresses. Due to inherited curved boundaries of membrane structures the orthogonal discretization is not possible near TMS edges. Evident by the name CST has constant strain inborn along all three sides, this makes the model analogous to geometrically nonlinear truss structural system. A linear strain displacement relationship is maintained throughout the finite element formulation procedure.

A plane CST element has two degrees of freedom in translational local axis (u, v)at every node. Hence this element type is bound to work well with 2D modelling problems and it fail to model 3D membrane surfaces. The introduction of third degree of freedom will impart another level of complexity in FE formulation. In consideration of the idea that the strain remains constant throughout the finite element, in order to adapt the change in degrees of freedom the change in the length of sides of CST element can be considered as another degree of freedom.

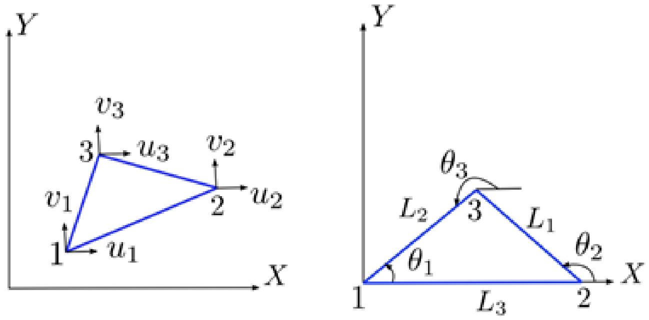

Figure 2 A typical CST element in local coordinate system

The representation of a typical CST finite element in a local coordinate system is displayed in the Figure 2 representing its properties with respect to same local coordinate system. The sides of the elements are denoted as $(i = 1,2,3)$ and the angles are denoted by $(\theta_i = 1,2,3)$ are measured from positive X-axis. The side $i = 1$ is considered to have parallel orientation with local X-axis. The Z-axis of local coordinate is therefore oriented orthogonally to XY-plane on local axis. If the strain developed in the sides of finite element is represented by $\epsilon_i, (i = 1,2,3)$ and the respective orthogonal strain denoted as $(\epsilon_x, \epsilon_y, \gamma_{xy})$ in that case the small strain model [Gosling, 1992], is recalled and the element side strain can be denoted as:-

$$\epsilon_x = \epsilon_x \cos^2\theta_i + \epsilon_x \sin^2\theta_i + \gamma_{xy} \cos\theta_i \sin\theta_i \qquad 3.1$$

The elongation of side length (δ) of CST element can be written in terms of unstrained length L_i $(i = 1,2,3)$, it is given by the expression :-

$$\{\delta\} = \{\delta_1 \quad \delta_2 \quad \delta_3\}^T == \{\epsilon_1 L_1 \quad \epsilon_2 L_2 \quad \epsilon_3 L_3\}^T \qquad 3.2$$

Elongation in each side of element will be denoted as:-

$$\epsilon_1 = \epsilon_x cos^2\theta_1 + \epsilon_x sin^2\theta_1 + \gamma_{xy} cos\theta_1 sin\theta_1 = \delta_1 L_1 \qquad 3.3\text{ (a)}$$

$$\epsilon_2 = \epsilon_x cos^2\theta_2 + \epsilon_x sin^2\theta_2 + \gamma_{xy} cos\theta_2 sin\theta_2 = \delta_2 L_2 \qquad 3.3\text{(b)}$$

$$\epsilon_3 = \epsilon_x cos^2\theta_3 + \epsilon_x sin^2\theta_3 + \gamma_{xy} cos\theta_3 sin\theta_3 = \delta_3 L_3 \qquad 3.3\text{(c)}$$

The matrix representation for same will be shown as :-

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{Bmatrix} \qquad 3.3$$

In the above expression $a_i = \cos^2\theta_i$, $b_i = \sin^2\theta_i$ and $c_i = \cos\theta_i \sin\theta_i$ for $i = 1,2,3$. The strain developed in the local coordinate system is represented by:-

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{\det[A]} \begin{bmatrix} (b_2c_3 - b_3c_2)L_1^{-1} & (b_3c_1 - b_1c_3)L_1^{-1} & (b_1c_2 - b_2c_1)L_1^{-1} \\ (a_3c_2 - a_2c_3)L_2^{-1} & (a_1c_3 - a_3c_1)L_2^{-1} & (a_2c_1 - a_1c_2)L_2^{-1} \\ (a_2b_3 - a_3b_2)L_3^{-1} & (a_3b_1 - a_1b_3)L_3^{-1} & (a_1b_2 - a_2b_1)L_3^{-1} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} \qquad 3.4$$

$$\det[A] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad 3.5$$

From the above relations, the strain-displacement equation is given as:

$$\{\varepsilon\} = [B]\{\delta\} \qquad 3.6$$

In the above expression $[B]$ represents strain-displacement relationship matrix that bridges the increase in length of finite element side $\{\delta\}$ with the orthogonal strain $\{\varepsilon\}$ and is represented as :-

$$[B]=\frac{1}{\det[A]}\begin{bmatrix}(b_2c_3-b_3c_2)L_1^{-1} & (b_3c_1-b_1c_3)L_1^{-1} & (b_1c_2-b_2c_1)L_1^{-1}\\(a_3c_2-a_2c_3)L_2^{-1} & (a_1c_3-a_3c_1)L_2^{-1} & (a_2c_1-a_1c_2)L_2^{-1}\\(a_2b_3-a_3b_2)L_3^{-1} & (a_3b_1-a_1b_3)L_3^{-1} & (a_1b_2-a_2b_1)L_3^{-1}\end{bmatrix} \quad 3.7$$

In local coordinate axis, the stress-strain relationship of material for plane stress problem is given as:-

$$\sigma=\begin{Bmatrix}\sigma_x\\\sigma_y\\\gamma_{xy}\end{Bmatrix}=\begin{bmatrix}d_{11} & d_{12} & 0\\d_{21} & d_{22} & 0\\0 & 0 & d_{33}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\\varepsilon_y\\\gamma_{xy}\end{Bmatrix}=\{\varepsilon\}[B]\{\delta\} \quad 3.8$$

In case of isotropic material, the component of elasticity matrix are $d_{11}=d_{22}=E/(1-\nu^2)$; $d_{12}=d_{21}=E/2(1+\nu)$. Where E and ν are the Young's modulus and poison's ratio of the material respectively.

The element stiffness matric cab be acquired now and represented as :-

$$[K_e]=[B]^T[E][B]\times V \quad 3.9$$

In the above expression $[K_e]$ is the element stiffness matrix associated to the finite element side elongation, which signifies that the finite element CST is replaced by imaginary cable elements depicting the sides of the triangle. V being volume of the CST element, the diagonal terms of element stiffness matrix $[K_e]$ are hence equivalent to axial stiffness EA/L of the pseudo cable element.

By applying similar algebraic rules, the geometric stiffness matrix $[K_g]$ for the finite element can be obtained conveniently. The relationship between rigid body rotation and axial force P is used to derive the geometric stiffness matrix for CST finite element. This is a standard obtained expression developed by Lewis, 2003 and is given as:-

$$[K_g]=P/L\begin{bmatrix}[I_3]-[C][C]^T & -[I_3]+[C][C]^T\\-[I_3]+[C][C]^T & [I_3]-[C][C]^T\end{bmatrix} \quad 3.10$$

In the above equation [C] is cable element's direction cosine vector, $[I_3]$ is the identity matrix. The force developed within the element side T_i (*i=1,2,3*) can be formulated in terms of finite element side extension{ δ} and element stiffness matrix $\left[K_e\right]$, it can be represented as :-

$$\{T\} = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = [B]^T [E][B] \times V = \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} \qquad 3.11$$

From the above mathematical relationships, the plane stresses, σ and element force, T_i can be associated and represented as :-

$$\{T\} = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = V \times [B]^T = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \gamma_{xy} \end{Bmatrix} \qquad 3.12$$

The geometric stiffness matrix $\left[K_g\right]$ and the elastic stiffness matrix $\left[K_e\right]$ are both associated to the strain-displacement relationship $[B]$ and the tension T_i developed in the side of finite element is taken as the axial force P_i of the cable element. The element and geometric stiffness matrix both are transformed into global axis coordinates in order to get the nodal deformation response of each finite element node. The transformation is done by using direction cosine vector $[C]$associated with the element sides of CST finite element. Finally the overall stiffness of a TMS system in global coordinates is given as the sum of geometric stiffness matrix and the elastic stiffness matrix in global axis coordinate system.

3.3 Newton-Raphson Method – application on TMS

Newton-Raphson method is considered as one of the convenient and most popularly used method for handling non-linear set of equations. The solutions for the non-linear equations are intended to be obtained using first derivatives of non-linear function with respect to the variable. To understand it through an example, let us consider a vector P_A which is a non-linear function and a force vector of another function u_A which is a displacement vector. And it can be written as:

$$(k_0 + k_{N_A})u_A = P_A \qquad 3.13$$

Where,

k_0 = constant and k_{N_A} =function of displacement vector u_A

$$k_{N_A} = f(u_A) \tag{3.14}$$

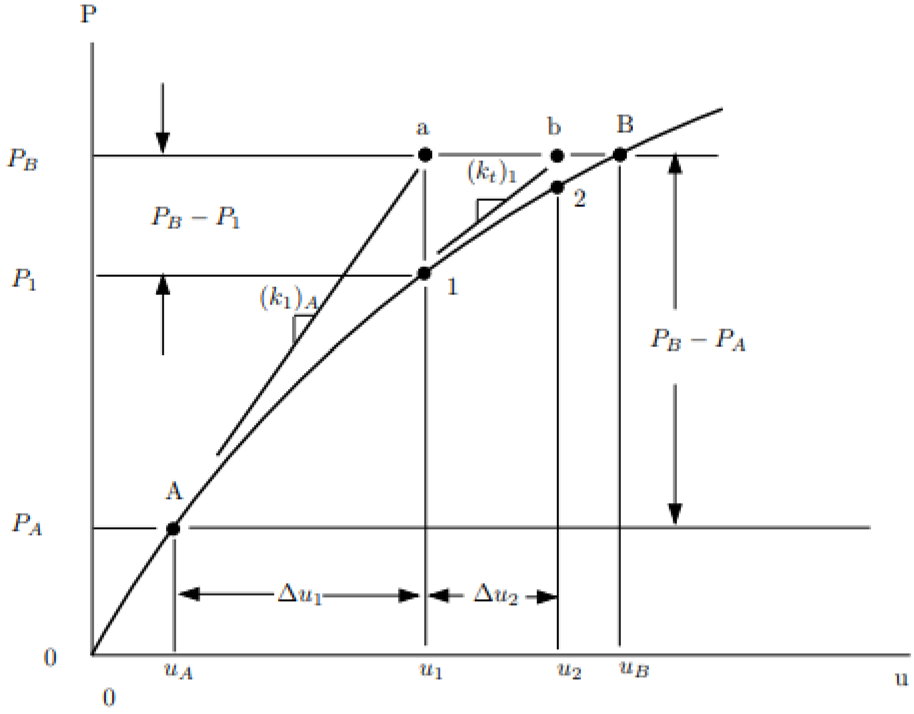

Figure 3 A general representation of Newton-Raphson solution procedure

In the problem the load vector has taken a new position P_B and consequently the displacement has reached to u_B. Taking function $P = f(u)$ and observing its Taylor's series expansion in $(u_B - u_A)$ we get,

$$f(u_B) = f(u_A) + \left(\frac{dP}{du_A}\right)(u_B - u_A) \tag{3.15}$$

Substituting $\Delta u_1 = (u_B - u_A)$ in the above equation and we obtain:

$$f(u_A + \Delta u_1) = f(u_A) + \left(\frac{dP}{du_A}\right)\Delta u_1 \tag{3.16}$$

Where, $$\frac{dP}{du} = \frac{d}{du}(k_0 u + k_N u) = k_0 + \frac{d}{du}(k_N u) = k_t$$

And k_t represents the tangent stiffness. We intend to find δu_1 so that $f(u_A + \Delta u_1) = P_B$, therefore knowing that $f(u_A) = P_A$ and the tangent matrix accessed at point A, hence it can be written that:

$$P_B = P_A + (k_t)_A \Delta u_1 \qquad 3.17$$

On solving further we get,

$$(k_t)_A \Delta u_1 = P_B - P_A \qquad 3.18$$

Where $P_B - P_A$ can be considered as a residual force or the force of imbalance. According to the Figure 3 the displacement vector is updated after computing Δu_1 and it is done as : $u_1 = u_A + \Delta u_1$. The new tangent stiffness (k_t) is obtained for the next iteration using the equation:

$$\frac{dP}{du} = \frac{d}{du}(k_0 u + k_N u) = k_0 + \frac{d}{du}(k_N u) = k_t \qquad 3.19$$

Taking $u = u_1$ and deriving new residual force or imbalance force $P_B - P_1$, and P_1 can be calculated from the equation:

$$(k_0 + k_N)u = P \qquad 3.20$$

Where $k_N = f(u)$ with respect to $u = u_1$. The distance after iteration is updated as $u_2 = u_1 + \Delta u$, in which the Δu_2 is obtained using the equation:

$$(k_t)_1 \Delta u_2 = P_B - P_1 \qquad 3.21$$

This above mentioned procedure for finding the solution for non-linear equation system continues. It is terminated till the displacement increments or the force increments are converged.

3.4 First order reliability method and limit state functions.

The foremost step in estimation of reliability of a TMS is to recognize and enumerate the uncertainties associated with the real behavior of a membrane structure. There are number of factors which impart uncertainty in the TMS structural system, primarily the uncertainties arises due to material behavior, manufacturing, fabrication and in-field construction conditions, human induced uncertainties, environmental related uncertainties. There are several statistical methods to obtain the uncertainties accurately but, some factors like, human factors are case-dependent uncertainties, hence it is hard to accurately measure the uncertainty in that case. The reliability analysis in this research has included the uncertainties related to material characteristics and loads applied. An important TMS structure will constitute of more than one material like, fabric membrane, steel, cables etc. each material has their own associated uncertainties and can be denoted as X in order to formulate it mathematically. In order to contain the complete uncertainty information of a TMS from a fabric properties point of view, six statistical variables are defined as follows:

Table 5: Definition of statistical variables

Statistical Variables	X_1	X_2	X_3	X_4	X_5	X_6
	E_f	E_w	ν_{wf}	G_{wf}	$\sigma_{ult.}^{warp}$	$\sigma_{ult.}^{fill}$

Where, E_f *and* E_w are the Young's modulus of elasticity in fill and warp directions respectively, ν_{wf} is the Poisson's ratio, G_{wf} represents shear modulus along warp and fill direction and $\sigma_{ult.}^{warp}$ *and* $\sigma_{ult.}^{fill}$ are ultimate strength (rupture strength) of the fabric. An another additional statistical variable identified by (Zhang Lei & Gosling, 2010) as an imposed load coefficient t_{load} with unit mean value:-

$$X_7 = t_{load} = \frac{F}{F_c} \qquad 3.22$$

Where F and F_c are applied load and deterministic design load respectively. The primary most statistical parameters for the characterization of a variable are mean μ, standard deviation σ, and the probability density function (PDF). All of these parameters are determined using test results combined with a routine statistical assessment along with the transformation of non-normal distributions of statistical variables.

The structural fabric used in making of TMS is an exclusively tension type composite, and it is made with the orthogonally weaved fiber yarns in two direction: warp and fill (weft). Hence it is more likely that the failure on the TMS surface will occur in the alignment to the yarn direction. The indication for the surface failure in TMS is wrinkle appearance, which signifies the loss of positive minimum principle stress.

The safety procedure associated with the stress in the fabric material can be denoted using three functions $G_1(X_S)$, $G_2(X_S)$, and $G_3(X_S)$ and these functions are expressed in terms of permissible stresses in both the directions (warp and fill).

$$G_1(X_{si}) = \sigma_{per}^{fill} - \sigma_{\max}^{fill} \quad 3.23(a)$$

$$G_2(X_{si}) = \sigma_{per}^{warp} - \sigma_{\max}^{warp} \quad 3.23(b)$$

$$G_3(X_{si}) = \sigma_{\min}^{p} - \sigma_{per}^{p} \quad 3.23(c)$$

And the condition of material failure is,

$$G_i < 0, \;\; i = 1,2, \quad 3.24$$

The permissible stresses are obtained using the ultimate strength and the safety factors as:-

$$\sigma_{per}^{fill} = f_f \sigma_{\max}^{fill} \quad 3.25$$

$$\sigma_{per}^{warp} = f_w \sigma_{\max}^{warp} \quad 3.26$$

Where f_w, f_f and $\sigma_{ult.}^{warp}$, $\sigma_{ult.}^{fill}$ are the safety factors and ultimate strengths in warp and fill direction respectively. The safety factors in Euro-code are defined and estimated according to the material degradation due to the impacts of environmental factors. $\sigma_{\max}^{fill}$ and $\sigma_{\max}^{warp}$ are maximum tension stresses in the fabric along fill and warp direction. $\sigma_{\min}^{p}$ corresponds to the minimum principal stress of the fabric membrane, and σ_{per}^{p} is the predefined lower limit of stress which in general signifies the percentage of the membrane surface pre-stress, and X_{si} is the relevant statistical variable.

The controlled deformation to a sensible level is one basic serviceability requirement in structural design. The serviceability limit for the deflection can be governed as:

$$G_4(X_s) = D_{allowed} - D_{\max} \quad 3.27$$

$D_{allowed}$ is deflection allowance and $D_{\max}$ = maximum deformation.

The TMS surface is required to avoid the excessive deformation in order to ensure the expected structural performance related to phenomena like ponding due to snow load. The structural geometry should be kept to accommodate positive drainage from all areas. The anticlastic curvature of membrane surface provides effective drainage in an un-deformed pre-stressed membrane, but large deformation especially in perpendicular to surface direction may result in ponding in some localized area on surface. For this reason the allowable deformation $D_{allowed}$ selection must not only consider the serviceability in general but should also be able compensate the ponding phenomena.

The failure surface or in other words, the limit state *i* is recognized with the condition $G_i = 0$. This limit state represents the boundary between safe and unsafe regions in the design parameter space. It also identifies the cases where the TMS can no longer comply with function for which it was planned. The Figure 4 below shows the failure surface and safe and unsafe regions with respect to statistical variables X_{s1} and X_{s2}.

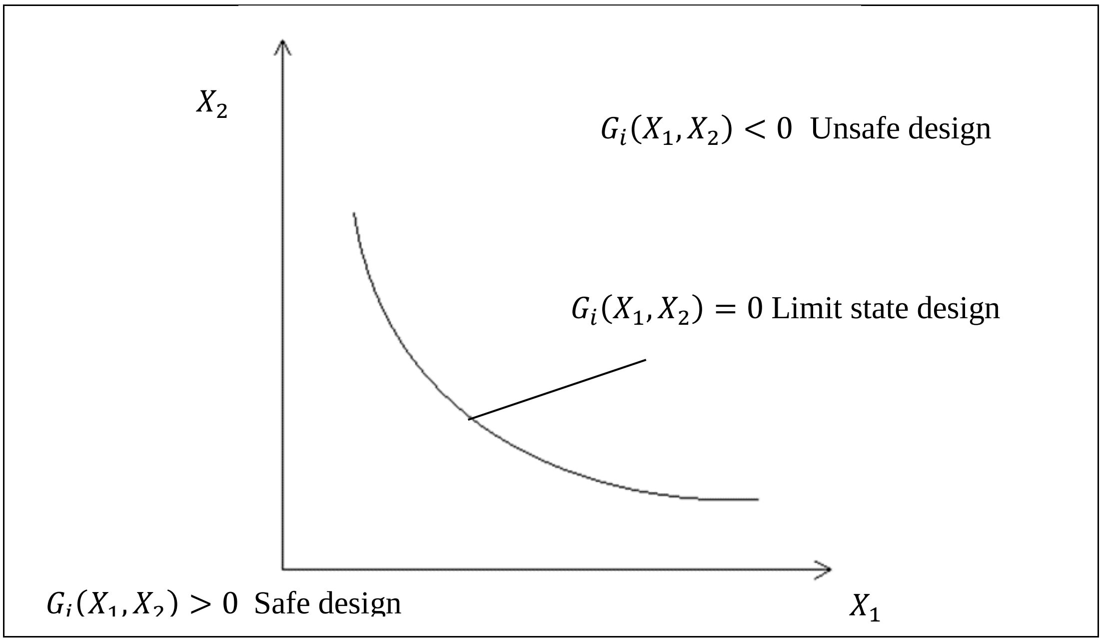

Figure 4 Representation of limit state function, safe and unsafe regions

For a given space with *n* variables, the probability of structural failure p_f can be calculated using the equation below:-

$$p_f = \int ..... \int_{g(X_s)<0} fx(x_{s1}, x_{s2,} ....., x_{s7}) dx_1 dx_2 ..... dx_n \qquad 3.28$$

$fx(x_{s1}, x_{s2,} ....., x_{sn})$ is the joint probability density function (PDF) for the statistical variables $X_{s1,} X_{s2}, X_{s3,} ....X_{sn}$, the integration is performed over the failure region $G(X_s) < 0$. A condition where it is assumed that the random variables are all statistically independent and the limit state function is a simple combination of design variables, in that case, the joint probability density function is likely to be replaced by the product of the individual PDF. The computation of p_f by above equation is the fundamental equation of reliability analysis. In practice the joint probability density function is often not accessible and the multiple integration for individual probability density function is also a tedious exercise. The first-order reliability method (FORM) makes the calculation of reliability simpler by using some analytical approximations, hence making equation 3.6 more compliant.

FORM can effectively estimate the probability of failure of a structural system based on the first and second moments of the random variables, by using first-order approximation with corresponding random variables. The limit state function formulation is generally written as:

$$G_i = f_i(X_{s1}, X_{s2}, ..., \mathrm{X}_{sn}) \qquad 3.29$$
$$i = 1 \rightarrow 4$$

In above equation f_i is the function of the random variables for *ith* limit state function. On expanding the expression of limit state function about the mean value using Taylor's series gives us:

$$G_i = f_i(\mu x_s) + \sum_{i=1}^{n} \frac{\partial f_i}{\partial X_{si}}(X_{si} - \mu x_{si}) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 f_i}{\partial X_{si}\partial X_{sj}}(X_{si} - \mu x_{si})(X_{sj} - \mu x_{sj}) + ... \qquad 3.30$$

The derivatives in the above equation ae evaluated at the mean values of the random variables ($X_{s1,} X_{s2}, X_{s3,} ....X_{sn}$), and μx_{si} represents the mean value of variable X_{si}. Selecting the linear terms from the series expansion, the first-order approximation function is obtained,

$$G_i \approx f_i(\mu x_s) + \sum_{i=1}^{n} \frac{\partial f_i}{\partial X_{si}}(X_{si} - \mu x_{si}) \qquad 3.31$$

Statistically independent random variables $X_{s1,} X_{s2}, X_{s3,} ....X_{sn}$ are assumed to be normally distributed, the first-order approximated function G_i is distributed normally too, and the approximated mean and variance for G_i can be written as :

$$\mu G_i \approx f_i(\mu_{x_{s1}}, \mu_{x_{s2}}, \ldots\ldots, \mu_{x_{sn}}) \qquad 3.32$$

$$\sigma_{G_i}^2 \approx \sum_{i=1}^{n} \left(\frac{\partial G_i}{\partial X_{si}} \right)^2 Var(X_{si}) \qquad 3.33$$

After all the statistical investigation about the limit state function we are able to write the probability of structural failure expression as:

$$p_f = P(G_i < 0) \qquad 3.34$$

Or if the information of cumulative distribution function (Φ) is available in that case the above equation will be written as :

$$p_f = \Phi\left(\frac{0 - \mu_{G_i}}{\sigma_{G_i}} \right) = 1 - \Phi\left(\frac{\mu_{G_i}}{\sigma_{G_i}} \right) = 1 - \Phi(\beta) \qquad 3.35$$

The structural failure probability calculated using the above equation is dependent on the ratio of the mean and the standard deviation of limit state function G_i. This ratio is a well-known statistical entity called the reliability index also known as the safety index, and it is denoted by the symbol *β*.

$$\beta = \frac{\mu_{G_i}}{\sigma_{G_i}} \qquad 3.36$$

The information of mean and standard deviation of the limit state function is required while calculating the safety index using above equation. The information about statistical parameters is not known explicitly, in particular when the analysis is done with the help of some finite element software using simulation techniques. The dependency of safety indices on limit state equation and its assumption regarding distribution of limit state makes it difficult to estimate the safety index. This shortcoming was sorted by the H-L method**,** a reduced coordinate system and reduced corresponding variables are defined and formulated as :

$$X_{si}^{'} = \frac{X_{si} - \mu_{X_{si}}}{\sigma_{X_{si}}} (i = 1, 2, 3..., n) \qquad 3.37$$

$X_{si}^{'}$ is a random variable with mean and standard deviation equal to zero. The above equation is used to transform the original limit state to reduced limit state, i.e. $G(X_s) \rightarrow G(X_s^{'})$. X_s and $X_s^{'}$ are the variables defined in original coordinate system and reduced coordinate system respectively. In H-L method the safety index is redefined in the reduced coordinate system, it is estimated as the minimum distance from the origin of the reduced coordinate axes. The H-L reliability index can be expresses as:

$$\beta_{H-L} = \sqrt{(x_s'^*)^t (x_s'^*)} \quad 3.38$$

The design point or checking point on limit state surface from origin is the minimum distance point. It is denoted by vector x_s^* in original coordinate system and $x_s'^*$ is reduced coordinate system. The purpose of these vectors is to represent the values of all the random variables $X_{s1,} X_{s2}, X_{s3,} ....X_{sn}$ at the design point location with corresponding coordinate system is used.

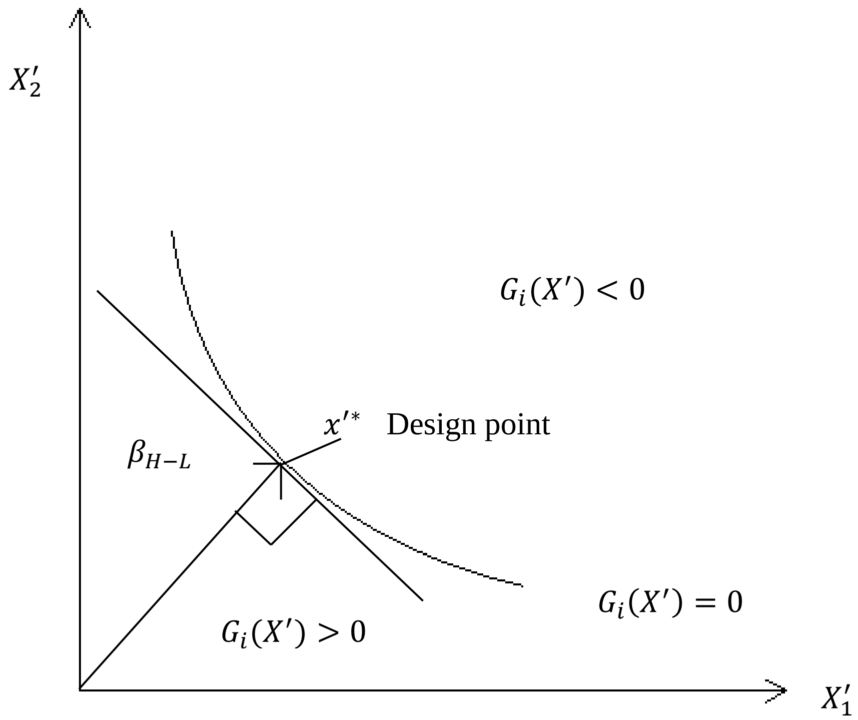

Figure 5 Hasofer-Lind reliability index: nonlinear limit state function

The Figure 5 illustrates that closer the $x'^*$to the origin, larger is the chances of failure of design. Hence it can be derived that the shortest distant point on the curve of limit surface indicates the most probable point of failure. The design point x'^* also represents the borderline combination of stochastic variables. Therefore the exploration of the perfect design point and estimating the minimum possible distance for non-linear limit state functions itself become a problem of the optimization. A distance D, is optimized as:

Minimize D = $\sqrt{x_s'^t x_s'}$, subjected to the condition $G_i(X_s) = G_i(X_s') = 0$,Where x_s' is the limit state function checking point coordinates in the reduced coordinate system. With the help of Lagrange multipliers it can be proposed that the minimum distance can be found when the safety index (β_{H-L}) gradient is equal to the gradient of the constraint function $G(X_s) = f_i(X_{s1,} X_{s2}, X_{s3,} ....X_{sn})$ =0. Hence the safety index can be written as,

$$\beta_{H-L} = -\frac{\sum_{i=1}^{n} x_{si}'^{*} \left(\frac{\partial f_i}{\partial X_{si}'}\right)^{*}}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial f_i}{\partial X_{si}'}\right)^{2*}}} \qquad 3.39$$

In the above equation $\left(\frac{\partial f_i}{\partial X_{si}'}\right)^{*}$ is the i^{th} partial derivative with coordinates $(x_{s1}'^{*}, x_{s2}'^{*}, x_{s3}'^{*}....x_{sn}'^{*})$ evaluated at the limit state design point. A basic algorithm by Rackwitz (1976) to evaluate the reliability index and the coordinates of the design points is described in the Figure 6 (Kiureghian & Ke, 1988)**:**

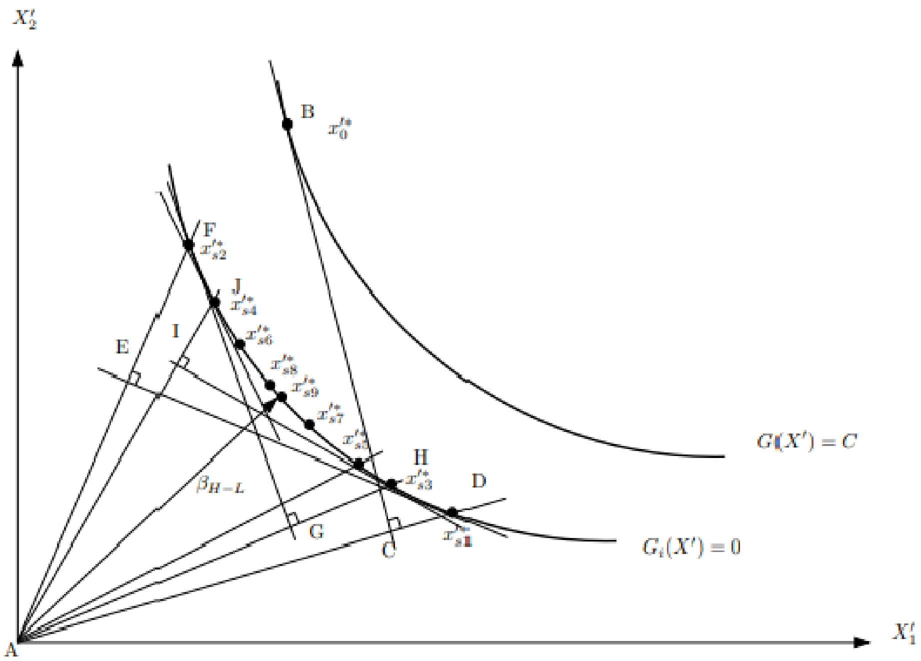

Figure 6 Basic FORM algorithm for reliability index calculation

According to this algorithm, the iteration starts with a design point B, which is usually the mean value of the limit state function, and the coordinates of this starting design point are not required to lie on the limit state surface. The tangent line BC is the representation of initial design point, its intersection with the limit state surface gives the first iteration of the design point i.e. $x_{s1}^{'*}$. The iteration continues with the updating of design point through tangent line for each design point after every iteration until the safety index converges to a minimum specified value.

The normal distribution of the random variables is generally incorporated in the FORM solution procedure. In case the non-normal distribution of the variables is used for reliability analysis, then these variables are required to transform into an equivalent normal distribution system. One of the transformation method developed by Rackwitz and Fiessier for non-normally distributed random variables uses a condition of cumulative density function (CDF) and the probability density function (PDF). The condition to apply this transformation is that the CDF and PDF of the actual variables and the equivalent normal random variables should be equal at the design points $(x_{s1}^{'*}, x_{s2}^{'*}, x_{s3}^{'*} .... x_{sn}^{'*})$ of the limit state surface.

3.5 Summary

For non-linear formulations and analysis of the TMS the principle of residual force is applied, which portrays that the residual force or the force of imbalance due to stress field in the state of equilibrium must approach to zero magnitude. In this study, a proficient finite element method to contain the large deformation behavior of TMS for load analysis as well as form finding state is outlined. The element used for the discretization of the TMS surface is CST, it is chosen due to its versatility in handling the non-linear finite element formulations. From the perspective of the convenience of implementation and the computational compatibility, the side length of the element is necessarily considered as a degree of freedom (D.O.F). It has been observed that in compliance with finite element technology standards, for an agreement to fulfill the Green's strains criteria, the CST formulation has been successfully achieving the benchmarks.

The finite element characteristics like element coordinates and stiffness equations of the CST elements are initially described in the coordinates of local axes. With the action of transformation matrices defined in the local coordinates for each element, the solution system is implemented in Global axes. The TMS are inefficient in comprehending even the slightest compressive stress. Hence the possibility of zero stress region consequently causing the wrinkling failure to the membrane surface.

Talking about the reliability analysis, the traditional/conventional structural systems, are designed using a factored material strength and loads in order to ensure a safe structural design with sufficient safety. The TMS are however likely to experience the change in both magnitude and distribution of the stresses even due to slight change in the loading. This kind of scenario induces the huge amount of uncertainty in analyzing the TMS behavior. For such condition the direct application of partial safety factors approach to the material characteristics and loading conditions will be very wrong, as the change in loading distribution and magnitude will alter the overall geometry of the structure. Therefore for the present study, a permissible stress design approach is adopted in which the ultimate tensile strength of the material is corrected using safety factors and compared with the stress field developed during the form finding and load analysis of TMS. The maximum safety factors being used worldwide are 9.5 in German guide, 8.0 in Japanese design code and 7.0 in IASS design recommendations. Finally the reliability assessment of the TMS in this study is achieved using a robust reliability tool called FORM.