


## CERTIFICATE

It is certified that the work contained in the thesis titled “*Analytical Investigation of Nonlinear Wave Propagation and Riemann problems in Hyperbolic Gaseous Systems*” by *Gaurav* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. degree.

  
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### DECLARATION BY THE CANDIDATE

I, **Gaurav**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Prof. Lal Pratap Singh** from **July, 2019** to **May, 2025** at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credit to the researchers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, thesis, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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### CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my/our knowledge.

  
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*To my beloved parents*

***Shri Shashikant Upadhyay***

***Shrimati Indu Upadhyay***

*Their unwavering support reached me, no matter the  
distance.*



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