Chapter 4

Prescribed-time adaptive backstepping control of a twin rotor helicopter

4.1 Introduction

In the preceding chapter, we delved into the adaptive prescribed-time control problem for nonlinear systems with parametric uncertainties in strict-feedback form. We employed a prescribed-time approach to finite-time stabilization, utilizing feedback with time-varying gains that tend to infinity as the prescribed convergence time is approached. Building upon these foundations, the focus of this chapter shifts towards the application of this approach to adaptive prescribed-time control for trajectory tracking in the context of the twin rotor helicopter system.

The twin rotor helicopter system presents a challenging and dynamic environment, making it an ideal testbed to evaluate the efficacy of our proposed adaptive prescribed-time control methodology. By extending the prescribed-time approach to trajectory tracking, we seek to achieve precise control over the helicopter's movement, even in the presence of parametric uncertainties. The utilization of feedback with time-varying gains enables the system to reach the desired trajectory within the prescribed time bounds, ensuring robust performance under various operating conditions.

The major contributions are:

(i) A prescribed-time adaptive controller utilizing the backstepping technique is designed to follow the desired pitch and yaw angular position independently.

(ii) Adaptive laws are employed to approximate the unknown parameters of the twin rotor helicopter systems.

(iii) The boundedness of all the signals of the closed-loop system is proved using the Lyapunov direct stability theory.

The rest of the chapter is structured as follows: Section II provides preliminaries that will be useful for obtaining the main results. In Section III, the main results are shown. Section IV validates the efficacy of the proposed scheme. Finally, Section V concludes this chapter.

4.2 Preliminaries and problem statement

The following results will be helpful in order to understand the concept of convergence in a given time, where the upper bound of the settling time can be prescribed.

4.2.1 Motivating example

We consider the following first-order system

$$\dot{z}(t) = u(t), \quad z(t_0) = z_0 \tag{4.1}$$

where $z(t) \in \mathbb{R}$ and $u(t) \in \mathbb{R}$. The key element in our proposed methodology is the utilization of the following time-varying feedback

$$u(t) = \begin{cases} -\gamma \frac{z(t)}{(t_p+t_0-t)}, & \forall t \in [0, t_p) \\ 0, & \forall t \in [t_p, \infty) \end{cases} \tag{4.2}$$

where, t_0 is the initial time, t_p is the desired time duration of convergence from initial time t_0 and $\gamma \in \mathbb{R}_{>1}$ is a tuning parameter. It is easy to prove the existence and uniqueness of the solution of this system, which ensures $z(t) = 0$ and $\dot{z}(t) = 0$ for all $t \geq t_p + t_0$.

Remark 4.1 *Utilizing the control law* (4.2)*, the solution to* (4.1) *is*

$$z(t) = \frac{z(t_0)}{(t_p)^\gamma}(t_p + t_0 - t)^\gamma \tag{4.3}$$

Further, from (4.2)*, we get*

$$u(t) = \frac{z(t_0)}{(t_p)^\gamma}(t_p + t_0 - t)^{\gamma-1} \tag{4.4}$$

From (4.3) *and* (4.4)*, the continuity and boundedness of* $z(t)$ *and* $u(t)$ *can be ensured, respectively. Furthermore, from* (4.4)*, one can observe that the boundedness of* $u(t)$ *is only ensured when* $\gamma > 1$*. Hence, it is recommended that* $\gamma > 1$*.*

The following Lemma lays out the Lyapunov characterization for a prescribed-time stable system.

Lemma 4.2 *If there is a positive real-valued continuously differentiable function* $V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ *and two functions* β_1 *and* β_2 *belonging to the class* $\mathcal{K}_\infty$ *functions, such that the following conditions are satisfied for the system represented by* (2.1)*:*

$$\beta_1(\|z\|) \leq V(t, z) \leq \beta_2(\|z\|), \quad \forall z \in \mathbb{R}^n \setminus \{0\} \tag{4.5}$$

$$V(t, 0) = 0, \quad \forall\, t \in [t_0, t_p + t_0) \tag{4.6}$$

$$\dot{V} \leq \begin{cases} -\gamma \frac{V(z)}{(t_p+t_0-t)}, & \forall t \in [t_0, t_p + t_0), \forall V \neq 0 \\ 0, & \forall t \in [t_p + t_0, \infty) \end{cases} \tag{4.7}$$

where $\gamma \in \mathbb{R}_{>1}$*. Then the origin* $z(t) = 0$ *of the system* (2.1) *exhibits prescribed-time stability.*

Proof. Firstly, we will establish the boundedness of $z(t)$ and the uniform stability of the origin $z(t) = 0$ for the system (2.1).

From equation (4.7), it can be observed that $\dot{V}(t, z) \leq 0$ for all $t \in [t_0, \infty)$ and $z(t) \in \mathbb{R}^n$. This implies that $V(t, z)$ is monotonically decreasing with respect to $t \in [t_0, \infty)$. Consequently, we have the inequality:

$$V(t, z) \leq V(t_0, z_0) \tag{4.8}$$

Combining equations (4.5) and (4.8), we obtain:

$$\|z(t)\| \leq \beta_1^{-1}(V(t, z)) \leq \beta_1^{-1}(V(t_0, z_0))$$

This ensures the boundedness of $z(t)$ for all $t \in [t_0, \infty)$.

To establish uniform stability, let $\epsilon \in \mathbb{R}_{>0}$ be given, and suppose there exists $\delta \in \mathbb{R}_{>0}$ such that $\beta_2(\delta) < \beta_1(\epsilon)$. Now, select the initial condition z_0 such that $\|z_0\| < \delta$. It follows:

$$\beta_1(\epsilon) > \beta_2(\delta) \geq V(t_0, z_0) \tag{4.9}$$

Since $\dot{V}(t,z) \le 0$ for all $t \in [t_0, \infty)$, we have $\beta_1(\epsilon) > \beta_2(\delta) \ge V(z(t;t_0,z_0)) \ge \beta_1(\|z(t;t_0,z_0)\|)$. As β_1 is class $\mathcal{K}_\infty$ function, this implies that $\|z(t;t_0,z_0)\| < \epsilon$ for all $\forall t \in [t_0, \infty)$. Thus, the equilibrium point of system (2.1) is uniformly stable.

Having established the uniform stability of the origin and the boundedness of the solution, we can now proceed to demonstrate the desired prescribed-time stability. To achieve this, we will utilize the comparison lemma [105] to establish the convergence of $z(t)$ to the origin within a pre-determined time frame that we have chosen beforehand.

Suppose that for all $t \in [t_0, t_p + t_0)$, $V(t,z)$ satisfies the differential inequality $\dot{V}(t) \le -\gamma \frac{V(z)}{(t_p+t_0-t)}$, where $\gamma \in \mathbb{R}_{>1}$. Let $\xi(t)$ represent the solution to the subsequent differential equation $\dot{\xi}(t) = -\gamma \frac{\xi(t)}{(t_p+t_0-t)}$. The solution of this differential equation can be expressed as:

$$\xi(t) = \frac{\xi(t_0)}{(t_p)^\gamma}(t_p + t_0 - t)^\gamma \tag{4.10}$$

From (4.10), it can be observed that at $t = t_p + t_0$, we have $\xi(t) = 0$. Therefore, for $t \ge t_p + t_0$, $\xi(t) = 0$ is maintained for all future times. The same conclusion holds true for $V(t,z)$, which establishes the prescribed-time stability. Thus, the proof is complete. ■

Remark 4.3 *This study diverges from the existing literature, which primarily focuses on ensuring the scaled state and boundedness of the control input (e.g., [17] and [109]). Consequently, they consider that the defined closed-loop time interval must be finite and cannot be infinite. In contrast, our study demonstrates that as the time approaches the prescribed time $t_p + t_0$, all closed-loop signals, including the control input, indeed converge to zero. This enables us to extend the solution beyond the terminal time. Additionally, the simplicity of the proposed control structure allows for its application in a broader range of scenarios and applications.*

Lemma 4.4 *[110] For the system* (3.1)*, if there exist two positive differentiable functions $V_1(z)$, $V_2(\tilde{\Phi})$ and class $\mathcal{K}_\infty$ functions β_1, β_2, β_3 and β_4 such that $\forall z(t) \in \mathbb{R}^n$, we have*

$$V = V_1(z) + V_2(\tilde{\Phi}) \tag{4.11}$$

$$\beta_1(\|z\|) \le V_1(z) \le \beta_2(\|z\|) \tag{4.12}$$

$$\beta_3(\|\tilde{\Phi}\|) \le V_2(\tilde{\Phi}) \le \beta_4(\|\tilde{\Phi}\|) \tag{4.13}$$

$$\dot{V} \le \begin{cases} -\gamma \frac{V_1(z)}{(t_p+t_0-t)}, & \forall t \in [t_0, t_p + t_0) \\ 0, & \forall t \in [t_p + t_0, \infty) \end{cases} \tag{4.14}$$

where $\gamma > 1$ is a design constant, then the equilibrium point of the system (3.1) *is prescribed-time stable.*

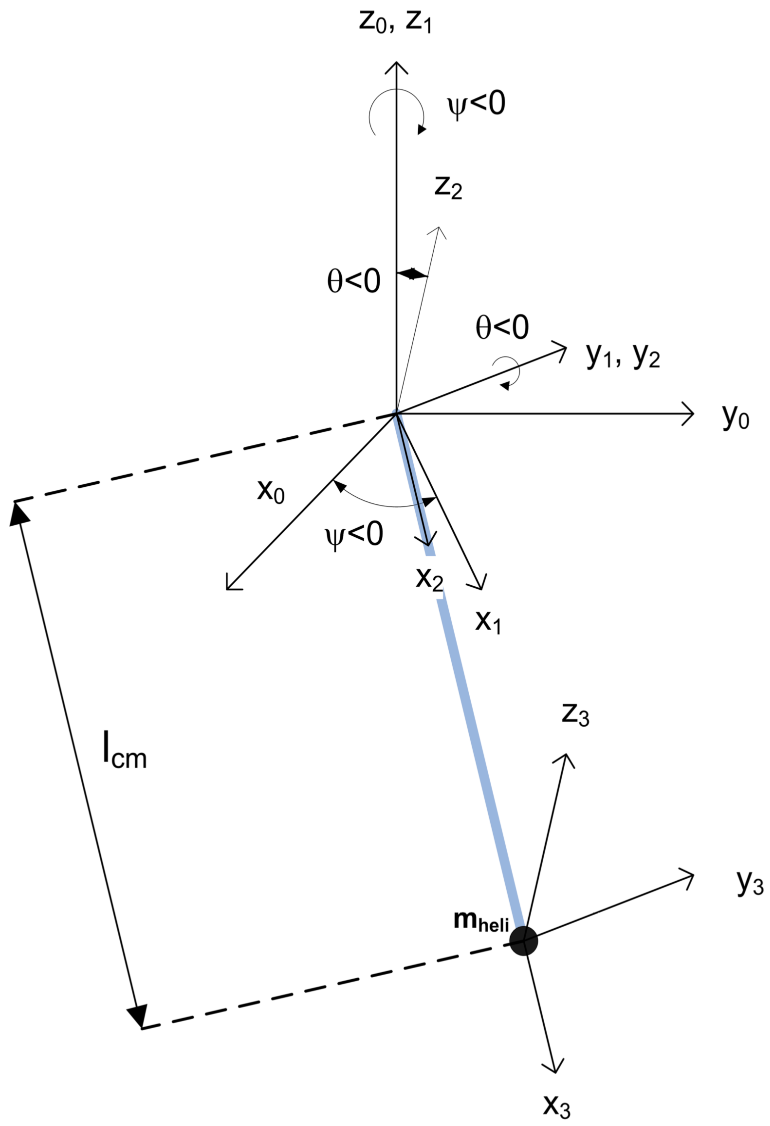

Figure 4.1: Free-body diagram of twin rotor helicopter model.

4.2.2 Twin rotor helicopter model

The twin rotor helicopter model setup is considered here. The free body diagram of a twin rotor helicopter model is shown in Figure 4.1. With the help of the Euler-Lagrangian formula, mathematical modeling is obtained as follows :

$$\begin{aligned}\ddot{\theta} = &-\frac{B_p\dot{\theta}}{J_p + m_h l^2} - \frac{m_h l^2 \dot{\psi}^2 \sin(\theta)\cos(\theta)}{J_p + m_h l^2} \\ &- \frac{m_h g l \cos(\theta)}{J_p + m_h l^2} + \frac{K_{pp}V_p}{J_p + m_h l^2} + \frac{K_{py}V_y}{J_p + m_h l^2} \end{aligned} \tag{4.15}$$

$$\begin{aligned}\ddot{\psi} = &\frac{2 m_h l^2 \dot{\psi} \sin(\theta)\cos(\theta)\dot{\theta}}{J_y} \\ &- \frac{B_y\dot{\psi}}{J_y} + \frac{K_{yp}V_p}{J_y} + \frac{K_{yy}V_y}{J_y} \end{aligned} \tag{4.16}$$

where $\theta, \dot{\theta}, \psi$ and $\dot{\psi}$ denote the pitch angle, pitch velocity, yaw angle and yaw velocity, respectively. V_p and V_y are the input voltages to the pitch and yaw motors. The detailed description of the system parameters is given in [111]. Now, the state variables and control are defined as

$$\begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix}, \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} u_p \\ u_y \end{bmatrix} \tag{4.17}$$

where u_p and u_y are the pitch and yaw motors control inputs, and they are specified as

$$u_p = K_{pp}V_p + K_{py}V_y \tag{4.18}$$

$$u_y = K_{yp}V_p + K_{yy}V_y \tag{4.19}$$

Rewriting equations (4.15) and (4.15) into the state space form as

$$\begin{aligned} \dot{z}_1(t) =& z_2(t) \\ \dot{z}_2(t) =& \Theta_1^\top(z)\Phi_1 + b_p u_p \\ \dot{z}_3(t) =& z_4(t) \\ \dot{z}_4(t) =& \Theta_2^\top(z)\Phi_2 + b_y u_y \end{aligned} \tag{4.20}$$

where the known nonlinear functions Θ_1 and Θ_2 are defined as

$$\Theta_1 = \begin{bmatrix} -z_2(t) \\ -\cos(z_1) \\ -z_4^2(t)\cos(z_1)\sin(z_1) \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} -z_4(t) \\ z_2(t)z_4(t)\cos(z_1)\sin(z_1) \end{bmatrix}$$

Additionally, the unknown constant vectors Φ_1 and Φ_2 are specified as

$$\Phi_1 = \frac{1}{J_p + m_h l^2} \begin{bmatrix} Bp \\ m_h g l \\ m_h l^2 \end{bmatrix}, \quad \Phi_2 = \frac{1}{J_y} \begin{bmatrix} B_y \\ 2m_h l^2 \end{bmatrix}$$

and unknown constants b_p and b_y are defined as $b_p = \frac{1}{J_p + m_h l^2}, \quad b_y = \frac{1}{J_y}$.

Since the aim is to develop a control law for u_y and u_p so that the outputs $z_1(t)$ and $z_3(t)$ follow the desired reference signals $z_{d1}(t)$ and $z_{d3}(t)$ respectively in a prescribed-time, which can be specified in advance. The following assumptions are made in order to attain the control objective.

Assumption 3 *Φ_1, Φ_2, b_p, and b_y are all positive constants that represent the unknown parameters.*

Assumption 4 *The first-order and second-order derivatives of the desired reference signals $z_{d1}(t)$ and $z_{d3}(t)$ are known, bounded and piecewise continuous.*

4.3 Prescribed-time adaptive backstepping control design

For the considered twin rotor helicopter model with uncertain parameters, we will use the idea of an adaptive backstepping technique to develop a prescribed-time feedback control law. We begin by applying the change of coordinates for the state variables as

$$x_1(t) = z_1(t) - z_{d1}(t) \tag{4.21}$$

$$x_2(t) = z_2(t) - \alpha_1(t) - \dot{z}_{d1}(t) \tag{4.22}$$

$$x_3(t) = z_3(t) - z_{d3}(t) \tag{4.23}$$

$$x_4(t) = z_4(t) - \alpha_2(t) - \dot{z}_{d3}(t) \tag{4.24}$$

where $\alpha_1(t)$ and $\alpha_2(t)$ are the virtual controllers and chosen as

$$\alpha_1(t) = -\gamma_1 \frac{x_1(t)}{(t_p + t_0 - t)} \tag{4.25}$$

$$\alpha_2(t) = -\gamma_3 \frac{x_3(t)}{(t_p + t_0 - t)} \tag{4.26}$$

where γ_1 and γ_3 are the positive design constant. Now the transformed dynamics can be written as:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) + \alpha_1(t) \\
\dot{x}_2(t) &= \Theta_1^\top(z)\Phi_1 + b_p u_p - \dot{\alpha}_1(t) - \ddot{z}_{d1}(t) \\
\dot{x}_3(t) &= x_4(t) + \alpha_2(t) \\
\dot{x}_4(t) &= \Theta_2^\top(z)\Phi_2 + b_y u_y - \dot{\alpha}_2(t) - \ddot{z}_{d3}(t)
\end{aligned} \tag{4.27}$$

We propose the following prescribed-time control law for the system (4.27)

$$u_p = \hat{\rho}_1 \bar{u}_p \tag{4.28}$$

$$u_y = \hat{\rho}_2 \bar{u}_y \tag{4.29}$$

with

$$\bar{u}_p = -x_1(t) - \Theta_1^\top(z)\hat{\Phi}_1 - \gamma_2 \frac{x_2(t)}{(t_p + t_0 - t)} + \dot{\alpha}_1(t) + \ddot{z}_{d1}(t)$$
$$\bar{u}_y = -x_3(t) - \Theta_2^\top(z)\hat{\Phi}_2 - \gamma_4 \frac{x_4(t)}{(t_p + t_0 - t)} + \dot{\alpha}_2(t) + \ddot{z}_{d3}(t)$$

The parameter updating laws are chosen as

$$\dot{\hat{\Phi}}_1 = \Xi_1 \Theta_1 x_2(t) \tag{4.30}$$
$$\dot{\hat{\Phi}}_2 = \Xi_2 \Theta_2 x_4(t) \tag{4.31}$$
$$\dot{\hat{\rho}}_1 = -\Upsilon_1 \bar{u}_p x_2(t) \tag{4.32}$$
$$\dot{\hat{\rho}}_1 = -\Upsilon_2 \bar{u}_y x_4(t) \tag{4.33}$$

where γ_2 and γ_4 are positive constants, Ξ_1 and Ξ_2 are the adaptive gain positive definite matrices, Υ_1 and Υ_2 are positive constants, $\hat{\Phi}_1$, $\hat{\Phi}_2$, $\hat{\rho}_1$ and $\hat{\rho}_2$ are the estimates of Φ_1, Φ_2, $\rho_1 = \frac{1}{b_p}$ and $\rho_2 = \frac{1}{b_y}$ respectively. Let $\tilde{\Phi}_1 = \Phi_1 - \hat{\Phi}_1$, $\tilde{\Phi}_2 = \Phi_2 - \hat{\Phi}_2$, $\tilde{\rho}_1 = \rho_1 - \hat{\rho}_1$ and $\tilde{\rho}_2 = \rho_2 - \hat{\rho}_2$ be the parameter estimation errors.

Theorem 1 *Considering the nonlinear twin rotor helicopter system* (4.15)-(4.16) *under the Assumption 3 and 4, with the proposed control law* (4.28)-(4.29) *and the parameter updating law* (4.30)-(4.33)*, the following Conclusions can be drawn:*

- *The prescribed-time tracking of the reference signals can be achieved, i.e.,* $\lim_{t \to t_p} \left[z_1(t) - z_{d1}(t) \right] = 0$, *and* $\lim_{t \to t_p} \left[z_3(t) - z_{d3}(t) \right] = 0$.
- *All signals of the closed loop system* (4.15)-(4.16) *are ensured to be bounded for all future time.*

Proof. The total candidate Lyapunov function is chosen as

$$V = V_1(x) + V_2(\tilde{\Phi}) \tag{4.34}$$

where $V_1(x) = \frac{1}{2}x_1^2(t) + \frac{1}{2}x_2^2(t) + \frac{1}{2}x_3^2(t) + \frac{1}{2}x_4^2(t)$ and $V_2(\tilde{\Phi}) = \frac{1}{2}\tilde{\Phi}_1^\top \Xi_1^{-1}\tilde{\Phi}_1 + \frac{1}{2}\tilde{\Phi}_2^\top \Xi_2^{-1}\tilde{\Phi}_2 + \frac{b_p}{2\Upsilon_1}\tilde{\rho}_1^2 + \frac{b_y}{2\Upsilon_2}\tilde{\rho}_2^2$.

Now, taking the time derivative of V along the solution to (4.27), we get

$$
\begin{aligned}
\dot{V} &= x_1(t)\dot{x}_1(t) + x_2(t)\dot{x}_2(t) + x_3(t)\dot{x}_3(t) + x_4(t)\dot{x}_4(t) \\
&\quad - \tilde{\Phi}_1^\top \Xi_1^{-1}\dot{\hat{\Phi}}_1 - \tilde{\Phi}_2^\top \Xi_2^{-1}\dot{\hat{\Phi}}_2 - \frac{b_p}{\Upsilon_1}\tilde{\rho}_1\dot{\hat{\rho}}_1 - \frac{b_y}{\Upsilon_2}\tilde{\rho}_2\dot{\hat{\rho}}_2 \\
&= x_1(t)(x_2(t) + \alpha_1(t)) \\
&\quad + x_2(t)\Big(\Theta_1^\top(z)\Phi_1 + b_p u_p - \dot{\alpha}_1(t) - \ddot{z}_{d1}(t)\Big) \\
&\quad + x_3(t)(x_4(t) + \alpha_2(t)) \\
&\quad + x_4(t)\Big(\Theta_2^\top(z)\Phi_2 + b_y u_y - \dot{\alpha}_2(t) - \ddot{z}_{d2}(t)\Big) \\
&\quad - \tilde{\Phi}_1^\top \Xi_1^{-1}\dot{\hat{\Phi}}_1 - \tilde{\Phi}_2^\top \Xi_2^{-1}\dot{\hat{\Phi}}_2 - \frac{b_p}{\Upsilon_1}\tilde{\rho}_1\dot{\hat{\rho}}_1 - \frac{b_y}{\Upsilon_2}\tilde{\rho}_2\dot{\hat{\rho}}_2
\end{aligned} \tag{4.35}
$$

Substituting the proposed control law (4.28) and (4.29) in (4.35), we obtain

$$
\begin{aligned}
\dot{V} =& -\gamma_1 \frac{x_1(t)}{(t_p + t_0 - t)} x_1(t) - \gamma_2 \frac{x_2(t)}{(t_p + t_0 - t)} x_2(t) \\
& -\gamma_3 \frac{x_3(t)}{(t_p + t_0 - t)} x_3(t) - \gamma_4 \frac{x_4(t)}{(t_p + t_0 - t)} x_4(t) \\
& + \Theta_1^\top \tilde{\Phi}_1 x_2(t) + \Theta_2^\top \tilde{\Phi}_2 x_4(t) - b_p \tilde{\rho}_1 \bar{u}_p x_2(t) \\
& - b_y \tilde{\rho}_2 \bar{u}_y x_4(t) - \tilde{\Phi}_1^\top \Xi_1^{-1}\dot{\hat{\Phi}}_1 - \tilde{\Phi}_2^\top \Xi_2^{-1}\dot{\hat{\Phi}}_2 \\
& - \frac{b_p}{\Upsilon_1}\tilde{\rho}_1\dot{\hat{\rho}}_1 - \frac{b_y}{\Upsilon_2}\tilde{\rho}_2\dot{\hat{\rho}}_2
\end{aligned} \tag{4.36}
$$

Further simplifying (4.36), one can get

$$
\begin{aligned}
\dot{V} =& -\gamma_1 \frac{x_1(t)}{(t_p + t_0 - t)} x_1(t) - \gamma_2 \frac{x_2(t)}{(t_p + t_0 - t)} x_2(t) \\
& -\gamma_3 \frac{x_3(t)}{(t_p + t_0 - t)} x_3(t) - \gamma_4 \frac{x_4(t)}{(t_p + t_0 - t)} x_4(t) \\
& - \tilde{\Phi}_1^\top \Xi_1^{-1}\Big(\dot{\hat{\Phi}}_1 - \Xi_1 \Theta_1 x_2(t)\Big) \\
& - \tilde{\Phi}_2^\top \Xi_2^{-1}\Big(\dot{\hat{\Phi}}_2 - \Xi_2 \Theta_2 x_4(t)\Big) \\
& - \tilde{\rho}_1 \frac{b_p}{\Upsilon_1}\Big(\dot{\hat{\rho}}_1 + \Upsilon_1 \bar{u}_p x_2(t)\Big) \\
& - \tilde{\rho}_2 \frac{b_y}{\Upsilon_2}\Big(\dot{\hat{\rho}}_2 + \Upsilon_2 \bar{u}_y x_4(t)\Big)
\end{aligned} \tag{4.37}
$$

Next, substituting the parameter updating law (4.30)-(4.33) in (4.37), it is straightforward to show that

$$
\begin{aligned}
\dot{V} =& -\gamma_1 \frac{x_1^2(t)}{(t_p + t_0 - t)} - \gamma_2 \frac{x_2^2(t)}{(t_p + t_0 - t)} - \gamma_3 \frac{x_3^2(t)}{(t_p + t_0 - t)} \\
& - \gamma_4 \frac{x_4^2(t)}{(t_p + t_0 - t)}
\end{aligned} \tag{4.38}
$$

which further implies

$$\dot{V} \leq -\gamma \frac{V_1(x)}{(t_p + t_0 - t)} \tag{4.39}$$

where, $\gamma = \min\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$.

Hence, the prescribed-time stabilization of the twin rotor helicopter follows from equations (4.39), Lemma 4.2 and Lemma 4.4. Further, utilizing the LaSalle-Yoshizawa theorem [105], boundedness of V can be ensured. This implies that $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ are bounded and prescribed-time stable and $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t) \to 0$ as $t \to t_p$, also $\hat{\Phi}_1$ and $\hat{\Phi}_2$ are bounded. Since $x_1(t) = z_1(t) - z_{d1}(t)$ and $x_3(t) = z_3(t) - z_{d3}(t)$, the desired reference signals tracking is also obtained in prescribed-time t_p. Hence, the upper bound of the settling time can be prescribed. Moreover, $z_1(t)$ and $z_3(t)$ are also bounded since $x_1(t)$ and $x_3(t)$ are bounded and since $z_{d1}(t)$ and $z_{d3}(t)$ are bounded by Assumption 4. From (4.25) and (4.26) it follows that the virtual controls $\alpha_1(t)$ and $\alpha_2(t)$ are also bounded. Similarly, the boundedness of the control inputs can be guaranteed from (4.28) and (4.29). Which completes the proof. ■

4.4 Result discussion

Using MATLAB/Simulink and the Quanser twin rotor helicopter model, the performance, and efficacy of the proposed prescribed-time adaptive backstepping control law are demonstrated in this section. Angles and angular velocities' initial conditions are set to zero, while a fixed sample duration of 0.001s is employed. The values of the design constants are taken as $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 18$, $\Xi_1 = \Xi_2 = 0.1$, and $\Upsilon_2 = \Upsilon_2 = 0.02$.

The simulations are performed considering the desired time of convergence as $t_p = 10s$. Firstly, a constant reference signal with an amplitude of 25° and 25° is applied to the pitch angle and yaw angle, respectively. We also compared our findings to the results of a related study [112] in order to demonstrate the superiority of the designed control scheme. The comparison results are displayed in Figure 4.2-4.3. Figure 4.2 is the evolution of pitch angle under constant reference 25° and Figure 4.3 is the yaw angle under constant reference 25°. From the obtained results, one can see that the proposed control scheme compared to [112] performs better with regard to convergence time. Figure 4.4 illustrates the required control effort generated by the proposed control laws (4.28) and (4.29).

Next, we have examined the tracking properties of the proposed control law under different initial conditions (under the same control parameters). The reference tracking of pitch angle $z_1(t)$ and yaw angle $z_3(t)$ under different initial conditions are shown in Figure 4.5 and Figure

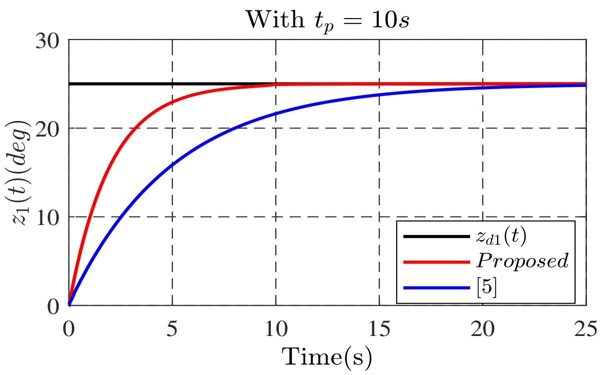

Figure 4.2: Pitch angle $z_1(t)$ under constant reference $z_{d1}(t)$.

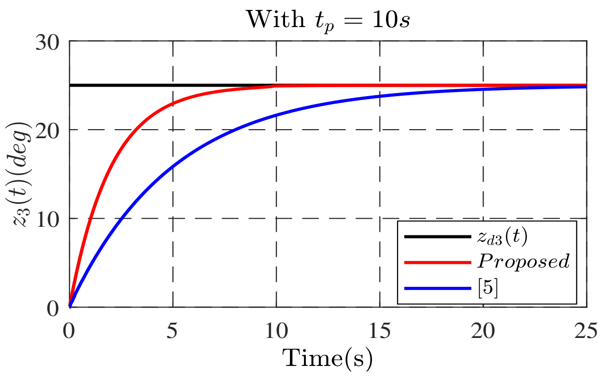

Figure 4.3: Yaw angle $z_3(t)$ under constant reference $z_{d3}(t)$.

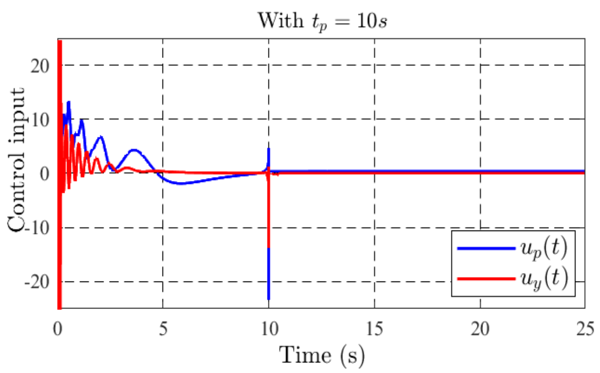

Figure 4.4: Control input.

4.6 respectively, from which one can observe that the convergence time is independent of initial conditions. Furthermore, to evaluate the tracking performance of the proposed control scheme, the time-varying reference signals are taken into account. For this, we take a sinusoidal signal with amplitude 10 and frequency 20Hz. Figures 4.7 and Figure 4.8 show the evolution of the desired pitch and yaw angles, respectively. From the aforementioned results, it is clear that the settling times for all cases are independent of initial conditions. Moreover, the upper bound of the settling time can be prescribed in advance.

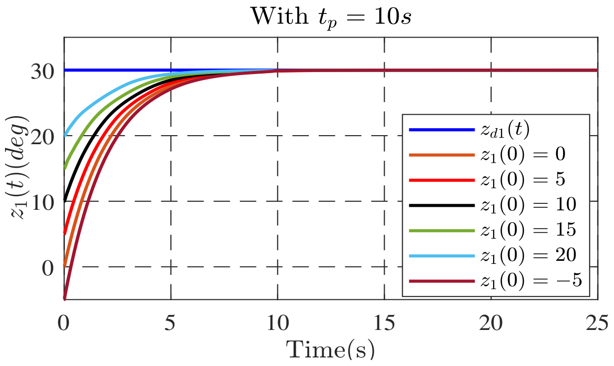

Figure 4.5: Pitch angle $z_1(t)$ with different initial conditions.

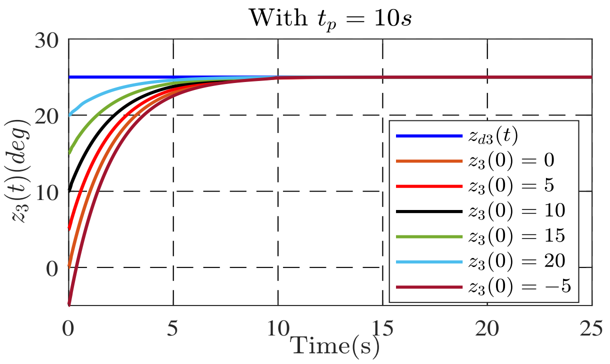

Figure 4.6: Yaw angle $z_3(t)$ with different initial conditions.

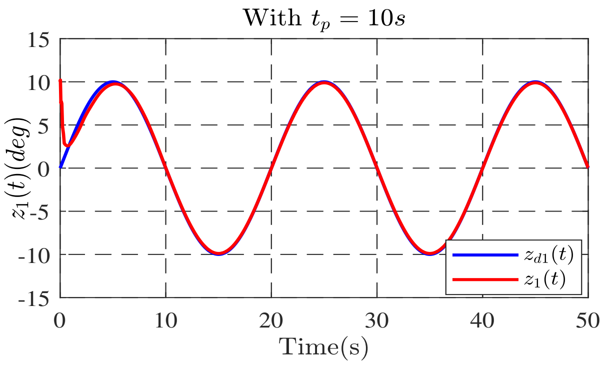

Figure 4.7: Pitch angle $z_1(t)$ under time-varying reference $z_{d1}(t)$.

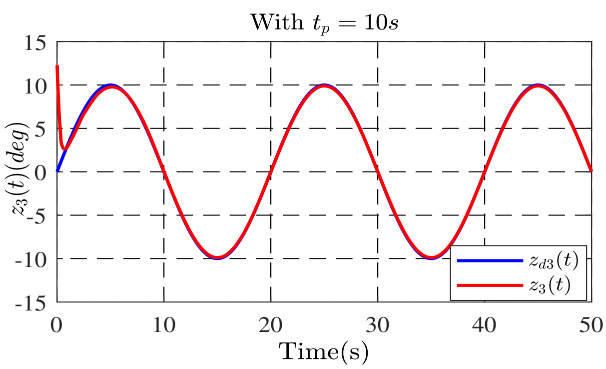

Figure 4.8: Yaw angle $z_3(t)$ under time-varying reference $z_{d3}(t)$.

Remark 4.5 *Please note that the proposed control scheme is valid till the prescribed time* t_p*. Similar to [113], a switched-gain strategy is utilized to maintain the tracking beyond the specified time.*

4.5 Conclusion

This chapter proposes a prescribed-time adaptive controller for an uncertain twin rotor helicopter that tracks pitch and yaw angle reference trajectories in a prescribed time. At first, the Euler-Lagrange equations are utilized to obtain the mathematical model of the twin rotor helicopter. Using Lyapunov theory, a theoretical demonstration of prescribed-time stabilization with adaptive backstepping is provided, guaranteeing the boundedness of all signals in the closed loop system and attitude tracking in the prescribed time. On a twin rotor helicopter, simulations are carried out to show the effectiveness and control potential of the designed control scheme.

Please note that although the proposed control law successfully tracks the desired reference within the prescribed time, there is a concern as the control input value increases near the specified time (see Figure 4.4). This escalation may lead to potential actuator failures in practical real-time applications. The upcoming chapter will concentrate on developing a constrained control scheme. This approach aims to restrict the control input as the specified time approaches, addressing the potential issues highlighted earlier.