CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Carbon nanotubes (CNTs) have emerged as a revolutionary material in the field of composite materials, known for their exceptional mechanical and thermal properties. These unique attributes make CNTs highly suitable for various applications, including aerospace engineering and automotive design. When combined with polymer matrices, CNTs can form composite materials that boast enhanced properties, making them a popular choice for advanced engineering solutions. This chapter delves into the development of CNT-reinforced composite beams resting on Pasternak elastic foundations, employing trigonometric shear deformation theory based on secant functions and the inverse hyperbolic sine function. The literature review focuses on the latest research concerning the structural responses of these composite beam structures. It discusses the various modelling approaches and solution methods used to address the governing equations related to their structural behavior.

Researchers have investigated the structural responses of CNT-reinforced composite beams to understand their behavior under different loading and boundary conditions. These studies have employed various modelling techniques to simulate the mechanical performance and stability of the beams. The chapter provides a comprehensive overview of these approaches, highlighting the strengths and limitations of each method. Moreover, the review explores the different solution methods used to solve the governing equations of the composite beams. These methods are crucial for accurately predicting the structural responses and ensuring the reliability of the composite materials in practical applications. By examining these modelling and solution techniques, the chapter aims to provide a thorough understanding of the current state of research in the field of CNT-reinforced composite materials.

2.2 Evolution of CNTs and their structural behavior

Carbon nanotubes (CNTs) have garnered significant interest from the scientific and engineering communities due to their exceptional properties such as high stiffness, impressive strength, and low density. Discovered by Iijima (Iijima, 1991) in 1991, CNTs are composed of hexagonal cells and exhibit remarkable tensile strength reaching up to 63 GPa, alongside a Young's modulus of approximately 1 TPa. These outstanding characteristics have driven extensive research into understanding and harnessing the potential of CNTs for various applications. In material science, different analytical and numerical methods have been applied to study CNTs and predict the properties of CNT-reinforced composites. The weighted average method has become popular for estimating composite properties like modulus of elasticity, bulk density, tensile strength, thermal conductivity, and electrical conductivity. Additionally, micromechanical models have been widely employed to delve deeper into the mechanics of these composites. Models such as the modified Halpin-Tsai, rule of mixture, modified rule of mixture, extended rule of mixture, Mori-Tanaka, Eshelby–Mori–Tanaka approach, and refined rule of mixtures have all contributed to more accurate analyses of CNT-reinforced composites' mechanical behavior. These models and methods have enabled researchers to gain a detailed understanding of the interaction between CNTs and the surrounding matrix in composites, leading to better predictions of the performance of the material under various conditions. The integration of CNTs into composites holds great promise for revolutionizing numerous industries by offering materials that are stronger, more durable, and capable of delivering unprecedented performance.

2.2.1 Mechanical performance of CNT-Reinforced composite structures

Researchers have undertaken numerous tests to explore the mechanical behavior of carbon nanotube (CNT) reinforced composites. Liu and Chen (2003) identified CNTs as ideal reinforcements for advanced nanocomposites due to their exceptional mechanical properties. Using a 3D nanoscale representative volume element (RVE) with FEM and the extended rule of mixing, they evaluated the effective axial Young's moduli of CNT-based composites. Kreupl et al. (2004) highlighted CNTs' broad application potential, while Yeetsorn (2004 explored their excellent mechanical, thermal, and electrical properties in zigzag and armchair forms. Methods like chemical vapor deposition, arc discharge, and laser ablation were used for CNT production.

Bakshi et al. (2007) conducted tensile tests on multi-walled CNT-reinforced composites produced by electrostatic spraying, showcasing their practical performance. Kulkarni et al. (2010) studied the tensile properties of CNT fiber-reinforced polymer composites, enhancing understanding of how CNTs improve composite material properties. Formica et al. (2010) studied the vibration properties of CNTRCs using a continuous model based on the Eshelby-Mori-Tanaka approach, offering insights into their dynamic behavior. Ma et al. (2010) examined how CNT functionalization, dispersion, surface, and interfacial properties affect the mechanical performance of CNT-based composites. Yu et al. (2011) examined CNT composites produced via the precursor infiltration and pyrolysis (PIP) method, using methyl trichlorosilane in CVD for fiber-matrix interface coating. Mechanical and thermal properties were evaluated through bending tests, single-edge notched beam tests, and the laser flash method, offering insights into CNTs' impact on composites. Lu and Hu (2012) used computational simulations to predict CNT mechanical properties, highlighting atomic arrangement's impact. They developed advanced 3D finite element models for armchair, zigzag, and chiral SWCNTs using molecular mechanics to address experimental challenges.

Odegard et al. (2002) used a representative volume element (RVE) based on continuum mechanics to relate the structural properties of nanostructured materials. Volkov et al. (2012) focused on mesoscopic and atomistic simulations to study the effects of bending and buckling on the thermal conductivity of CNT materials. Grace (2013) examined various types of CNTs, including SWCNTs and multi-walled carbon nanotubes (MWCNTs), along with their geometrical arrangements, such as armchair, chiral, and zigzag configurations. Hu et al. (2013) conducted uniaxial tensile tests to investigate the mechanical properties of CNTs, while Laine et al. (2014) used three-point bending tests to assess the transverse shear stiffness of composite sandwich polymer foam cores. Additionally, ultrasonic testing has been a valuable method for predicting the mechanical behavior of CNT-reinforced composites.

2.2.2 Structural responses of CNT-reinforced composite structure with analytical approach.

Recent research has extensively analyzed the bending, buckling, and free vibration responses of composite beams reinforced with functionalized and uniformly graded carbon nanotube (CNT) material. Wuite and Adali (2005) conducted benchmark research to examine the static responses of CNTRC beams using classical beam theory. Their findings indicated that varying stacking sequences in the components can affect the material properties. Murmu and Pradhan (2009) used Eringen's nonlocal elasticity theory and Timoshenko beam theory to investigate the buckling behavior of nine single-walled CNTs (SWCNTs) embedded in an elastic medium. Thai (2012) explored the buckling, bending, and vibration responses of nanobeams using a nonlocal shear deformation beam theory, while Yas and Samadi (2012) conducted a free vibration analysis of functionally graded (FG) CNT-reinforced composite beams on an elastic foundation with the Timoshenko beam theory. Lin and Xiang (2014) conducted a free vibration analysis on FG CNT-reinforced composite beams using both first-order shear deformation theory (FSDT) and third-order shear deformation theory (TOSDT), and employing the kp-Ritz method for solutions. Mayandi and Jeyaraj (2015) performed a bending analysis on composite beams with FG CNTs under non-uniform thermal loads. Lei et al. (2016) focused on the bending analysis of FG-CNTR composite beams using FSDT, utilizing the kp-Ritz method as the solution technique. Kumar and Srinivas (2017) studied the static and free vibration behavior of FG-CNTRC polymer beams and hybrid laminated composites with CR polymer sheets reinforced by CNTs using numerical analysis. These studies collectively advance the understanding of the mechanical behavior of CNT-reinforced composites, showcasing their potential in various structural applications.

Wattanasakulpong and Ungbhakorn. (2013) investigated the bending, buckling, and free vibration responses of CNTRC beams on an elastic foundation using an analytical solution technique. Fantuzzi et al. (2017) analyzed the free vibration of arbitrarily shaped functionally graded CNT-reinforced composites (FG CNTRC), contributing to the understanding of their dynamic behavior. This research, along with earlier studies understanding the mechanical performance of CNT-reinforced composites, emphasizing the significant influence of CNT distribution and orientation on the structural behavior of these advanced materials. These studies collectively enhance the potential for CNT-reinforced composites in various engineering applications, highlighting their superior stiffness, strength, and dynamic response characteristics.

The studies mentioned focus on various analytical and computational approaches to understanding the mechanical behavior of carbon nanotube (CNT)-based materials under different loading conditions. Popov et al. (2000) utilized a lattice dynamical model to analyze the elastic properties of CNT lattices. Odegard et al. (2003) developed a constitutive model for polymer composites reinforced with CNTs. Chen and Liu (2004) employed a representative volume element (RVE) approach in continuum mechanics to determine the effective mechanical properties of CNT-based composites. Sears et al. (2006) investigated buckling behavior in multi-walled (MWNTs) and single-walled nanotubes (SWNTs) under compressive stresses using molecular dynamics (MD) simulations and compared their findings with continuum models. Vodenitcharova and Zhang (2006) utilized the Airy stress function to study buckling and bending in nano-composite beams reinforced with SWCNTs, highlighting the enhanced load-bearing capacity with increased CNT reinforcement. Sun and Liew (2008) employed higher-order gradient continuum mechanics and meshless techniques to examine the bending and buckling behavior of SWCNTs. These studies collectively contribute to a deeper understanding of how CNTs affect the mechanical properties and structural performance of composite materials at the nanoscale.

Giannopoulos et al. (2008) developed a finite element formulation to compute the mechanical response of zigzag and armchair SWCNTs, calculating shear and Young's modulus across various nanotube radii. Guo et al. (2008) used atomic-scale FEM to analyze the bending and buckling behavior of SWCNTs under various loads. Zhang et al. studied the buckling responses of CNTs using FEM, offering insights into their stability and collapse mechanisms. Mohammedi et al. (2011) used non-local elasticity theory to study the buckling behavior of double-walled CNTs embedded in an elastic medium under axial compression, highlighting the impact of non-local effects on their mechanical response. Shima (2011) examined the mechanical behavior of CNTs under various loads (compression, bending, tension, torsion) and developed a nonlinear model for CNT-based composites. Ayatollahi et al. (2011) used finite element-based molecular mechanics to evaluate the nonlinear behavior of zigzag and armchair SWCNTs under axial, bending, and torsional loads. Yas and Samadi (2012) used the Timoshenko beam theory to analyze the free vibrations and buckling behavior of functionally graded SWCNTs resting on an elastic foundation, exploring their dynamic and stability characteristics. Yan et al. (2012) investigated the buckling behavior of SWCNTs using a moving Kriging interpolation and the higher-order Cauchy-Born rule to predict their mechanical response, focusing on structural stability under various loading conditions.

Ansari et al. (2012) studied the bending behavior of single-walled silicon carbide nanotubes using density functional theory, exploring their structural response under mechanical loads. Shen and Xiang (2013) analyzed the post-buckling behavior of SWCNT-reinforced nanocomposite cylindrical shells under thermomechanical pressure, using von Karman nonlinearity and HSDT shell theories to model their response. Lei et al. (2013) used the element-free kp-Ritz method to analyze the vibration of functionally graded SWCNTs reinforced in matrices, exploring the impact of different distribution types on structural dynamics. Rangel et al. (2013) developed an analytical approach using finite element methodology to characterize the elastic properties of armchair-type SWCNTs, highlighting their superior mechanical performance.

Benjeddou et al. (1997) developed a finite element model to analyze sandwich beams, focusing on extension and shear actuation mechanisms under various loading conditions. Han and Elliott (2007 used energy minimization and molecular dynamics (MD) simulations to study the elastic properties and mechanical response of CNT-reinforced composites at the nanoscale. Shen (2009) applied higher-order shear deformation theory (HSDT) with von Karman nonlinearity to analyze the nonlinear bending of simply-supported FG-CNTRCs under thermo-mechanical loading, focusing on stability and deformation. Shen and Zhang (2010) studied thermal buckling and post-buckling of FG-CNTRCs using a micromechanical model with molecular dynamics, the Eshelby-Mori-Tanaka technique, and the extended rule of mixtures. Zhu et al. (2012) used FSDT-based finite element simulations to analyze the vibration and bending behavior of FG-CNTRCs under mechanical loads.

2.2.3 Structural analysis of carbon nanotube reinforced composite structure with computational methods

Researchers have utilized various computational modeling techniques to explore the effects of carbon nanotube (CNT) reinforcement on the mechanical behavior of nanocomposites. Chen and Liu (2003) employed 3D finite element modeling to predict the effective mechanical response of CNT composites, focusing on understanding their structural integrity and performance under different loading conditions. Jeong et al. (2005) used finite element analysis to study the effect of CNT distribution on nanocomposite mechanical properties, aiming to optimize dispersion for improved strength and stiffness. Della and Shuo (2008) used the Mori-Tanaka approach to model CNT-reinforced ultra-high molecular weight polyethylene, predicting and analyzing their mechanical response under various stresses. Georgantzinos et al. (2009) used finite element modeling to analyze rubber-like materials reinforced with single-walled CNTs, focusing on their impact on the stiffness and elasticity of rubber-based composites. Jiang (2010) used molecular simulations to calculate the cohesive strength of CNT composite interfaces, focusing on bonding and interface properties. Montazeri and Naghdabadi (2010) employed molecular dynamics to study the behavior of CNT composites at the atomic scale, analyzing interactions between CNTs and the matrix under different loads.

Arani et al. (2011) used finite element modeling to analyze the flexural response of CNT-reinforced laminated composites, focusing on the impact of CNTs on bending characteristics and mechanical performance. Farsadi et al. (2012) used 3D modeling to study carbon nanotube composites reinforced with corrugated CNTs, focusing on their structural integrity and mechanical properties under different loading conditions. Malekzadeh and Shojaee (2013) used first-order shear deformation theory to analyze CNT-reinforced quadrilateral laminated sheets, focusing on predicting critical buckling loads and modes under various loading conditions. Moradi Dastjerdi et al. (2013) used a meshless technique to analyze axisymmetric functionally graded nanocomposite cylinders reinforced with wavy SWCNTs, studying how the wavy morphology affects their natural frequencies and dynamic behavior. Shahrbabaki and Alibeigloo (2014) used finite element analysis to study the effect of nanotube distribution and volume fraction on the in-plane frequencies of rectangular beams, aiming to optimize nanotube-reinforced structures for better vibrational performance. Mayandi and Jeyerad (2015) used finite element simulations to model the vibrational, bending, and buckling behavior of functionally graded CNT-reinforced polymer beams under thermal conditions, focusing on the impact of temperature variations on structural performance and stability.

2.2.4 CNT-Reinforced composite structure on elastic foundation

Ajayan et al. (1994) pioneered the research on carbon nanotubes (CNTs) as a structural component, demonstrating their exceptional properties that sparked further investigation into practical applications. This foundational work led subsequent researchers, such as Odegard et al. (2003), Griebal and Hamaekers (2004), and Mokashi et al. (2007), to explore various aspects of CNT-reinforced composites (CNTRCs) for structural enhancement. Yas and Samadi (2012) extended this research using Timoshenko beam theory to analyze the vibration characteristics of CNTRCs supported by Winkler-Pasternak elastic foundations. They studied how different reinforcement distribution patterns, volume fractions, and side-to-thickness ratios impact the vibrational behavior of these composite structures, highlighting the FG-X reinforcement distribution as optimal for achieving higher frequencies. Kutlu and Omurtag (2012) studied the vibration response of angular and elliptical CNTRC plates with elastic foundation sandwiches, focusing on their bending behavior to optimize CNTRC beam design for improved mechanical performance. Wattanasakulpong and Ungbhakorn (2013) conducted static analysis on CNTRC beams using Winkler-Pasternak elastic foundations, exploring how variations in reinforcement distribution, volume fraction, and side-to-thickness ratios affect the structural stiffness. Their findings underscored the effectiveness of the FG-X reinforcement distribution pattern in enhancing stiffness and structural integrity. Shen and Xiang (2013) studied the nonlinear behavior of CNTRCs with Winkler elastic foundations using high-order shear deformation theory (HSDT), focusing on how nonlinear effects impact their mechanical response and stability under various loading conditions.

Carbon nanotube reinforced composite (CNTRC) materials have gained prominence in engineering structures due to their exceptional mechanical and physical properties. Researchers have focused on analyzing these materials, considering factors such as reinforcement distribution patterns, the volume fraction of carbon nanotubes (CNTs), the Winkler spring constant factor, and side-to-thickness ratios, which significantly influence their static and dynamic behavior. Shen and Xiang (2014) studied CNTRC beams with Winkler elastic foundations, analyzing various CNT reinforcement distributions. They found that the FG-A pattern caused the highest transverse deflection, while the FG-X pattern resulted in the lowest, helping to understand how CNT distribution affects deformation and stiffness. Furthermore, Shen and Xiang (2014) studied the impact of temperature on the dynamic behavior of CNTRC shells with Winkler elastic foundations. They found that increasing CNT volume fraction and Winkler spring constant improved the dynamic response, while higher temperatures negatively affected the vibration characteristics, highlighting the role of environmental conditions.

Additionally, Duc et al. (2017) studied the buckling response of conical shells with elastic foundations, extending the understanding of CNTRC structures to complex geometries and loading conditions, and highlighting how shape and foundation characteristics affect buckling behavior. Ke et al. (2010) applied the Ritz method within Timoshenko beam theory to analyze the free vibrations of CNTRC beams. Rafiee et al. (2013) studied the large amplitude vibration of FG-CNTRC beams with piezoelectric layers using a physical neutral surface concept. Ke et al. (2013) conducted free vibration and stability analysis for FG-CNTRCs.

Ansari et al. (2014) investigated the vibration behavior of FG-CNTRC beams under forced conditions. Nejati et al. (2016) analytically investigated the buckling and vibration of FG-CNTRC beams under axial loading. Yang et al. (2015) focused on the thermal and voltage-induced buckling of slender FG-CNTRC beams, while Mirzaei and Kiani (2015) examined thermal buckling in sandwich beams with FG-CNTRC face sheets. Wu et al. (2015) explored the vibration and buckling of sandwich beams with a stiff core and FG-CNTRC face sheets. Wu et al. (2016) used the Newton-Raphson adaptive technique and first-order shear deformation theory for post-buckling analysis of CNTRC beams. Further, Kiani (2016) used the Ritz technique to analyze the thermal post-buckling response of a sandwich beam made up of CNTRC surface sheets and a rigid mass at its center. Mirzaei and Kiani (2016) studied the vibration response of a sandwich beam with a CNTRC-facing layer, using the Ritz technique and Hamilton's principle to analyze its dynamic behavior. Shi et al. (2017) developed a semi-analytical method using the Fourier technique and FSDT for analyzing the free vibration of FG-CNTRC beams. Ebrahimi and Farazamandnia (2017, 2018) applied this approach to study the thermo-mechanical behavior of sandwich FG-CNTRC beams. Ghorbani Shenas et al. (2017) proposed an HSDT-based Ritz method for analyzing the free vibration of twisted FG-CNTRC beams under thermal conditions. Kiani and Mirzaei (2019) explored the nonlinear stability of sandwich beams with FG-CNTRC face sheets under thermal loading and elastic foundation conditions.

Mohseni and Shakouri (2019) later examined free vibration and buckling in FG-CNTRC beams with varying thicknesses on elastic foundations. More recently, Talebizadehsardari et al. (2020) investigated the bending behavior of FG-CNTRC curved beams using FSDT combined with non-local strain gradient theory (NSGT). Xu (2021) explored how the distribution and orientation of carbon nanotubes (CNTs) impact the behavior of functionally graded (FG) nanocomposite beams using First-Order Shear Deformation

Theory (FSDT) and two-node finite elements. Fan and Wang (2015, 2016 (a, b)) investigated the nonlinear bending and buckling of FG-CNT-reinforced cracked beams with embedded piezoelectric layers, utilizing Higher-Order Shear Deformation Theory (HSDT). Shen et al. (2013, 2015, 2017, 2020) performed a nonlinear analysis of FG-CNT-reinforced composite beams subjected to thermal effects and supported on an elastic foundation, employing a micromechanical model with HSDT. Tagrara et al. (2015) introduced a new trigonometric SDT for analyzing bending and buckling in FG-CNT-reinforced beams. Salami (2016, 2018) researched the bending and free vibration of sandwich beams with soft cores and FG-CNT-reinforced face sheets, applying an extended higher-order panel theory. Pouresmaeeli and Fazelzadeh (2017) used the Galerkin method to analyze the buckling of FG-CNT-reinforced beams with various imperfections. Mohammadimehr and Alimirzaei (2017) numerically investigated the buckling behavior of functionally graded carbon nanotube reinforced composite (FG-CNTRC) microbeams using Reddy's Higher-Order Shear Deformation Theory (HSDT). Concurrently, Alibeigloo and Liew (2015) developed an elasticity-based solution using the Differential Quadrature Method (DQM) to analyze the bending and free vibration of FG-CNTRC beams with piezoelectric layers. Later, Wu et al. (2017) also conducted a thermal post-buckling analysis of imperfect FG-CNTRC beams using FSDT. Ebrahimi and Farazmandnia (2017) conducted a thermomechanical vibration analysis of sandwich beams with FG-CNTRC face sheets, applying HSDT. Chaudhari et al. (2017) developed a stochastic HSDT-based model for the nonlinear bending analysis of CNTRC beams under thermal conditions. Additionally, Khelifa et al. (2018) examined the buckling of simply supported FG-CNTRC beams on an elastic foundation using a refined shear deformation beam theory that accounts for stretching effects.

Mohammadimehr et al. (2018) compared different theories, including Classical Plate Theory (CPT), Third-Order Beam Theory (TBT), and Reddy's Higher-Order Shear

Deformation Theory (HSDT), to assess the bending, buckling, and free vibration responses of FG-CNTRC beams. Daikh et al. (2020) introduced a new hyperbolic shear deformation theory to explore the static behavior of simply supported cross-ply CNT-reinforced composite nanobeams under various loading conditions. More recently, Karami et al. (2019) applied hyperbolic shear deformation theory to investigate the non-local buckling of FG-CNTRC curved beams. Taati et al. (2020) published findings on the free vibration behavior of FG-CNTRC beams using a nonlocal theory based on FSDT. Keshtegar et al. (2021) analyzed the dynamic stability of CNT-reinforced hybrid nanocomposite beams using Extended Shear Deformation Theory (ESDT), deriving governing equations with the Differential Quadrature Method (DQM) and Bolotin's method. Civalek et al. (2021) explored various shear deformation theories for the free vibration analysis of CNTRC micro-beams. Hadji et al. (2022) developed a new refined beam theory for analyzing the free vibration of carbon nanotube reinforced composite beams under different boundary conditions. Karamanli and Vo (2021) numerically examined the bending, vibration, and buckling of CNTRC and graphene nanoplatelet-reinforced composite (GPLRC) beams using a finite element model based on Shear and Normal Deformation Theory (SNDT). Garg et al. (2022) conducted a bending and free vibration analysis of symmetric and asymmetric functionally graded CNT-reinforced sandwich beams with a soft core, using a higher-order zigzag theory.

Research findings indicate that arranging carbon nanotubes (CNTs) in a functionally graded manner across the composite beam, known as the FG-X reinforcement pattern, helps minimize transverse deflection during bending. Furthermore, authors have found that employing the Inverse Hyperbolic Shear Deformation Theory (IHSDT) by Grover et al. (2013) proves effective in accurately predicting how the composite beam responds both statically and dynamically. These insights underscore the practical benefits of optimizing CNT distribution and using advanced theoretical models to enhance the performance and reliability of composite structures.

2.3. Development of beam theories

Beam theory is a key part of structural analysis, helping engineers predict how structures like beams will behave under different loads. The development of beam theory has taken place over many centuries, with contributions from various scientists and engineers who have improved our understanding of how beams bend, deform, and support weight. Timoshenko (1921) advanced the Euler-Bernoulli theory by incorporating shear deformation and rotational effects, enhancing its accuracy for shorter, thicker beams. As the understanding of material strength progressed, more sophisticated methods emerged to analyze beams under complex loading conditions, accounting for material variations and plastic deformation.

The Finite Element Method (FEM) was developed alongside advances in computer technology, providing a powerful numerical approach for analyzing complex structures. FEM enables engineers to study beams with diverse shapes and loading conditions, including scenarios where traditional beam theory is insufficient.

Beam theory has evolved through ongoing research, refining models and exploring new approaches for improved accuracy and applicability. Modern computational methods, including artificial intelligence and machine learning, offer promising tools to enhance beam analysis and design, enabling more precise and efficient solutions. Additionally, integrating beam theory with materials science and optimization methods allows engineers to better understand material behavior and optimize beam designs for improved performance and durability.

In structural analysis, deriving the governing equation is essential for understanding the behavior of beams and shell structures under bending, vibration, and buckling. Typically expressed as partial differential equations (PDEs) or ordinary differential equations (ODEs), this equation mathematically describes how structures deform in response to external forces. An exact analytical formulation of the governing equation involves no assumptions and accurately represents physical phenomena. In contrast, an approximate formulation introduces simplifications to make the model computationally feasible, but may introduce numerical errors due to these assumptions. Once the governing equation is established, the next step is to find solutions. Closed-form analytical solutions are exact solutions derived directly from the governing equation and boundary conditions. They provide precise results without additional error, assuming the boundary conditions are accurately defined and the assumptions in the model hold true across the entire domain of interest. In contrast, numerical solutions involve computational techniques to approximate solutions, especially for complex or irregular geometries where exact analytical solutions may not be feasible. While numerical methods are versatile and can handle a wide range of scenarios, they inherently introduce discretization errors due to their approximation nature. These errors can be controlled and minimized through appropriate mesh refinement and convergence criteria during the numerical solution process.

In recent years, the use of composite materials, which have different properties in different directions, has led to further refinement of beam theory. Nonlinear analysis, which deals with large deformations and non-elastic behavior, is another area of ongoing research. Additionally, the integration of smart materials that can change their properties in response to environmental conditions has opened new possibilities for beam theory, especially in structures that can adjust their shape or stiffness in real-time.

In numerical solutions methodology, results are obtained at discrete points, introducing errors due to interpolation between these points. This contrasts with analytical solutions, which provide continuous solutions without interpolation errors. However, numerical methods are favored for their practicality and efficiency, especially when closed-form analytical solutions are challenging or impossible to obtain. Literature on composite beams, beams, shell structures, and carbon nanotube reinforced composites (CNTRC) extensively covers the derivation of governing equations for bending, vibration, and buckling. These equations are formulated as partial differential equations (PDEs) or ordinary differential equations (ODEs), depending on the physical phenomena being studied. The second part of the literature review focuses on solution methodologies. It begins with elasticity solutions, which involve deriving solutions directly from the governing equations using analytical techniques. These solutions are exact and do not introduce additional errors, assuming accurate boundary conditions and idealized assumptions. Modelling beams using beam theories is another key aspect discussed in the literature. Beam theories provide simplified mathematical models that describe the behavior of beams under various loading conditions.

2.3.1 Modelling of beams

Elasticity solutions for beams, plates, and shells begin with 2D equilibrium equations, which are converted into strains and then displacements, resulting in six unknowns for analysis. Saint-Venant (1856) provided an exact solution for an anisotropic cantilever beam. Silverman (1964), Hasin (1967), Gerstner (1968), Rao and Ghosh (1979), and Cheng et al. (1989) developed exact solutions for shear deformable laminated composite beams, while Smith (1986) studied buckling and vibrations in orthotropic beams. Dischinger (1932), Cheng and Pei (1954), Herrman (1964), Iyengar and Prabhakara (1968), Pagano (1969, 1970), and Esendemir et al. (2006) contributed to solutions for continuous beams and composite plates. Kardomateas (1995, 2001) derived 3D solutions for transversely isotropic rods, and Chen et al. (2003, 2004) developed elasticity solutions for free vibration in laminated beams.

In the realm of structural mechanics, beam theories provide insight into the deformation behavior of slender structures subjected to external loads. Consider a beam a fundamental structural element. When forces act upon it, it undergoes bending and torsion. Our analysis focuses on the x-coordinate (along the length of beam) and the z-coordinate (through its thickness). The displacements (*u, v, w*) are functions of these coordinates, with *v* being negligible. Essentially, beam theories allow us to anticipate beam responses across various scenarios. In beam theories, such as classical beam theory (CBT), first-order shear deformation theory (FSDT), and higher-order theories, the primary focus is on the deformation modes occurring within the midplane of the beam. These deformation modes are expressed as mathematical functions of the thickness coordinate, and they represent the essential unknowns in the analysis. By defining these deformation modes at the midplane, typically denoted as (*U* and W) for the in-plane displacements, the behavior of the beam can be effectively described. These functions capture how the beam deforms under various loading conditions and constraints. Once these deformation modes are determined, the stresses and strains throughout the thickness of the beam can be computed. This process allows engineers to understand how forces and moments applied to the beam translate into internal stresses and deformations, aiding in the design and analysis of structural components.

2.3.1.1 Beam theories

Beams are essential structural elements used in bridges, buildings, aircraft wings, and countless other applications. Beam theory provides a framework for understanding their behavior under various loads. The development of beam theory generally involves the following typical assumptions.

- The internal displacements (*U*) in a beam are assumed to vary linearly with its thickness. This means the displacements at any arbitrary point *P* can be described in terms of a constant stress mode at a specific depth *z* from the midplane, combined with a rotational mode at the midplane. This rotational mode is related to a linear function of the thickness coordinate (*z*), allowing for a simplified mathematical representation of the deformation of the beam.

- The transverse displacement (W) remains constant through the thickness of the beam, implying that the length of a normal cross-section does not change in the thickness direction. This assumption simplifies the analysis by ensuring that the deformation in the thickness direction is uniform, allowing the focus to remain on the in-plane and bending deformations.

Using these assumptions, we can express the displacement components as follows.

$$U(x, z, t) = u_0(x, t) + z\theta_x(x, t)$$
$$W(x, z, t) = w_0(x, t) \qquad 2.1$$

Where u_0, w_0, and θ_x represent the displacement modes and rotational deformations at the midplane. The equation in Eq. (2.1) describes the deformation profile of the first-order beam theory, which will be elaborated upon in the following sections.

2.3.1.2 Classical Beam Theory

Classical Beam Theory (CBT) (Bernoulli and Euler, 1744), is a cornerstone in beam bending analysis. Here cross-sections initially perpendicular to the neutral axis remain plane and perpendicular after deformation. The theory, which neglects shear deformation and rotary inertia, is accurate for thin beams but less so for thicker ones. Rayleigh enhanced CBT by incorporating rotary inertia. The development of beam theory began with Galileo (1638) and evolved through contributions from Barre de Saint Venant (1856), Love (1944), and Timoshenko (1953). Researchers like Krieger (1950), Burgreen (1951,1958), Eringen (1952), and McDonald (1955) addressed nonlinear vibration in beams, while Boley (1963) examined CBT's accuracy for variable-section beams. Rissone and Williams (1965) studied nonuniform cantilever beam vibrations, and Srinivasan (1965, 1966) used the Ritz-Galerkin method for vibration analysis of simply supported beams. Later advancements included studies by Evensen (1968) on boundary conditions, Bennett and Eisley (1970) on large-amplitude beam motion, and MacBain and Genin (1973) on support flexibility effects. Barbero et al. (1993) explored buckling in composite beams, while Sherbourne and Kabir (1995) investigated lateral buckling in fibrous composites. Kim and Kim (2001) used Fourier series to analyze beams with general restraints, and Boay and Wee (2008) examined the coupling effects in laminated composite beams.

2.3.1.3 First Order Shear Deformation Theory

The First Order Shear Deformation Theory (FSDT) builds on Classical Beam Theory (CBT) but relaxes the assumption that transverse normals remain perpendicular to the midplane during deformation. By allowing transverse shear deformations, FSDT provides a more accurate representation of thick beams. Researchers such as Timoshenko (1921), Dengler and Goland (1951), Anderson (1953), Mindlin and Deresiewicz (1954), Huang (1961), Cowper (1966), Whitney and Pagano (1970), Murty (1970), Aalami and Atzori (1974), Hutchinson (1981), Mucichescu (1984), Librescu and Reddy (1987), Rychter (1988), Renton (1991), Raman and Davalos (1996), Gruttmann and Wagner (2001), Reddy (2004), Kocaturk and Simsek (2005), Mechab et al. (2008), Kennedy et al. (2011), have used FSDT or Timoshenko beam theory (TBT) for the structural analysis of beams, benefiting from its improved accuracy over CBT for thick beam structures.

2.3.1.4 Higher-Order Shear Deformation Theories

Higher-order Shear Deformation Theories (HSDTs) are advanced formulations that enhance Classical Beam Theory (CBT) and First Order Shear Deformation Theory (FSDT). They aim to balance the mathematical intricacies required for precise responses from 2D formulations with the limitations of CBT and FSDT in accurately modeling isotropic and traditional composite beam structures. HSDTs incorporate higher-order terms in the displacement field equations, providing a more detailed representation of the transverse shear strains and stresses, thus offering improved accuracy for both thin and thick beam structures without the full complexity of 2D analysis.

In the literature, Higher-Order Shear Deformation Theories (HSDTs) are classified into polynomial-based higher-order theories (PHSDTs) and non-polynomial-based higher-order theories (NHSDTs). The primary distinction between these two classes lies in their use of polynomial and non-polynomial mathematical functions to describe the displacement components in relation to deformation modes. Researchers such as Bickford (1982), Levinson (1985), Petrolito (1995), Eisenberger (2003), Aydogdu (2009), Ghugal (2010), Sayyad (2011, 2012), Carrera et al. (2011, 2012, 2012), Ghugal and Dahake (2013) have employed HSDTs for structural analysis of beam structures.

The non-polynomial-based higher-order shear deformation theory (HSDT) incorporates various non-polynomial shear strain functions, such as secant, sine, and tangent functions, to capture the non-linearity of transverse shear stresses through the thickness, while using fewer field variables compared to traditional HSDTs, which typically rely on polynomial functions. The First-Order Shear Deformation Theory (FSDT) lacks the necessary deformation modes to accurately model thick CNT-reinforced sandwich beams and is generally used for thin beams where shear deformation is not significant. Although polynomial-based HSDTs include higher-order deformation modes (such as membrane and bending), they require a large number of higher-order terms, which increases computational costs. In contrast, trigonometric shear deformation theory accounts for the non-linearity of shear deformation using a single non-polynomial function, the secant function, within the kinematic field, leading to more efficient results with reduced computational effort. Moreover, the non-polynomial-based HSDT naturally satisfies the traction-free conditions for transverse shear stresses at the top and bottom surfaces of the beam. In contrast, most polynomial-based HSDTs do not inherently consider these conditions, and they are sometimes artificially enforced.

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_0}{\partial x}(x,t) + f(z)\theta_x(x,t) \quad 2.2a$$

$$w(x,z,t) = w_0(x,t) \quad 2.2b$$

in which $f(z) = g(z) + \Omega z$.

2.3.2 Advanced beam theories for modeling of multi-layered structures

The subsequent section presents an extensive review of the literature concerning the modeling of multi-layered composite beam behavior.

2.3.2.1 Equivalent Single Layer (ESL) approach

The Equivalent Single Layer (ESL) approach offers a streamlined method to extend single-layered beam theories to more complex multi-layered structures without introducing excessive computational complexities. In ESL, the number of primary variables required remains constant and is equal to the total number of field variables in the original kinematic model. This consistency helps maintain computational efficiency while ensuring that the essential characteristics of each layer, such as displacement and strain fields, are accurately represented.

Equivalent Single Layer (ESL) theories stem from the method of hypotheses. In these displacement-based theories, a single expansion for each displacement component is employed across the entire thickness of the laminate. ESL theories find widespread use in analyzing the bending behavior of simply supported isotropic, laminated composite, and sandwich beams. These theories fall into different categories, including classical beam theory, first-order shear deformation theory, second and third-order shear deformation theories (based on Taylor series expansion), and higher-order shear deformation theories that incorporate functions like parabolic, trigonometric, hyperbolic, exponential, and mixed forms in the displacement field.

In the study of laminated composite and sandwich beams, researchers such as Reddy (1984), Khdeir and Reddy (1997), Soldatos and Watson (1997), Zenkour (1999), Mistou et al.

(1999), Naik (2000), Reddy et al. (2001), Matsunaga (2001), Arya et al. (2002), Liu and Soldatos (2002), Ferreira et al. (2004), Kroker and Becker (2010), Zenkour et al. (2010 a, b), Sayyad and Ghugal (2011, 2015), Vo and Thai (2012), Sayyad et al. (2014, 2015), Cernescu and Romanoff (2015), El-Nady and Negm (2012) and Pawar et al. (2015). Nazargah et al. (2015), Ghugal and Shikhare (2015), and Nguyen and Nguyen (2015) have worked on displacement-based higher-order shear deformation theories. These theories incorporate trigonometric functions in terms of the thickness coordinate to account for the effects of transverse shear and normal deformations in the bending analysis of laminated composite and sandwich beams.

Researchers such as Banerjee and William (1995), Banerjee (1998, 2001), Banerjee and Sobey (2005), Howson and Zare (2005), Banerjee et al. (2007) and Damanpack and Khalili (2012) have devised an exact dynamic stiffness matrix method for analyzing the vibration behavior of composite and sandwich beams. Krishna Murty (1985), Soldatos and Elishakoff (1992), Zenkour (1997), Aydogdu (2009), have developed higher-order shear deformation theories for the free vibration analysis of laminated composite and sandwich beams. These theories incorporate either polynomial or non-polynomial shape functions to account for transverse shear and normal deformations. The investigations involve both analytical and numerical methods. Additionally, Abramovich (1992), Abramovich and Livshits (1994), Abramovich et al. (1996), Aydogdu (2006, 2007), Vo and Inam (2012), Giunta et al. (2014), Smyczynski and Blandzi (2015) have studied the free vibration and buckling behavior of symmetrically and anti-symmetrically laminated composite beams.

2.3.2.2 Layer Wise (LW) approach

In the Layer Wise (LW) approach for analyzing multi-layered laminated composite beams, each layer is treated independently with its kinematic field expansions. These expansions can take various forms, such as Classical Laminate Beam Theory (CLBT), First-order Shear Deformation Theory (FSDT), and Higher-order Shear Deformation Theories (HSDTs), which may be polynomial or non-polynomial in nature. This method allows for a detailed representation of the displacement and strain fields within each discrete layer of the composite beam. Researchers Krajcinovic (1972), Swift and Heller (1974), Davalos et al. (1994), Aitharaju and Averill (1999), Shimpi and Ghugal (1999,2001), Arya et al. (2002), Shimpi and Ainapure (2002), Tahani (2007), Ghugal and Shinde (2013), Afshin and Behrooz (2015) have employed the LW approach to investigate and analyze the structural behavior of such beam configurations.

2.3.2.3 Zigzag (ZZ) approach

The ZZ approach in structural analysis, particularly for multi-layered laminated composite beams, diverges from traditional ESL methods like LW by introducing additional unknowns at the interfaces of the layers. These unknowns are incorporated into the 3D displacement components using a piecewise linear interpolation along the thickness coordinate. This method effectively generates discontinuities in transverse shear strains, which are crucial for accurately modeling the behavior of such composite structures.

Carrera's (2003) historical review explored zig-zag theories used in analyzing multilayered laminated structures. Interestingly, Lekhnitskii (1935) was the first to apply zig-zag theory to solve an elasticity problem related to layered beams. Subsequently, researchers like Xavier et al. (1994), Cook and Tessler (1998), Aitharaju and Averill (1999), Icardi (1999, 2001(a, b), 2003), Di Sciuva and Icardi (2001), Di Sciuva et al. (2001), Arya et al. (2002), Arya (2003), Kapuria et al. (2003 (a, b), 2004 (a, b, c, d)), Kapuria and Alam (2005), Vidal and Polit (2006), Tessler et al. (2007 (a, b), 2009), Zhen and Wanji (2008), Di Sciuva et al. (2010), Chakrabarti et al. (2011), Youzera et al. (2012), Iurlaro et al. (2015) and Tessler (2015) developed higher-order zig-zag approach to analyze and understand the mechanics laminated composite and sandwich beams.

2.4 Solution schemes

In this section, various analytical solution techniques used to solve the governing equations of beam analysis are discussed. Among these, the Navier-based analytical method stands out for its ability to provide exact solutions for beam structures under diaphragm-supported boundary conditions. This method has been widely adopted in research, as evidenced by studies such as those by Mistou et al. (1999), Naik (2000), Sayyad (2011,2012), Giunta et al. (2011, 2013), Vo and Thai (2012), Ghugal and Shikhare (2015), Nguyen and Nguyen (2015), Kulkarni et al. (2015), Punera and Kant (2017), Avcar and Mohammed (2018), Singh and Sahoo (2020), Lezgy-Nazargah (2020), Turan and Kahya (2021), and Nguyen et al. (2022) which apply Navier's solution to analyze static and dynamic behaviors of multilayered composites, functionally graded materials (FG), and carbon nanotube (CNT)-reinforced beam, plate and shells. Navier's solution, however, is limited to scenarios with diaphragm-supported boundary conditions. Other analytical methods are also prominent in the literature, each tailored to handle different boundary conditions and complexities. For instance, the Galerkin method, as explored by Srinivasan (1965, 1966), Apetre et al. (2008), Ansari et al. (2014), Karami et al. (2019), and Daikh and Zenkour (2020), leverages a weighted residual approach to derive solutions for diverse boundary conditions. Similarly, the power series solution method, as studied by Shariyat and Alipour (2013), and the Ritz method, investigated by Aydogdu (2005), Ke et al. (2010), Jedari (2016), Wu et al. (2016), and Nguyen et al. (2017), are effective for obtaining analytical solutions under various boundary conditions. These methods often involve assuming specific forms of boundary conditions in one direction, transforming the partial differential equations (PDEs) into higher-order ordinary differential equations (ODEs) for the primary variables. Subsequently, a state-space approach is utilized to convert these higher-order ODEs into a system of first-order ODEs, which are then solved to obtain the desired structural responses. This systematic approach allows researchers to accurately predict the static and dynamic behaviors of complex composite structures using rigorous mathematical frameworks tailored to specific boundary conditions and structural configurations.

Numerical methods are essential for solving the complex governing equations that describe the behavior of beams, plate, and shell structures in structural analysis. These methods aim to transform the continuous partial differential equations (PDEs) or ordinary differential equations (ODEs) into a set of algebraic equations, making computation feasible. This transformation involves discretizing the domain and making approximations that convert the infinite-dimensional solution space of differential equations into a finite-dimensional space of algebraic equations. The finite element method (FEM) is particularly prominent in engineering and scientific disciplines for its versatility and effectiveness. In FEM, the domain of the structure is divided into smaller subdomains or elements. Within each element, the field variables (such as displacements or stresses) are approximated using polynomial shape functions and nodal coordinates. These shape functions capture the variation of the field variables within the element. To apply FEM, the strong form of the governing equations, which includes equilibrium equations and constitutive relations, is first transformed into an equivalent weak form. This weak form is derived using principles like the principle of virtual work or weighted residuals. The approximate solutions, defined by the polynomial shape functions and nodal values, are then substituted into this weak form to derive elemental equations for each element. Once elemental equations are obtained for all elements, they are assembled to form a global system of equations that represents the entire structure. This global system accounts for interactions between elements and across the entire structure, providing a discretized representation of the original continuous problem. Solving this global system yields the values of primary variables (such as displacements and stresses) throughout the structure under specified loading and boundary conditions.

In structural analysis, advanced numerical methods like the Extended Finite Element Method (XFEM) and Discrete Singular Convolution (DSC) are used to address challenges such as crack discontinuities and dynamic responses. XFEM enhances FEM by incorporating enrichment functions near cracks, allowing for accurate predictions without mesh refinement during crack propagation. It has been successfully applied to cracked composite and functionally graded beams. DSC approximates derivatives at grid points and has been used to analyze the free vibration of laminated composites and FG beams, offering improved flexibility and accuracy. These methods provide enhanced predictive capabilities for complex structural behaviors.

2.5 Key insights from the literature review

This literature review provides a comprehensive evaluation of carbon nanotube (CNT) reinforced composite beams, focusing on their manufacturing processes, mechanical properties, interface characteristics, and potential applications. The analysis underscores significant advancements in the mechanical strength, stiffness, thermal conductivity, and electrical conductivity offered by CNT-reinforced composites, positioning them favorably for diverse industrial applications. However, several challenges remain, including achieving uniform dispersion of nanotubes, ensuring robust interfacial bonding, addressing scalability issues, and optimizing cost-effectiveness. Moving forward, future research needs to prioritize standardizing production methods, assessing environmental and health impacts, exploring synergies with other nanomaterials, and assessing the economic viability of CNT-reinforced composite beams. These steps are crucial to fully exploit the potential of these materials in practical applications and to pave the way for their widespread adoption across industries. Overall, the critical insights gleaned from this literature review provide a solid foundation for advancing the development of novel, high-performance composite materials, guiding researchers towards addressing existing challenges and exploring new frontiers in CNT-reinforced composites.

2.6 Motivation and literature gap

It is observed from the detailed literature review that the displacement-based shear deformation theories are broadly classified into Equivalent single layer theory (ESLT), Layer wise theory (LWT), and Zigzag theory (ZZT). In ESLT, the laminate is treated as a single equivalent anisotropic layer. ESLT further breaks down into classical beam theory (CBT), first-order shear deformation theory (FSDT), and higher-order shear deformation theories (HSDT). CBT, based on Kirchoff's assumptions, produces inaccurate results by neglecting transverse shear deformation. To address this limitation, FSDT was introduced, incorporating a shear correction factor. However, FSDT can also yield inaccurate predictions due to the dependence of the shear correction factor on material properties and boundary conditions. To overcome these challenges, HSDT was introduced, eliminating the need for shear correction factors. The higher-order shear deformation theory (HSDT) can be further classified into Polynomial Shear Deformation Theory (PSDT) and Non-Polynomial Shear Deformation Theory (NPSDT). In PSDT, the displacement field undergoes variation through Taylor's series expansion, whereas NPSDT is a recently developed theory that incorporates a non-polynomial shear strain function.

An extensive literature review reveals a gap in research regarding the structural analysis of functionally graded carbon nanotube reinforced composite beams using non-polynomial shear deformation theory based on secant function and inverse hyperbolic sine functions. This study addresses that gap by employing a trigonometric shear deformation theory which utilizes secant and inverse hyperbolic sine functions to represent the nonlinearity of transverse shear stresses through the beam thickness. This approach offers a distinct advantage by using fewer field variables compared to existing higher-order shear deformation theories (HSDTs), which are typically polynomials. The first-order shear deformation theory (FSDT) is adequate for thick beams where shear deformation is minimal but is inadequate for thick carbon nanotube reinforced composite beams due to its lack of necessary deformation modes. In contrast, traditional HSDTs require numerous higher-order terms to accurately model membrane and bending deformation modes, leading to increased computational costs. The trigonometric shear deformation theory efficiently models the nonlinearity of shear deformation using a single non-polynomial function, the secant function, within the kinematic field. This approach yields more efficient results at a lower computational cost. Additionally, it naturally satisfies the traction-free conditions of transverse shear stresses at the top and bottom surfaces of the beam, a feature that the most polynomial-based higher-order shear deformation theories (HSDTs) often fail to account for or must enforce artificially.

In this study, the trigonometric shear deformation theory is employed to model functionally graded carbon nanotube reinforced composite beams. The structural responses are evaluated using analytical approaches. Various carbon nanotube reinforced distributions are considered, including uniformly distributed (UD) and three types of functionally graded (FG) distributions: FG-O, FG-X, and FG-V. The analytical approach utilizes Hamilton principle to derive governing differential equations, which are then solved using Navier's solution technique. The analytical approach is employed to determine the deflections, stresses, buckling load, and natural frequency of functionally graded carbon nanotube reinforced composite beams. These analyses are conducted for various span thickness ratios, carbon nanotube distributions, and loading conditions. The ESL models show discontinuous stress values at interfaces because of the through-thickness variations of transverse shear stresses, which do not align with the 2D variations of these stresses. This discrepancy arises because ESL models use global functions of the thickness coordinate for the entire thickness of laminated composite beams in the kinematic expansions. Consequently, the constitutive relations predict the transverse shear stresses.

Understanding the structural analysis of advanced composite beams on an elastic foundation is crucial for grasping the complex soil-structure interaction. This analysis is essential for structures such as retaining walls, building foundations, bridges, roads and railroads, which depend on the elastic medium supporting them. The behavior of these structures under various loads is influenced by the supporting elastic foundation. Therefore, comprehending soil-structure interaction is vital for enhancing structural safety and ensuring reliable design.

Based on the literature review, the bending, buckling, and free vibration responses of functionally graded carbon nanotube reinforced composite beam structures on Pasternak elastic foundation have not yet been modeled using the trigonometric shear deformation theory, which employs secant and inverse hyperbolic sine functions. The current study investigates the bending, buckling, and free vibration responses of functionally graded carbon nanotube reinforced composite beam structures on Pasternak elastic foundation, using a trigonometric shear deformation theory that incorporates secant and inverse hyperbolic sine functions. The analysis considers various factors such as the reinforcement distribution patterns of CNTs through the beam thickness, the volume fraction of CNTs, the Winkler spring constant factor, the shear layer constant factor, and the length-to-thickness ratio. The mathematical model and results presented are validated numerically by comparison with previously published results in the literature.

2.7. Objective and scope of the present work

Based on the observations from the previous section, the present work aims to accomplish the following key objective:

Objective

The objective of this study is to develop new non-polynomial shear deformation theories based on trigonometric functions such as the inverse hyperbolic sine function and the secant function and to implement the analytical solution for the structural analysis of functionally graded carbon nanotube reinforced composite beams supported by Pasternak elastic foundation.

Scope

- To develop an analytical model employing non-polynomial shear deformation theories that utilize trigonometric functions such as the inverse hyperbolic sine function and the secant function for the functionally graded carbon nanotube reinforced composite beams.
- To explore the structural analysis i.e., bending, free vibration, and buckling behavior of functionally graded carbon nanotube reinforced composite beams based on different span-to-thickness ratios, various loading conditions, CNT volume fractions and distributions, as well as the Winkler spring and shear layer constants. This investigation will use an analytical approach based on the previously mentioned non-polynomial mathematical models.
- To develop an analytical model incorporating non-polynomial shear deformation theories specifically the inverse hyperbolic sine function and secant function for analyzing functionally graded carbon nanotube reinforced composite beams resting on Pasternak elastic foundation. Additionally, the model for the elastic foundation will consider both horizontal and vertical stiffness.
- To examine the structural responses i.e., bending, free vibration, and buckling responses of functionally graded carbon nanotube reinforced composite beams resting on a Pasternak elastic foundation.

2.8 Summary

This chapter deals with an extensive literature review on the application of trigonometric shear deformation theories, particularly those based on secant and inverse hyperbolic sine functions, to develop CNT-reinforced composite beams resting on Pasternak elastic foundation. It provides a thorough exploration of carbon nanotubes (CNTs) and the various composite beams developed using them, focusing on the mechanical properties of CNTs and their interactions with polymer matrices and other composite materials. The review covers different methods for developing CNT-reinforced composite beams, examining the advantages and disadvantages of each, with an emphasis on higher-order shear deformation theory. Additionally, it discusses the types of Pasternak elastic foundations and their influence on the structural behavior of these composite beams. The analytical approach is used to characterize the behavior of CNT-reinforced composite beams are analyzed, highlighting their effectiveness and limitations. The review also touches on the broad range of applications for these beams, assessing their potential for future use and identifying the challenges that need to be addressed. In conclusion, the chapter points out several promising areas for future research in the development and application of CNT-reinforced composite beams, indicating the ongoing advancements and the necessity for further investigation to overcome existing challenges and enhance the performance of these innovative materials.