

PREFACE

The classical derivative is local operator, but fractional derivatives are non-local. Hence, the latter are good enough to provide a mechanism to incorporate the hereditary properties of many physical phenomena. Fractional partial differential equations (FPDEs) are partial differential equations that involve fractional derivatives. FPDEs are widely used in fluid mechanics, optical fibers, plasma physics, mathematical biology, viscoelasticity, etc. In this thesis, the fractional advection-diffusion equation (FADE), fractional telegraph equation, and fractional Burgers equations are considered and numerical methods are investigated to solve them.

Fractional generalization of the diffusion equation was introduced to describe the anomalous kinetics of a simple dynamical system with chaotic motion. FADE is used in groundwater hydrology to model the movement of passive tracers carried by fluid flow in a porous medium. FADE, in which time derivative is of fractional order, arises from power-law particle residence time distribution and describes particle motion with memory in time. Another considered linear PDE is telegraph equation which is used in the fields of signal analysis, diffusion process of chemicals, heat transfer theory in thermodynamics, wave propagation, transportation charged particles, etc. Some processes in nature are governed by time-fractional telegraph equations, such as the anomalous diffusion processes in blood flow experiments.

A Burgers equation is a non-linear PDE whose solution gives a travelling wave containing front sharpening. This equation describes non-linear propagation with diffusion effects and is a mathematical model of traffic flow. The generalization of the Burgers equation describes the non-linear phenomena more accurately. Fractional Burgers equation models the unidirectional propagation of non-linear acoustic waves through a pipe filled with gas.

This thesis mainly focuses on the FPDEs generated by time-fractional derivatives such as Atangana-Baleanu derivative and generalized fractional derivative. The researchers and scientists investigated many definitions of fractional derivative, like Caputo, Grünwald-Letnikov, Hadamard, Riemann-Liouville, Riesz, etc. These definitions have shown significant roles in fractional calculus but still have some limitations. They contain a singular kernel, which is a restriction in modelling the behaviour of viscoelastic material, electromagnetic systems, etc. The generalizations of fractional derivatives and integrals are studied by many authors and in different ways. In 2012, Agrawal proposed the definition of generalized fractional operators using scale function and weight function. With the scale function, the assumed domain can be changed, and the weight function is used to extend the kernel in operators. With the help of them, a model can be made more flexible. In 2015, Caputo and Fabrizio proposed a new definition for the fractional derivative in term of exponential kernel to solve the problems of non-local dynamical system. Atangana and Baleanu modified the Caputo-Fabrizio derivative using the Mittag-Leffler function as the kernel, which is non-singular and non-local. The benefit of a non-singular kernel is that it does not lead the addition of artificial singularity into the models. Also, it describes the two different waiting times distribution, which is an ideal waiting time distribution, as noted in the biological phenomena like in the spread of cancer.

As the presence of non-integer order derivatives makes the differential equations too complex to be solved analytically, and hence the numerical methods becomes important to study such problems. It is challenging to provide more accurate numerical results, especially for the newly proposed fractional derivatives. This research work aims to develop some high order numerical approximations for Atangana-Baleanu and generalized fractional derivatives. The finite difference method (FDM) is used to maintain simplicity and provide the proficiency of the scheme. For achieving a high order of convergence, Taylor series expansion is used, followed by FDM. The scope of the thesis is to form the numerical schemes for FPDEs like the FADE,

fractional telegraph equation, and fractional Burgers equation; and to study the stability of proposed methods; to provide the experimental analysis to support theoretical statements.