

PREFACE

This dissertation is focused on noncommutative aspects of the study of direct projective modules and their generalizations. These generalizations include semi-simple direct projective modules, finite direct projective modules, and pure direct projective modules. Throughout the work, our aim is to discover new properties and characterizations of these modules in respect of several rings.

The dissertation comprises five chapters. In the first chapter, we provide preliminary notions and results that we will use in the subsequent chapters.

In **Chapter 2**, we discuss several properties of *direct projective modules*. We characterize direct projective modules in terms of SSP (Summand Sum Property), SIP (Summand Intersection Property), Endoregular, and Hopfian modules. We also introduce the concept of semi-simple direct projective modules, which is a generalization of direct projective modules, and present several properties of them.

Chapter 3, introduces the concept of *finite direct projective modules*, which is a generalization of direct projective modules. We offer a counter-example to show that this generalization is strict. Additionally, we study the properties of finite direct projective modules in relation to their summands. We also characterize von Neumann regular rings in terms of the endomorphism ring of finite direct projective modules. Finally, we establish connections among Rickart Modules, D_3 modules, direct projective modules, finite direct projective modules, and endoregular modules.

In **Chapter 4**, we define the notion of *finite direct projective covers and envelopes* of modules. Here, we identify a condition under which a finite direct projective cover is equivalent to a projective cover of a module. We also characterize the finite direct projective cover of a module over a semi-perfect ring and semi-regular

ring. Finally, we discuss the finite direct projective envelopes of modules and find equivalent conditions with respect to semi-simple Artinian rings and S -rings.

In **Chapter 5**, using the concept of purity in modules we defined another generalization of direct projective modules as *pure direct projective modules* which is the dual notion of pure direct injective modules. We provide some properties with respect to direct sums and direct summands of pure direct projective modules. We also establish connections among pure direct projective modules, pure Rickart modules, and endoregular modules. Finally, we characterize von Neumann regular rings with respect to pure direct projective modules.