Mathematical Models for the Treatment of Swallowing Disorders through Oesophageal Catheterisation

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Mathematical Sciences

by

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CERTIFICATE

It is certified that the research work contained in this thesis entitled "**Mathematical models for the treatment of swallowing disorders through oesophageal catheterisation**" by "**Anupam Kumar Pandey**" has been carried out under my supervision and that this work has not been submitted elsewhere for any degree.

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15-07-2024

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Date :

Place :

(Anupam Kumar Pandey)

List of Figures

Symbols

α	Peripheral layer thickness
$\bar{Q}$	Time averaged volume flow rate in the laboratory frame
δ	Wave number
ϵ	Radius of catheter in the non-dimensional form
λ	Wavelength
μ	Viscosity ratio
ϕ	Amplitude of wall displacement in the non-dimensional form
Ψ/ψ	Streamfunction in the laboratory/wave frame
ρ	Density
a	Cylindrical tube radius
a_1	Inner cylindrical tube radius
a_c	Radius of the catheter
b	Wave amplitude in the dimensional form
c	Wave velocity
f_a/F_a	Friction force at the tube wall in the dimensional/non-dimensional form
f_c/F_c	Friction force at the catheter wall in the dimensional/non-dimensional form
H/h	Wall displacement in the dimensional/non-dimensional form
H_1/h_1	Interface in the dimensional/non-dimensional form
k	Amplitude dilation parameter
p	Pressure
q	Instantaneous volume flow rate in the wave frame
Re	Reynolds number
T	Average thickness of the core layer

t	Time
U/u	Axial velocity in the laboratory/wave frame
V/v	Radial velocity in the laboratory/wave frame
Z/R	Axial and Radial coordinate in the laboratory frame
z/r	Axial/Radial coordinate in the wave frame
ω	Gradient parameter
m	Micropolar parameter
N	Coupling number

DEDICATED

To
My Beloved Family & Friends