

# Mathematical Models for the Treatment of Swallowing Disorders through Oesophageal Catheterisation



*Thesis submitted to*

**Indian Institute of Technology  
(Banaras Hindu University) Varanasi**

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**Doctor of Philosophy**

*in*

***Mathematical Sciences***

*by*

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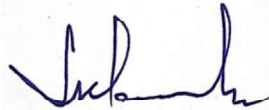
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15-07-2024

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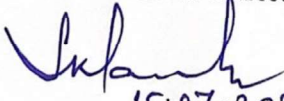
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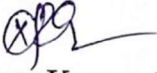
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# Symbols

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$\alpha$	Peripheral layer thickness
$\bar{Q}$	Time averaged volume flow rate in the laboratory frame
$\delta$	Wave number
$\epsilon$	Radius of catheter in the non-dimensional form
$\lambda$	Wavelength
$\mu$	Viscosity ratio
$\phi$	Amplitude of wall displacement in the non-dimensional form
$\Psi/\psi$	Streamfunction in the laboratory/wave frame
$\rho$	Density
$a$	Cylindrical tube radius
$a_1$	Inner cylindrical tube radius
$a_c$	Radius of the catheter
$b$	Wave amplitude in the dimensional form
$c$	Wave velocity
$f_a/F_a$	Friction force at the tube wall in the dimensional/non-dimensional form
$f_c/F_c$	Friction force at the catheter wall in the dimensional/non-dimensional form
$H/h$	Wall displacement in the dimensional/non-dimensional form
$H_1/h_1$	Interface in the dimensional/non-dimensional form
$k$	Amplitude dilation parameter
$p$	Pressure
$q$	Instantaneous volume flow rate in the wave frame
$Re$	Reynolds number
$T$	Average thickness of the core layer

---

$t$	Time
$U/u$	Axial velocity in the laboratory/wave frame
$V/v$	Radial velocity in the laboratory/wave frame
$Z/R$	Axial and Radial coordinate in the laboratory frame
$z/r$	Axial/Radial coordinate in the wave frame
$\omega$	Gradient parameter
$m$	Micropolar parameter
$N$	Coupling number

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