Chapter 4

Optimal Location of Accident Relief Facilities in a Railway Network- Multi-Objective Approach.

4.1. Introduction :

In this chapter, we apply the concept of network vulnerability and link importance presented in the Chapter 3 for formulating mathematical model for decision making on location of the relief facilities over a railway network.

The paramount importance and vastness of the railway's network makes it vulnerable to accident, derailment, natural calamity, and sabotage, that may cripple its entire operation. The Indian railway has experienced few unfortunate accidents in the past years. An average of 110 accidents took place every year between 2013-2018, in which around 990 people were killed and 1,500 injured (Dubey Vivek 2019.)

As already discussed in the first chapter (Introduction), it is known that to mitigate the impact of accidents or disasters, the Indian Railway has a dedicated fleet of rolling stock nominated for housing re-railing and rescue equipment. These rolling stocks are kept in preparedness to move at a short notice. Collectively they are called Accident Relief Equipment. Accident Relief Trains (ART) is a dedicated rake kept in preparedness for re-railing and rescue operation. Similarly, there are dedicated rakes for extending medical assistance to the passengers in the case of an accident involving passenger trains. These rolling stocks have extensive medical facilities including a small operation theatre with trained doctors nominated for emergency service. They are called Accident relief medical equipment (ARME). In addition to above, the Indian railway also has heavy capacity cranes (140T cranes) to tackle accidents involving heavy infrastructure or capsized rolling stock like a locomotive or a loaded wagon.

At present, the Indian railway has 175 ART and 162 ARMV located at important junctions throughout the country (Indian railway safety directorate 2020).

Primarily, the location of these facilities should be chosen such that it ensures the availability of relief at the accident site within the least response time. This is a challenging problem because of the following reasons. *First*, the exact location and magnitude of the accident are uncertain, and almost impossible to predict. *Second*, the number of relief facilities are limited. *Third*, relief facilities can only be positioned at locations that meet the practical concerns about the availability of the eligible human resource, maintenance facility, and other necessities like a spare locomotive to haul the rake to the accident site etc. *Fourth*, the optimal assignment of different types of relief trains across the network is challenging because several (uncertain) attributes need to be incorporated. *Fifth*, siting of relief facilities should take into consideration multiple competing objectives. At present, the relevant decisions are based only on the ability of a relief facility to provide radial coverage.

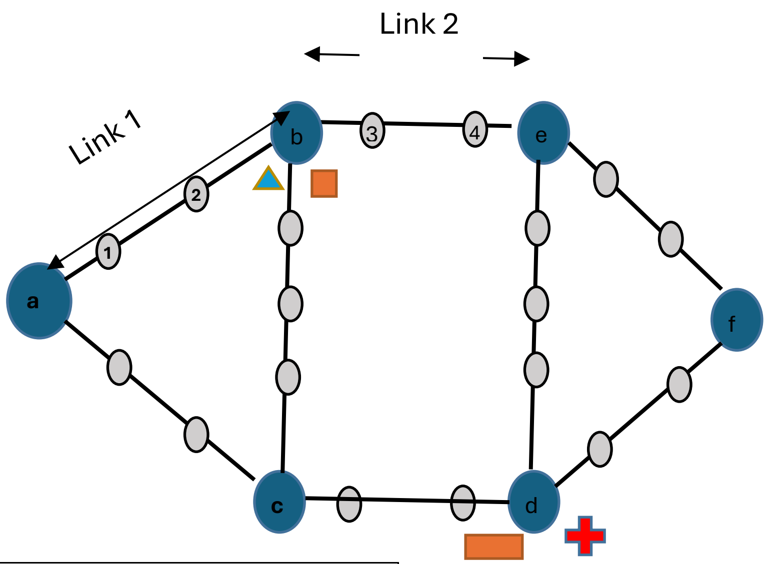
In the present research, we undertake the challenging problem of determining the optimal location for different types of accident relief facilities in the Indian railway network following the concept of cooperative coverage. A model for a multi-objective optimization program is presented with conflicting objectives . The complex problem is solved for the best solution through use of iterative Augmecon (Mavrotas and Florios 2013). The applicability of this research to the practice is demonstrated through a case study of the rail network of the North Central zone of Indian railways. It is also shown that the proposed plan of reconfiguring location of the relief facilities would help increase the coverage of the nodes and coverage of the links of the considered network.

The rest of the chapter is organized as follows. The problem description is given in Section 4.2. Section 4.3 contains the multi-objective mathematical model defined as an optimization program. Section 4.4 explains solution methodology and the Augmented ε-constraint method for solving the multi-objective optimization problem. A case of the NCR zone of Indian Railways is presented in Section 4.5. Section 4.6 presents the computational experiments and managerial insights. Finally, Section 4.7 concluded the work.

4.2. Problem Description :

The railway network considered in our problem can be represented as an undirected Graph $G = (N, A)$ where N represents the set of all nodes (i.e., stations) in the network and A represents the set of all arcs connecting the stations. Let N represent the set of stations that are normal stations or junctions. Then, we have a series of continuous arcs connecting the two possible nodes in the set S and such a combination of arcs are termed as 'links' in this work. A link thus represents a set of nodes/stations in a segment. Each link will have unique characteristics in terms of track specifications, the flow of passenger and freight traffic, and vulnerability. So, based on the above factors the importance of link varies in the network. The railway network can be presented in terms of Links and selected stations N or in terms of links and all nodes/vertices A. A symbolic railway network in our study is as shown in Figure 4.1.

Note that an accident can occur anywhere on the railway network, and hence each point on every link of the network is a potential demand point that might need accident relief facilities. That is each point on the network is assumed to have an equal probability of the accident. To maintain tractability, and without loss of generality, we assume all the stations on a link as potential demand points. If needed, intermediate points between the stations can be considered as node points. Thus, all the nodes on each of the links will represent demand points in the network and are denoted by set I. In the event of an accident, a demand node may require different relief facilities depending on the magnitude and scale of the accident. Three different relief facilities considered in this study are the Accident Relief Train (ART), Accident Relief Medical Van (ARMV) and Crane that provide specific services in case of accidents. The ART is equipped with the necessary tools and equipment along with the required manpower for restoration of the track while the Crane needs to be deployed in the case of major accidents to lift the rolling stock or to move heavy assets. The ARMV has designated doctors, paramedic staff, medical facility and operation theatre, to be deployed in case of major accidents involving passengers. These relief facilities need to be located at the stations suitable for stabling them. The potential stations for siting the relief facilities are chosen based on functional, technical, and operational requirements of the respective facility for regular operation and maintenance. The availability of spare locomotive, crew and other manpower requirements is also considered while considering the facility location to avoid any additional cost of keeping and maintaining the facility. Further, the ART, ARMV and the Crane are considered as separate units, which can be demanded and operated separately or in a combination of one another. Certain strategic restrictions are also imposed on siting the relief facilities. For example, there is a limitation on the maximum number of each facility type due to budgetary limitations. Also, one potential location can have a maximum of one facility of each type. Similarly, the crane cannot be sited independently, and it needs to be coupled with an ART. These constraints must be respected while finding a suitable location for siting the facilities.



Legend			
1	Node		ART
a	Potential station		AMRV
			Crane

Figure 4.1: An Illustrative Example of Network Considered in The Study.

A demand node will get the required service if there is a relief facility available that can provide the required service to the node within a certain time or distance limits. In other words, a node will be 'covered' if it can be reached by an ART, ARMV or Crane within the predefined time limits from the point where these relief facilities are sited. It is important to note that a node might need more than one relief facility, or it can be 'covered' by more than one facility site. Thus, it is assumed that the demand nodes are getting cooperatively covered by the potential relief facilities. Following the terminology of cooperative coverage (Berman et al., 2011), we say that the relief facilities emit a signal that decays over distance and the demand node will receive such signal from all facilities. The nodes will get covered if the sum of signals received by it from all facilities exceeds a predefined threshold value. Thus, given the number of relief facilities of each type to be located and a set of potential locations for siting these facilities in the network, the problem is to ensure that: a) links are covered as per their importance; b) overall coverage of the network gets maximized; c) redundancy (i.e., additional availability of relief facilities) for the demand nodes gets maximized; and d) overall response time to all the nodes in the network gets minimized.

The resulting problem is referred to as the accident relief facility location problem (ARFLP), which is modelled as a multi-objective integer optimization program as outlined in the next section.

4.3 Optimization Program:

	Sets and indices :
I	Set of demand nodes in the network indexed by i
L	Set of links in the network indexed by l
I_l	Set of demand nodes on link l

J Set of potential locations (stations) for siting relief facilities indexed by j

F Set of relief facilities indexed by f, F = {ART, ARMV, Crane}

Parameters :

E Importance of link l (calculated as per the procedure given in Section 5)

T_i Travel time between nodes i and j by facility f

P Limit on number of facilities of type f that can be sited

θ The threshold value for a facility of type f for coverage

ϕ Coverage function

$$\phi\left(T_{ij}^f\right) = \{\frac{\left(T_{max}^f - T_{ij}^f\right)}{T_{max}^f}, \quad if\ T_{ij}^f \leq T_{max}^f\ 0, \quad otherwise$$

where T_{Max}^f is the maximum permissible time to provide service by relief facility f

α Binary parameter

$$\alpha_{ij} = \{1, \quad if\ \phi\left(T_{ij}^f\right) > 0\ 0, \quad otherwise$$

Decision Variable :

$$x_j^f = \{1, \quad if\ facility\ type\ f\ is\ located\ at\ node\ j\ \ 0, \quad otherwise$$

$$y_i = \{1, \quad if\ node\ i\ is\ covered\ \ 0, \quad otherwise$$

$$w_l = \{1, \quad if\ link\ l\ is\ covered\ \ 0, \quad otherwise$$

$$u_{ij}^f = \{1, \quad if\ node\ i\ is\ assigned\ to\ location\ j\ for\ facility\ f\ 0, \quad otherwise$$

$$v_{ij}^f = \{1, \quad if\ node\ i\ can\ be\ responded\ from\ location\ j\ with\ facility\ f\ 0, \quad otherwise$$

Mathematical Model:

Objective Functions

$$\max \ \sum_{l \epsilon L} E_l w_l \quad (1)$$

$$\max \sum_{i \epsilon I} \sum_{j \epsilon J} \sum_{f \epsilon F} \phi\left(T_{ij}^f\right) x_j^f \quad (2)$$

$$\max \sum_{i \epsilon I} \sum_{j \epsilon J} \sum_{f \epsilon F} u_{ij}^f \quad (3)$$

$$\min \sum_{i \epsilon I} \sum_{j \epsilon J} \sum_{f \epsilon F} T_{ij}^f v_{ij}^f \quad (4)$$

Constrains:

$$\sum_{j \epsilon J} \phi\left(T_{ij}^f\right) x_j^f \geq \theta^f y_i\,, \ \ \forall\, (i, f) \in (I, F) \quad (5)$$

$$w_l \leq y_i\,, \ \ \forall\, (i, l) \in \ (I_l, L) \quad (6)$$

$$\sum_{j \epsilon J} x_j^f \leq P^f, \ \forall\, f \in F \quad (7)$$

$$x_j^f \leq 1, \ \forall (j,f) \in (J,F) \tag{8}$$

$$u_{ij}^f \leq \alpha_{ij} x_j^f, \ \forall (i,j,f) \in (I,J,F) \tag{9}$$

$$v_{ij}^f \leq x_j^f, \ \forall (i,j,f) \in (I,J,F) \tag{10}$$

$$\sum_{j \in J} v_{ij}^f = 1, \ \forall (i,f) \in (I,F) \tag{11}$$

$$x_j^{Crane} \leq x_j^{ART}, \forall \ j \in J \tag{12}$$

$$x_j^f \in \{0,1\}, \quad \forall (j,f) \in (J,F) \tag{13}$$

$$y_i \in \{0,1\}, \quad \forall i \in I \tag{14}$$

$$u_{ij}^f \in \{0,1\}, \quad \forall (i,j,f) \in (I,J,F) \tag{15}$$

$$v_{ij}^f \in \{0,1\}, \quad \forall (i,j,f) \in (I,J,F) \tag{16}$$

The first Objective function, eq. (1), maximizes the importance of all the links that can be covered, while the second objective function, eq. (2) attempts to maximize total coverage in the network. The objective function (3) aims to maximize the assignment of all the nodes to the potential rescue facilities thereby increasing the desired redundancy in the system, whereas (4) assigns the demand nodes to the potential relief facilities such that the overall first response time in the network is minimized. Constraint (5) depicts the requirement of coverage of a node through the cooperative coverage function, while (6) ensures that a link will be covered only if all the nodes of that link are covered. Constraint (7) limits the number of facilities of each type that can be located. Constraint (8) requires that at the most one facility of each type can be located at the potential stations. Allocation of a demand node to the potential relief facilities that are within the maximum travel limits is ensured by Constraint (9). Constraint (10) and (11) allocate a demand node to the nearest open facility while Constraint (12) ensures that the relief facility Crane cannot be sited independently, and it needs the presence of an ART at the potential location. Finally, Constraints (13)-(16) define the nature of the decision variables.

4.4. Solution Methodology – Interactive AUGMECON Method :

The model presented in Section 4.3 is a multi-objective optimization (MOO) problem with four conflicting objectives. The goal is to find a set of representative non-dominated solutions and quantify their trade-offs to satisfy various objectives of the problem as per subjective preferences of decision-makers. There are several exact methods, heuristics and evolutionary approaches that can be implemented in a-priori, generative (posteriori) or interactive style to solve the MOO problem (Emmerich and Deutz 2018) . In the case of interactive methods, the DM is involved in every step of the search process till a desirable solution is obtained. The Tchebycheff based approaches have been popularly used as interactive search methods in MOO (Kim and Kim 2006; Reeves and MacLeod 1999; Steuer 1989). In this research, we follow the Augmented ε-constraint Method (AUGMECON) proposed by Mavrotas (Mavrotas 2009) in an interactive fashion to solve the ARFLP. This method is influenced by the Interactive Weighted Tchebycheff method of Steur (Steuer 1989). However, the interactive AUGMECON offers contraction of criteria space instead of weight space and more flexibility over the Interactive Weighted Tchebycheff method (Programming and Administration 1983).

As the name indicates, the interactive AUGMECON is an implementation of AUGMECON in an iterative fashion with the engagement of the DM in each iteration. The basic AUGMECON is an improved version of the conventional $\boldsymbol{\varepsilon}$-constraint method. In AUGMECON, a MOO problem is converted into an equivalent single objective optimization problem by selecting only one objective function to be optimized and considering the remaining objectives into the constraints. With different combinations of the limits on the objective functions that are considered in the constraint, AUGMECON generates a set of non-dominating solutions. These solutions cannot be compared to each other for their utility in absence of higher-level information. The desired information can be obtained in consultation with the DM. The involvement of the DM allows implementing the AUGMECON in an interactive style as explained schematically in Figure 4.2.

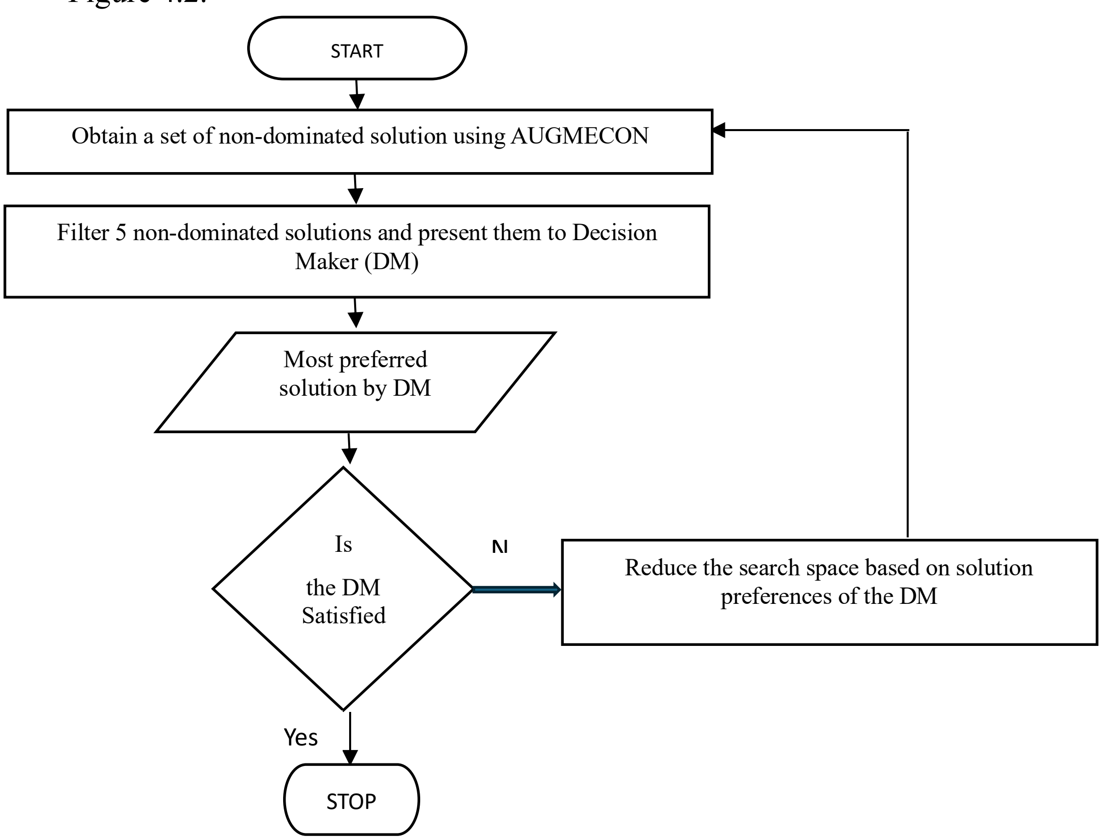


Figure 4.2: Schematic Presentation of Interactive AUGMECON Procedure.

Algorithm 1: Interactive AUGMECON

Step 1: Obtain initial payoff matrix.

Step 2: Select appropriate grid points and run **ARFLP-AUGMECON**. Obtain the set of non-dominated solutions.

Step 3: Filter five different solutions from the set of non-dominated solutions ensuring equal distribution of solution-on-solution space following **Algorithm 2**.

Step 4: Present the filtered solutions to the decision-maker and select the most preferred solution by the decision-maker.

Step 5: If the decision-maker is satisfied, **STOP**. Otherwise, go to Step 6.

Step 6: Prune the search space by imposing the lower bounds on the objective functions that are calculated as:

$$LB_j^{i+1} = Z_{*j}^i - \alpha^i (Z_{*j}^i - Z_j^{min})$$

where:

LB_j^{i+1} lower bound of the jth objective function, in the next (i + 1) iteration.

Z_{*j} the j^{th} element of the selected Pareto optimal solution's criterion vector in the i^{th} iteration.

Z_j^{min} the minimum of the j^{th} objective function over the entire Pareto set as obtained from the initial payoff table.

α^i Contraction parameter. It takes values in [0,1] and controls the rate of search space contraction. In our case, $\alpha^i = 0.35$.

Step 7: With the updated lower bounds on the objective functions, reconstruct the payoff table and then go to Step 2.

Figure 4.3: Procedure for the Interactive AUGMECON

The procedure of interactive AUGMECON is explained in Algorithm 1 given in Figure 4.4. As explained in Algorithm 1, the procedure starts with the initial payoff table and applies AUGMECON to obtain a set of non-dominated solutions. For implementing AUGMECON in the case of ARFLP, we consider Objective (1) that is maximizing the coverage of links according to their relative importance as the only objective function. The remaining objectives (2), (3) and (4) from ARFLP are converted into constraints. Then, the modified problem for AUGMECON is as follows.

ARFLP-AUGMECON :

$$\max \ \sum_{l\epsilon L} E_l w_l + \delta \left(\frac{s_2}{R_2} + \frac{s_3}{R_3} + \frac{s_4}{R_4}\right) \tag{17}$$

subject to

$$\sum_{i\epsilon I} \sum_{j\epsilon J} \sum_{f\epsilon F} \phi\left(T_{ij}^f\right) x_j^f - s_2 = E_2 \tag{18}$$

$$\sum_{i\epsilon I} \sum_{j\epsilon J} \sum_{f\epsilon F} u_{ij}^f - s_3 = E_3 \tag{19}$$

$$\sum_{i\epsilon I}\sum_{j\epsilon J}\sum_{f\epsilon F}T_{ij}^{f}v_{ij}^{f}+s_4=E_4 \tag{20}$$

$$s_2, s_3, s_4 \geq 0 \tag{21}$$

Constraints (5) to (16) from ARFLP

The additional variables in the model are the non-negative surplus and slack variables to consider the maximization and minimization type objectives in the original problem. Here s_2 and s_3 are surplus variables while s_4 is a slack variable. Constraints (18) and (19) requires that the value of Objective expressions (2) and (3) of ARFLP should be at least as large as the right-hand side (RHS) values E_2 and E_3 of these constraints, respectively. Similarly, Constraints (20) imposes a requirement that the value of Objective expressions (4) of ARFLP should not exceed its RHS E_4. AUGMECON allows setting different combinations of the RHS values by partitioning them as grid points over the range of maximum and minimum achievable values of these objective functions. This range of objective functions (2), (3) and (4) is indicated by R_2, R_3 and R_4, respectively, and can be obtained from the values in the payoff table which is constructed popularly through the Lexicographic approach. The algorithm tries to minimize the values of surplus/slack variables in the constraints (18)-(20). It can also be noted that a ratio of surplus/slack variable S_n to the range R_n of the n^{th} objective function in ARFLP is also considered as a part of the objective function (17) in AUGMECON of along with a multiplier δ which is usually in the range of $10^{-3}\ to\ 10^{-6}$. This additional term in the objective helps in exploring the strongly non-dominated solutions.

The model ARFLP-AUGMECON needs to be solved separately for each combination of RHS values $E_2, E_3\ and\ E_4$. Thus, if the range of each of the three objectives is divided into five grid points, then the AUGMECON suggests solving the problem [P1] for 5×5×5=125 times. For the detailed iterative procedure of the algorithm with an early exit strategy, we refer the reader to Mavrotas (Mavrotas 2009).

The output of AUGMECON is a set of non-dominated solutions. This set is filtered by following the procedure given in Algorithm 2 given in Figure 4.5.

Algorithm 2 : Filtering Procedure

Input: $\Theta = \{1,2, \dots k\}$ Set of non-dominated solutions from AUGMECON and $|\Theta| = K$.

Let, Ω be the set of filtered solutions. Initially, $\Omega = \{\quad\}$.

Step 1: Normalize the solutions obtained using the formula

$$f(n_i) = \frac{f_i(max) - f_i(n)}{f_i(max) - f_i(min)}$$

where

$f(n_i)$	normal value of the i^{th} objective function at n^{th} iteration
$f_i(max)$	maximum value of the i^{th} objective function
$f_i(min)$	minimum value of the i^{th} objective function
$f_i(n)$	value of the i^{th} objective function at n^{th} iteration

Step 2: Calculate utility value of each solution using formula

$$\underline{f}(n_i) = \frac{1}{m}\sum_{i=1}^{m} f(n_i)$$

where m is the total number of objective functions in the problem.

Step 3: Arrange the K solutions in the non-increasing values of utility $\underline{f}$.

Step 4: Let k_{max}, k_{min} represent the solutions with the maximum and the minimum value of utility $\underline{f}$. Include these solutions to the set Ω.

Step 5: Find the solution k_{avg}, closest to the average value of the utility. Set $k_{avg} \in \Omega$.

Step 6: Take the average of the utility of solutions for the interval from k_{max} and k_{avg} and select the solution closest to this value for its inclusion in the set Ω.

Step 7: Repeat the same process of Step 6 for the solution interval from for the interval from k_{avg} and k_{min}.

Step 8: Report the set of filtered solutions Ω.

Figure 4.4: Procedure for Filtering of Non-dominated Solutions

In Algorithm 2, all solutions in the obtained set of non-dominated solutions are first normalized considering maximum and minimum values in the set. A utility value is then calculated for each solution and then the solutions are arranged in non-increasing order of their utility. From the sorted list of solutions, the extreme solutions (with maximum and minimum utility) are selected along with the average solutions as given in Steps 4-7 of Algorithm 2. Following Algorithm 2, five most diverse solutions are presented to the decision-maker (DM) in every iteration. The DM will select the best solution out of five presented to him. If the DM is fully satisfied with the selected solution, the process stops. Otherwise, as given in Step 6 of Algorithm 1 the decision space is contracted as per the decision maker's choice and the algorithm is continued till the DM is satisfied with the solution obtained.

4.5. Case study : North Central Railway Zone of Indian Railways

To demonstrate the applicability of the proposed model to the practice, we have taken real-time data of North Central Railway, a very critical network as part of a bigger network of Indian railways. Indian railway is undoubtedly one of the largest railways in the world. For administrative purposes, the Indian railways have been divided into 16 zonal units, out of which North Central Railway (NCR) (North central railways accident manual 2019) is an important unit that we have selected for our study. The North Central Railway is known as the workhorse of Indian Railways(NCR 2016) that span over 5 important provinces of India including parts of Uttar Pradesh, Haryana, Rajasthan and Madhya Pradesh. It comprises critical routes on the Indian railways network defining the sides and diagonals of the Golden Quadrilateral of the railway network of tracks. It extends from Ghaziabad (excl.) in the north to Mughal Sarai (excl.) in the east on New Delhi- Howrah trunk route and from Palwal (excl.) to Bina (excl.) on New Delhi-Mumbai-Chennai corridor. The NCR has around 3062 route km of broad gauge comprising predominantly, double line- electrified sections and 6366.77 km of total track length. It has a total of 443 stations, out of which 202 are at the mainline. Administratively, it is subdivided into Prayagraj (Erstwhile Allahabad), Jhansi and Agra divisions. Thus, in terms of the size of the rail network, the NCR zone itself surpasses several countries in the world. The map of the NCR zone is as shown in Figure 4.6.

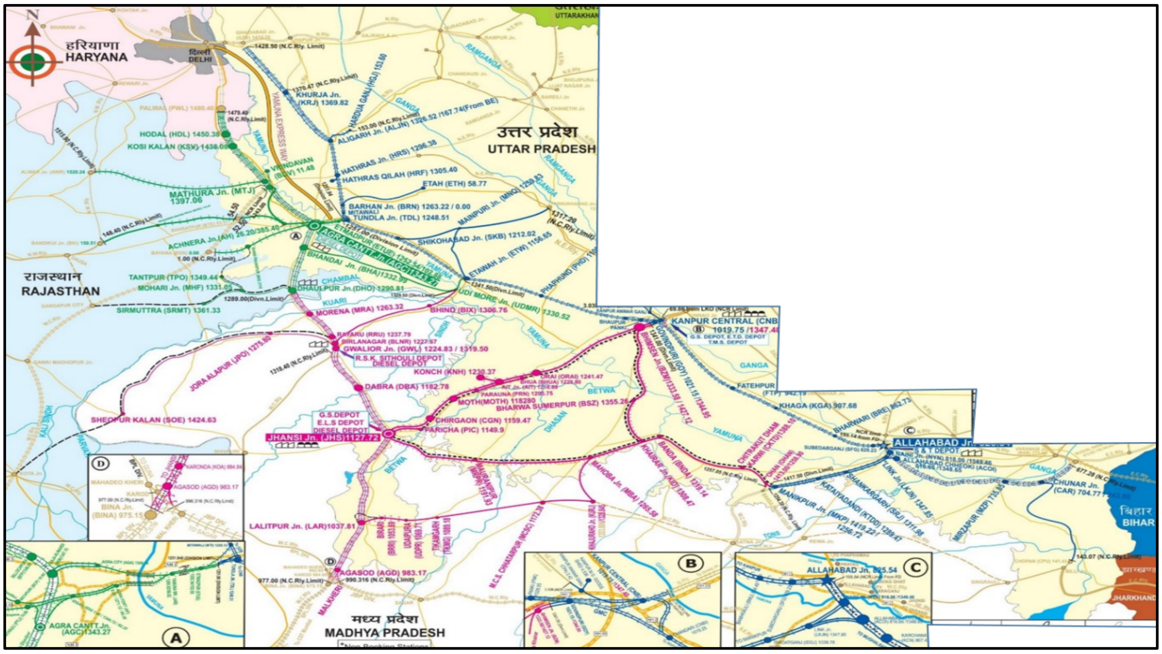


Figure 4.5: Division Map of NCR Zone, Indian Railways

Table 4.1: Input Parameters for the Case Study

Parameter	**Description**	
Number of nodes (stations) in the network	105	
Number of links	132	
Type of accident relief facilities	ART, ARMV and Crane	
	ART	13
Number of accident relief facilities to be sited	ARMV	12

	Crane	7
The average speed of the relief facilities	75 kmph	
	ART	3
Maximum allowable time (hours)	ARMV	2
	Crane	4
	ART	0.6667
Threshold value (θ^f)	ARMV	0.5
	Crane	0.75
Number of candidate sites for the relief facilities	Small instance	16
	Large instance	33

The NCR zone forms a corridor for the trains to and from all directions of the spread of Indian railways. It is the most critical and vulnerable part of the bigger network of Indian railways. Its relative importance and criticality in the overall transportation network in the country have encouraged us to consider it for our study purpose. A summary of input parameters for the case study of the zone is given in Table 4.1

From the data obtained from the NCR officials, 53 important links in the zone are initially considered in the study to calculate the link importance. However, some of the links were as long as 228.760 km with an average length of 68.982 km. Therefore, we modify the link network by augmenting the intermediate stations such that the new link lengths are less than the average. In the modified network, we have 132 links and 105 nodes. It can be noted that the NCR zone shares track with the adjacent zonal units of Indian railways including Northern Railways (NR), Northwestern railways (NWR), West-central railways (WCR) and Eastern central railways (ECR). Therefore, some of the major adjacent stations from these zones are also considered in the study for completeness of the network. After defining the potential locations and demand points in the network, the shortest distance between all the points in the network is obtained using Floyd-Warshall Algorithm (Amoako 2019) using Python pandas (McKinney 2022) and functionalities available with the NetworkX library (NetworkX 2014.) . The minimum distance is used to calculate the coverage function. Further, to determine the maximum allowable response time of the facilities, the concept of the golden hour is popularly followed (Cowley, Ems, and States 2015). However, the time for the departure of accident relief facilities has been stipulated by railway authorities (North central railways accident manual 2019) as 30 minutes during the day and 45 minutes during the night. This time is reckoned from the time of ordering the relief facility to the time of leaving it from the shed. Therefore, the concept of maximum time for coverage has been adopted for more practical applicability of the same.

The existing layout of the relief facilities for the NCR zone is as shown in Table 4. 2.

Table 4.2 : Current Location of the Accident Relief Facilities

Sr. No.	Station name	Station Code	Zone	Relief facilities		
				ART	ARMV	Crane

1	Prayagraj	PRYJ	NCR	✓	✓	
2	Kanpur	CNB		✓	✓	✓
3	Banda	BANDA		✓	✓	
4	Tundla	TDL		✓	✓	
5	Jhansi	JHS		✓	✓	✓
6	Agra	AGC		✓	✓	✓
7	Mathura	MTJ		✓	✓	
8	Gwalior	GWL		✓	✓	
9	Deendayal Upadhyay	DDU	ECR	✓	✓	✓
10	Jaipur	JPU	NWR	✓	✓	✓
11	Bina	BINA	WCR	✓	✓	✓
12	Chopan	CPU	WCR	✓		
13	New Delhi	NDRL	NR	✓	✓	✓

In addition to the 13 stations that currently host relief facilities, the railway official suggested considering three more stations - Khajuraho (KURJ), Ghaziabad (GZB), and

Mahoba (MBA) as the set of candidate locations. Further, facilities available at the stations from other zones are treated as fixed as the decision-maker does not have any control over them. This problem instance is called 'Small Instance' in our study.

After completion of the 'small instance' the results were presented to the DM for evaluation. Some managerial insights were revealed when mathematical results were interpreted and translated to actual effect on the ground. The exercise was repeated after discussion with DM and considering more stations as possible places for facility location. This problem instance with more numbers of possible locations is called a "Large Instance' in our study. With these inputs, the proposed MOO model is solved using interactive AUGMECON in two different settings. In the first setting, 11 candidate facility locations that are suggested by railway authorities are considered. This is the case of 'Small Instance'. In the second setting, we consider 17 more candidate locations in the network in addition to the locations in the first setting to broaden the scope. This is the case of 'Large instance'. The new candidate location was selected as the junction stations in the network, keeping the possibility of other infrastructure development and ease of operation at these points in view.

Results of the computational experiments performed for the case are presented in the next section.

4.6. Results and Discussion

All the experiments are conducted on a personal computer equipped with Intel core i-7 2 GHz processor, 8 GB RAM, and Windows 10 Operating System. The model and solution procedure for AUGMECON is implemented using Python in GUROBI 9.1, a state-of-the-art math programming solver with Jupyter IDD. The results of the computational experiments are as follows.

4.6.1 Results of Interactive AUGMECON for the Small Instance :

The first step of interactive AUGMECON is to obtain an initial payoff table for getting the maximum and minimum values of objective functions that are considered in the constraints. The Payoff table is typically constructed using the lexicographic or hierarchical optimization approach. In this approach, a priority is set to each objective and the optimization is carried out in the order of the priority. While optimizing for one objective, only those solutions are considered that do not degrade the objective values of higher-priority objectives. Thus, to apply the lexicographic approach in our case for the four objectives in model ARFLP, each objective is set a top priority, and the remaining objectives are set the second priority one by one. The result is a 4×4 matrix that is the payoff matrix as given in Table 4.3.

Table 4.3: Initial Payoff Table for Small Instance

	Objective 1	**Objective 2**	**Objective 3**	**Objective 4**
Objective 1 (Importance of all the links)	**41.63**	433.09	988	246.66
Objective 2 (Total coverage)	28.51	**487.63**	1039	307.56
Objective 3 (Redundancy in the system)	34.28	454.06	**1065**	294.48
Objective 4	38.81	411.18	958	**245.22**

(Overall, first response time)				
Min	28.52	411.18	958	245.22
Max	41.63	487.63	1065	307.56
Range	13.11	76.45	107	62.34

Each row of the Payoff table is obtained by setting the respective objective as a top priority objective in the lexicographic approach. The value is row 1 and column 2 indicates the optimal value of Objective function 2 when it is optimized after the Objective function 1. The remaining values are obtained similarly. The boldfaced values across the diagonal represent the ideal solution vector. The individual objective function value in the ideal solution vector represents the maximum/minimum value that can be achieved if the objective is optimized individually and thus impose a bound on the objective function value. It should be noted that Objectives 1, 2 and 3 are maximization types, whereas Objective 4 is of minimization type.

After getting the necessary inputs from the payoff table, the next step in interactive AUGMECON is to run ARFLP-AUGMECON. For ARFLP-AUGMECON, Objective function 1 (link coverage) is considered as the main objective function to be optimized while the rest of the objective functions (2, 3, and 4) are considered in the constraints, as explained in Section 6. The range for objective functions (2, 3, and 4) is calculated from the Payoff table as the difference in the maximum and minimum values of the respective objective functions. The range is further divided into 4 equal intervals which lead to 5 grid points for each of the objective functions. For example, the maximum and minimum value for Objective function 2 are 487.63 and 411.18, respectively. Thus, it has a range of

76.45 which is divided into 4 equal intervals. This leads to a difference of 19.11 in consecutive intervals. Then, the five grid points obtained for Objective 2 are 411.18, 430.29, 449.4, 468.51 and 487.63. These grid points are nothing but the values of E_2 that is the RHS of Constraint (18). Similarly, five grid points each are obtained for Objective function 3 and 4 that corresponds to E_3 and E_4 in Constraints (19) and (20), respectively. Each combination of the grid points gives different RHS values of the ARFLP-AUGMECON constraints. Thus, we have 5 × 5 × 5 = 125 possible combinations of the right-hand side values for the constraints in ARFLP-AUGMECON. In other words, the mathematical model ARFLP-AUGMECON defined in Section 6 is run 125 times by changing the RHS of Constraints (18)-(20) of it and the solutions are recorded. It is noted that GUROBI 9.1 is very efficient in solving the problem instances. Each run of the AUGMECON is solved in less than a minute. The non-dominated solutions obtained are presented in Table 4.4.

Table 4.4: Set of Pareto-optimal Solutions by AUGMECON

Sr. No.	Objective 1 (Importance of all the links)	Objective 2 (Total coverage)	Objective 3 (Redundancy in the system)	Objective 4 (Overall, first response time)
1	41.63	433.10	988.00	294.73
2	41.63	433.10	988.00	282.63

3	41.63	432.31	988.00	257.69
4	41.44	448.59	1004.00	295.09
5	41.44	448.59	1004.00	307.51
6	41.44	448.59	1004.00	270.16
7	41.44	448.59	1004.00	295.09
8	41.44	448.59	1004.00	282.01
9	41.23	454.67	1025.00	307.56
10	40.40	453.80	1057.00	307.55
11	40.40	453.80	1057.00	295.02
12	40.40	453.80	1057.00	307.57
13	40.40	453.80	1057.00	295.09
14	40.36	462.33	1051.00	295.08
15	40.36	462.33	1051.00	282.63
16	40.36	462.33	1051.00	282.61
17	40.36	462.33	1051.00	307.57
18	40.36	462.33	1051.00	257.45
19	38.65	476.18	1048.00	291.61
20	38.65	476.18	1048.00	307.55

It should be noted that the values of the Objective functions are curtailed up to two decimal spaces for the compact presentation. Although the Objective function values do appear the same in many solutions, it is not the case. For example, the actual value of Objective function 1 in solutions 1, 2 and 3 is 41.63397126, 41.63377717 and 41.63376685, respectively. In all these solutions, improvement in one objective is associated with the degradation of some of the objective functions. The values obtained for different objective functions are critically analysed for the trade-offs and the correlation. Following heat map is plotted from the results as shown in Figure 7.

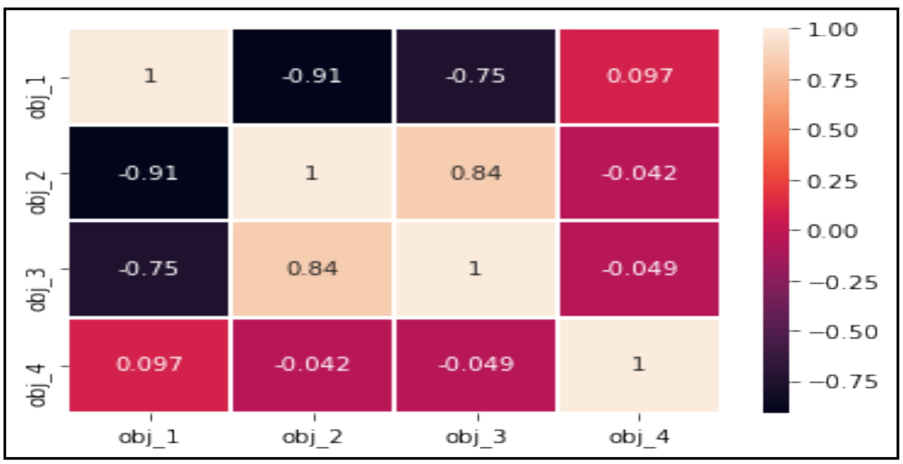


Figure 4.6: Heat Map for Correlation Coefficients of Objective Functions

It is noted from Figure 4.7 that objective1 varies strongly but negatively correlates with objective 2 and objective 3, indicating that efforts to increase the value of one will have a detrimental effect on the values of another objective. Similarly, objective 2 is positively correlated to objective 3 indicating that these objectives can simultaneously increase or reduce. However, objective 4 is very weakly correlated with the other three objectives. Impact of which is considered deliberately by the decision-maker while picking the solution for the next stage iteration process as described above. The correlation as depicted through this heat map was explained to the decision-maker (DM) to help him decide the choice of solution for the next stage.

This is the point in the study that seeks intervention from the DM who plays a prime role in the decision process. In our case, the DM is an officer in the Indian railways with over 20 years of experience in the service. In every iteration, a set of Pareto-optimal solutions are presented to the DM and the solution space is modified in the next iteration based on his input regarding preference(s) of the solutions. An attempt is made to reach a solution that the DM agrees upon. In the first iteration, a set of 20 non-dominated solutions are obtained as given in Table 4.4 after running ARFLP-AUGMECON. From this pool of non-dominated solutions, 5 representative solutions are filtered using Algorithm 2 and presented to the DM. From the presented solutions, the DM chooses a solution that minimizes the overall response time in the network (Objective 4) and at the same time achieves the largest number of redundantly covered links (Objective 3) without compromising much on the value of overall coverage (Objective 2) and coverage of important links (Objective 1). Solution number 18 in Table 4 exhibits these characteristics, and it is the choice of the DM. However, the DM wants to further explore the possible solutions. Therefore, the process of obtaining the payoff table, running ARFLP-AUGMECON to get a set of non-dominated solutions, filtering and presenting 5 solutions to the DM and seeking his inputs is continued. After the choice of a solution by DM in an iteration, the search space is contracted by imposing the lower bounds on individual objective functions. The lower bounds are calculated using the solution vector of the choice of DM and the global minimum values of each of the objective functions and used for constructing the payoff table in the next iteration. A reduction factor of 0.35 is used for the contraction of solution space. In our case, the entire process of interactive AUGMECON is repeated for three iterations, the details of which are presented in Table 4.5. The DM is satisfied with a solution having Objective function values as 41.23, 454.67, 1025, 249.39 for Objectives 1, 2, 3 and 4, respectively, at the end of iteration 3 and process stops.

Table 4.5: Summary of Solutions in Interactive AUGMECON

			Objective Function Values				
Iteration Number	**Description**		**Objective 1** (Importance of all the links)	**Objective 2** (Total coverage)	**Objective 3** (Redundancy in the system)	**Objective 4** (Overall, first response time)	**Remark**
1	Ideal solution (from payoff table)		**41.63**	**487.63**	**1065**	**245.22**	
	Solutions presented to DM	Solution 1	41.44	448.59	1004	295.09	
		Solution 2	41.63	433.09	988	282.63	
		Solution 3	40.36	462.33	1051	282.63	
		Solution 4	**40.36**	**462.33**	**1051**	**257.45**	Selection by DM
		Solution 5	38.64	476.17	1048	291.60	
	Lower bound*		**36.21**	**444.43**	**1018.45**	**274.88**	
2	Ideal solution		41.23	468.56	1048	246.73	

	(from payoff table)						
		Solution 1	41.23	454.67	1025	263.26	
		Solution 2	**41.23**	**454.67**	**1025**	**253.58**	Selection by DM
	Solutions presented to DM	Solution 3	40.36	462.21	1040	256.68	
		Solution 4	40.36	462.33	1050	243.69	
		Solution 5	39.73	468.56	1056	258.99	
	Lower bound*		**36.78**	**439.45**	**1001.55**	**272.47**	
3	Ideal solution (from payoff table)		**41.44**	**476.17**	**1056**	**243.45**	
		Solution 1	41.44	448.59	1025	263.27	
		Solution 2	**41.23**	**454.67**	**1025**	**249.39**	Selection by DM
	Solutions presented to DM	Solution 3	41.23	454.67	1025	263.86	
		Solution 4	40.36	462.33	1051	256.61	
		Solution 5	40.36	462.33	1051	247.68	

***Upper bound in case of Objective 4**

Several insights can be drawn from the results. We observed that in the interactive AUGMECON process, such as:

1. The decision-maker has consistently tried to increase the importance of the section covered by choosing an option with a higher objective function value than the current value for

objective function (objective1) while keeping a check on the total response time in the network.

2. It is also found that the solution selected by the DM resembles quite a lot with the existing scenario of the location of relief facilities. The locations for ART and ARMV are the same in this solution and the current plan. Only, the allocation of the crane has changed at one place from the current station JHS to BANDA station.

3. The existing plan of location of relief facilities could cover a total of 89 nodes out of 105 and 102 link sections out of 132 considered in our case study. It seems that the current method of thumb rule of radial coverage has worked well with the allocation of these facilities.

4. The proposed relocation of relief facilities resulted in the coverage of 91 nodes and 105 links. Therefore, the existing plan and proposed solution for siting the relief facilities do not differ much in the case of any comparison matrix.

5. We realized the roots of this issue are in the candidate locations suggested by the railway authorities. As mentioned before, only 11 locations were selected from the NCR zone based on their importance in the network, size of the station and required infrastructure and resources to host the relief trains.

   With this realization, we proposed to enlarge the search space with the addition of a few more candidate locations for siting the relief facilities and re-run the AUGMECON. The selection of new candidates is based on the fact that all the candidates are junction stations on the network providing flexibility of operation. The results are explained in the following sub-section.

4.6.2 Results of the Interactive AUGMECON For the Large Instance

For the large instance, we have considered 17 more junctions viz. CAR, NYN, MKP, LAR, DHO, BHA, AH, MHF, AWR, PWL, ALJN, SKB, MNQ, ETW, UDMR, BZM, KID, in addition to 16 junctions suggested by the railway authorities for the small instance. With the additional input, we repeat the interactive AUGMECON process whose results are explained below.

Table 4.6: Modified Payoff Table for the Large Instance

	Objective 1 (Importance of all the links)	**Objective 2** (Total coverage)	**Objective 3** (Redundancy in the system)	**Objective 4** (Overall, first response time)
Objective 1 (Importance of all the links)	**44.13**	426.06	967	238.16
Objective 2 (Total coverage)	26.07	**605.74**	1236	463.63
Objective 3 (Redundancy in the system)	26.07	561.79	**1264**	458.47
Objective 4	40.78	399.59	951	**234.95**

(Overall, first response time)				
Min	26.07	399.59	951	234.95
Max	44.13	605.74	1264	463.63
Range	18.06	206.15	313	228.69

The payoff matrix for the ARFLP-AUGMECON obtained using lexicographic approach is as given in Table 4.6. In comparison to the initial payoff table for small instance (Table 3), the ideal solution vector has shown a great improvement with the modified input. The first iteration of ARFLP-AUGMECON for the modified inputs has produced a set of 33 non-dominated solutions that are summarized in Table 4.7.

Table 4.7: Set of Non-dominated Solutions with Modified Input

	Objective Function Values			
Non-dominated solutions	**Objective 1** (Importance of all the links)	**Objective 2** (Total coverage)	**Objective 3** (Redundancy in the system)	**Objective 4** (Overall, first response time)
Solution 1	44.13	442.90	970	463.3

Solution 2	44.13	442.52	972	348.1
Solution 3	44.13	440.48	973	292.1
Solution 4	44.04	466.00	1030	406.4
Solution 5	44.04	453.96	1032	349.3
Solution 6	44.04	455.21	1031	291.9
Solution 7	42.26	483.57	1108	349.2
Solution 8	42.13	516.22	1108	292.1
Solution 9	34.67	538.18	1186	406.5
Solution 10	34.67	521.58	1187	349.3
Solution 11	44.12	452.80	1009	463.6
Solution 12	44.12	452.42	1011	405.9
Solution 13	44.12	451.17	1012	349.2
Solution 14	44.12	452.42	1011	292.1
Solution 15	44.04	466.00	1030	463.6
Solution 16	44.04	455.21	1031	349.3
Solution 17*	**44.04**	**460.78**	**1030**	**292.1**
Solution 18	42.26	483.57	1108	406.4
Solution 19	42.26	481.91	1108	347.9

Solution 20	42.13	516.22	1108	292.1
Solution 21	34.67	538.18	1186	406.4
Solution 22	34.67	521.89	1186	349.3
Solution 23	42.37	503.26	1087	292.1
Solution 24	42.13	515.80	1108	292.0
Solution 25	34.67	515.93	1189	406.5
Solution 26	34.67	514.78	1186	349.3
Solution 27	35.96	555.82	1150	463.6
Solution 28	35.96	555.74	1151	406.4
Solution 29	35.96	557.11	1146	349.2
Solution 30	35.96	555.74	1151	349.3
Solution 31	34.42	557.98	1196	349.3
Solution 32	26.63	572.76	1265	463.6
Solution 33	44.04	455.21	1031	291.9

Out of these, solution 17 has been selected by the DM as the most preferred solution and the DM is satisfied with this solution. This solution widely differs as compared to the current plan and the solution of small instances. These solutions are compared based on the location plans as given in Table 4.8.

Table 4.8: Comparative Location Plan for Relief Facilities

		Existing			Small instance			Large instance		
Sr. No.	**Station**	**ART**	**ARMV**	**Crane**	**ART**	**ARMV**	**Crane**	**ART**	**ARMV**	**Crane**
1	PRYJ	✓	✓		✓	✓				
2	CNB	✓	✓	✓	✓	✓	✓	✓	✓	✓
3	BANDA	✓	✓		✓	✓	✓			
4	TDL	✓	✓		✓	✓		✓	✓	
5	GZB									
6	JHS	✓	✓	✓	✓	✓		✓	✓	
7	AGC	✓	✓	✓	✓	✓	✓	✓	✓	
8	MTJ	✓	✓		✓	✓				
9	GWL	✓	✓		✓	✓		✓		
10	KURJ									

11	MBA			
12	CAR			
13	NYN		✓	
14	MKP	✓		
15	LAR			
16	DHO	✓		
17	BHA	✓		✓
18	AH			
19	MHF			
20	AWR		✓	
21	PWL			
22	ALJN			
23	SKB			
24	MNQ			
25	ETW		✓	
26	UDMR			
27	BZM			

28	KID	✓	✓

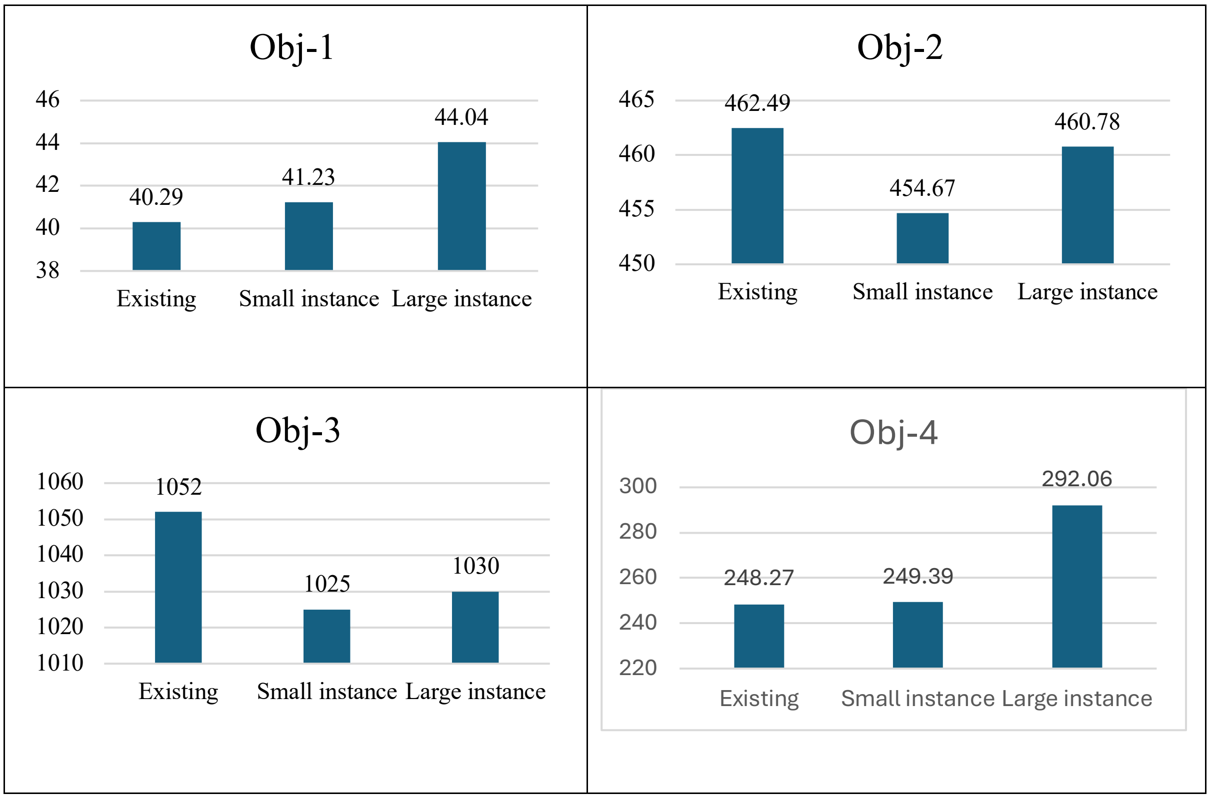


Figure 4.7: Trade-off on the Objective Functions

A summary of the comparison of these solutions with respect to the current plan based on various parameters such as nodes covered, number of links covered and the redundancy for different facilities is given in Figure 4.8 and Table 4.9.

Table 4.9: Comparative Different Solutions

Solution	No. of nodes covered (out of 105)	No of links Covered (out of 132)	Nodes covered with redundant facilities		
			ART	ARMV	Crane
Existing plan	89	102	446	266	340
Small instance	91	105	446	266	313
Large instance	96	113	480	237	313

The results from Table 4.9 and Figure 4.8 reveal there is no significant difference in the small instance and the existing layout of relief facilities. One crane has been relocated while keeping the rest of the facilities at the same location. Such rearrangement has resulted in a small improvement (2.33%) in value of Objective 1. While the values of remaining objectives have degraded as compared to the existing plan by 1.7%,2.57%,0.45% for Objectives 2,3 and 4 respectively.

On the contrary, the results of large instance are significantly different as compared to both the existing plan and small instance. In the case of large instance, Objective function 1 (coverage of links as per the importance) is increased by 9.28 % from its counterpart in the existing plan. It has resulted in an increase in the number of covered nodes by 7.87 % and the number of covered links by 10.78% compared to the existing plan. However, this betterment in Objective function 1 has resulted in degradation in the remaining Objectives 2, 3 and 4. Nevertheless, the reduction in Objective function values of large instance for Objectives 2 and 3 are as low as 0.37% and 2.09% as compared to the existing plan. Further analysis also indicates that the proposed solution of large instance is providing better redundancy in the case of ART. On an average, each covered node in the network receives the service from 3.26 ARTs, 2.5 ARMV and 5 Cranes. Therefore, even the reduced redundancy as compared to the existing plan does not affect much. The only significant degradation is observed in the case of overall response time in the network which is increased by 17% for the large instance. Such an increase in travel time is justified due to increased coverage of nodes and links of the network. It is seen that significant improvement in the desired objectives is obtained without much compromise made through the utilization of the iterative process and consultation.

Further, for the given threshold limit, if the primary objective is to provide 100% coverage to all the demand nodes in the network, the problem can be seen as a cooperative set covering problem. It will have an additional constraint that $y_i \geq 1, \; \forall \; (i) \in I$ that is each demand node is completely covered. The set covering model is solved to determine the optimal number of ART, ARMV and Cranes to provide the complete coverage. The results are summarized in Table 4.10.

Table 4.10: Location of Relief Facilities to Provide Complete Coverage.

Existing				
Sr. No.	**Station**	**ART**	**ARMV**	**Crane**

1	PRYJ			
2	CNB		✓	
3	BANDA			
4	TDL		✓	
5	GZB			
6	JHS	✓	✓	✓
7	AGC	✓		✓
8	MTJ			
9	GWL			
10	KURJ		✓	
11	MBA	✓		
12	CAR	✓		✓
13	NYN	✓	✓	
14	MKP			
15	LAR			
16	DHO	✓	✓	
17	BHA			
18	AH			
19	MHF			
20	AWR		✓	
21	PWL			
22	ALJN			
23	SKB			
24	MNQ			
25	ETW			
26	UDMR		✓	
27	BZM	✓		✓
28	KID		✓	
Newly sited		**7**	**9**	**4**
Existing		**5**	**5**	**4**

Total	**12**	**14**	**8**

Thus, the optimal number of ART, ARMV and Crane to provide the 100% coverage of the network are 12, 14 and 8, respectively. It is worth noting that the existing numbers of these relief facilities are 13, 12 and 4, respectively. Thus, 2 more ARMV and 4 more Cranes can provide complete coverage of the network. This can help in future investment decisions for the relief equipment taking the budgetary constraints in view. It is also interesting to note that if the number of candidate locations is kept limited to 16 only (as done in the small instance), then the set covering model becomes infeasible. That is, it might not be possible to cover all the nodes with a limited number of candidate locations. Thus, this work also necessitates the development of infrastructure at the additional stations.

4.7 Conclusion :

The location of accident relief facilities is one of the most important aspects of emergency preparedness for the railways and it can prove itself vital to restoring operation and saving precious lives at the time of the accident/disaster. In the case of Indian Railways, these decisions are made based on the potential of providing radial coverage to the demand points. However, the presence of different conflicting objectives and several strategic concerns demand a mathematical approach to make the optimal decisions. The present work has proposed a multi-objective integer programming model for this problem. The proposed model is solved using the AUGMECON in an interactive fashion. Set of non-dominated solutions are presented to the decision-maker and with his inputs, the desired areas are further explored. The engagement of the decision-maker at every stage has made the exercise more relevant and shown the potential of the model to cope with various requirements of the decision-maker. This mechanism and the model have the potential to prove itself as a handy tool and an intelligent assistant to the authorities' making decisions. The model guarantees the best possible solution for all possible combinations of the location of the assets.

The model is applied to a representative network and the results are testimony for the achievement of the desired objective. Various insights revealed during analysis have opened new possibilities. Our model has not only ensured the optimum use of the resources but also thrown us the realization that the choice of potential facility location suggested by the railway authorities is also restrictive. Therefore, compelled the exploration of new and unexplored possibilities at a place which were not considered as probable a place due to other operational constraints. Further analysing the network as a set covering model to assure complete coverage for a given threshold limit, it is learnt that the available facilities are lesser than the actual requirement of the facilities.

The analysis has thrown open a completely new dimension of the problem for further research and study from different perspectives. This problem can now be addressed as an optimization problem with the objective to optimize between the 'new location to be opened' at a completely underdeveloped location by bearing the cost of development of the facility or locating more facilities at the developed and currently suitable place that entails the investment in procured of assets to enhance the coverage of the network. Further, the network considered in this work is a part of a bigger network of Indian railways. Future research on a holistic consideration of the entire network of Indian railways is a way ahead for the study. This avenue also calls for specialized solution approaches such as evolutionary algorithms to efficiently solve the problem due to the resulting size of the network. In addition to this, uncertainty related to the railway accidents and actual deployment of the relief facilities needs to be explored further. The probability of accident on the links of the rail network can be considered from the past data and the model can be revisited using the two-stage stochastic programming or robust programming approach.

In addition to the uncertainty of the accident, several other practical considerations can be modelled efficiently using simulation study. We reserve these two aspects as future works. This work can also be extended as a part of a broader disaster management plan that includes several players in addition to the railroad operators. Such a perspective can open avenues for emergency preparedness for other essential items, determining optimal placement of emergency health facilities in addition to the relief types of equipment, use of information and communication technology and internet of things in improving the response to the accident sites. The next chapter of this study is further extension of the same problem and is solved differently by a two stage stochastic programming approach for optimal location of relief facilities on the same railway network.

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