Chapter 7

Summary and Future scope

7.1 Summary

This thesis examines non-linear wave propagation and its properties in various gas dynamic regimes. We use several analytical methods in different material media to explore the fundamental characteristics of propagating waves governed by a quasi-linear hyperbolic PDE system. The effect of various differential constraints on the propagating waves and their behavior is analyzed. In addition, it deals with classical and non-classical solutions of the Riemann problem for one-dimensional homogeneous and non-homogeneous quasi-linear hyperbolic systems. This thesis has the following major contributions:

- To study the shock waves in a two-dimensional supersonic planar and axisymmetric non-ideal gas flow with a magnetic field.
- To determine the effect of dust particles on the evolution of weak discontinuity in the two-dimensional supersonic flow of van der Waals gas.

- The behaviour of analytical Solution of a Hyperbolic system of PDEs with the magnetic field.
- To Investigate the phenomenon of the concentration and cavitation in the Riemann solutions for a non-homogeneous logarithmic equation of state with the magnetic field.
- To determine the structure of the solution of the Riemann problem for the one-dimensional compressible hyperbolic system under the influence of external force.

The first chapter is introductory and gives an overview of the work done in the field of nonlinear wave propagation, including their applications and methodology. We have also briefly discussed mathematical theory and their fundamental properties. We describe the basic features of non-linear waves and their propagation. This study briefly reviews the physical properties of dusty, non-ideal, and radiating gases. Moreover, some results are included in this chapter that we will need in the subsequent chapters.

The second chapter analyzes the shock formation in a two-dimensional steady supersonic flow of non-ideal gas with a magnetic field for the planar and axisymmetric flows. It is shown that the governing equations describing the non-ideal gas flow with a magnetic field is hyperbolic in nature. Further, using the theory of the propagation of wavefronts defined by weak shock, we derived the transport equations for shock waves, which lead to the determination of shock formation distance and provide the conditions of shock formation. It is also established that the shock wave formation is affected by the presence of a magnetic field, non-ideal parameter, and upstream flow Mach number $M_0 > 1$ is shown.

In the third chapter, we investigate the impact of the dust particles on the shock formation in two-dimensional steady supersonic flow of the composition of van der Waals gas and small solid dust particles for the planar, cylindrically symmetric, and spherically symmetric cases. It is shown that the governing equations describing the van der Waals gas with dust particles is hyperbolic in nature. Further, by employing the method of wave-front analysis, we have derived the transport equations for a shock wave, which lead to the derivation of shock formation distance and provide the relations of shock formation. Also, it is determined as how to the shock formation distance is affected by the presence of dust particles , parameter of van der Waals gas and upstream flow Mach number ($M_0 > 1$) by using MATHEMATICA 11 software.

In the fourth chapter, we are concerned with the solution of the Riemann problem for the hyperbolic system with the logarithmic equation of state and magnetic field. The formation of vacuum states and delta shock waves as the magnetic field and pressure vanish, have been discussed. Firstly, the Riemann problem for the magnetogasdynamic system is solved. Further, we have constructed the solutions for the pressureless and vanishing magnetic field system (that is, transport equations). In the context of vacuum states and delta-shock, it is shown that the solution of the Riemann problem consisting of two shocks which converges to the delta shock wave (delta-shock) solution of transport equations, and the Riemann solutions consisting of two rarefaction waves which converges to the vacuum state solution (that is, the intermediate state between two-contact discontinuity solutions of the transport equations). Hence, it is proved that the solutions of the system with the logarithmic equation of state and magnetic field converge to the solution of the corresponding system as the magnetic field and pressure vanish, which shows that the solution of the Riemann problem for the hyperbolic system with the logarithmic equation of state and magnetic field is stable.

The fifth chapter analyzes the phenomena of concentration and cavitation in the Riemann solution for the non-homogeneous hyperbolic system with the logarithmic equation of state and magnetic field. Firstly, we introduce a new state variable for the velocity to modify the non-conservative system into a conservative one and constructively solved the Riemann problem for the modified system. Further, the Riemann solutions for the transport equations are investigated as pressure and magnetic field vanish. It is proved that the Riemann solution for the non-homogeneous hyperbolic system with the logarithmic equation of state and magnetic field having two shock waves converges to the delta shock wave solution of the transport equations as pressure and magnetic field vanish. It is also proved that the Riemann solution for the non-homogeneous hyperbolic system with the logarithmic equation of state and magnetic field having two rarefaction waves converges to the contact discontinuity solution of the transport equations as pressure and magnetic field vanish.

In the sixth chapter, the analytical solution to the Riemann problem for a one-dimensional non-ideal flow of dusty gas with external force is presumed to be a continuous function of time. We explicitly obtain the elementary wave curves to the one-dimensional non-ideal flow of dusty gas with external force and determine these wave curves in the form of characteristics. The exhaustive calculations were performed for the elementary wave solutions, such as the rarefaction wave, shock wave, and contact discontinuity. We examine the influence of dust particles on density, velocity of flow, and shock speed and their implications on the solution of the Riemann problem. Also, adding external force implies that all solutions are not self-similar.

7.2 Future Scope

Our study is focused on nonlinear wave propagation governed by Euler's system of partial differential equations. Among our main contributions are a study of propagating waves in different gas regimes and a discussion of the influence of various parameters on the solution obtained using some analytical methods. This section provides future work to consolidate the study presented in this thesis. Our study is considered a one-dimensional and two-dimensional system of nonlinear partial differential equations in gas dynamics. It is possible, however, to extend this analysis to nonlinear partial differential equations in gas dynamics in two or higher dimensions. The key research areas identified here can be the focus of future research. A future project involves a deeper analysis of particular mechanisms, new proposals to try different methods, or curiosity. There are some highlights of the proposed extensions of the work made in the thesis given as follows:

- In this thesis, we focused only on the two-dimensional wave propagation in various gas regimes and the behavior of the waves affected by some parameters. One may study the wave interaction problems in different gas regimes governed by quasilinear PDEs' hyperbolic system, which involves much more analysis and challenging tasks.

- In the entire thesis, we have used some analytical methods to study the wave propagation phenomena. There are many numerical techniques available by which one can investigate these phenomena for various systems of hyperbolic PDEs.

- Using the asymptotic and wavefront analysis method, study of the evolutionary behavior of nonlinear waves propagating in higher-dimensional flow in different gaseous media may be performed.

- In this thesis, we have studied the classical elementary waves (shock waves, rarefaction waves, and contact discontinuities) and delta-shock for the hyperbolic system. One can extend these ideas to discuss delta-shock interactions with weak discontinuities, which may lead to the development of an interesting concept, an open problem.

- In this thesis, we focused only on the one-dimensional Riemann problems. One may study two-dimensional Riemann problems admitting delta-shock and nonlinear wave interactions for the hyperbolic system of conservation laws, which is a challenging and open problem.

- Specific problems related to the Generalized Riemann Problem (GRP) are also interesting research areas. We can use the Differential Constraint Method to study the GRP for various models.

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